The Importance of Equity in Mathematics Education
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#### Abstract

The extent to which equitable practices can be attained in education depends heavily on educators, teaching methods, and support for individual students' needs. Equity is a concept that is defined by the NCTM as supplying "high expectations and strong support for all students."


 (NCTM, 2000) Students that can learn and expand on their knowledge of mathematics in equitable environments are more likely to embrace and accomplish a sense of the importance of mathematics in relation to their own education. In this study, we will analyze the characteristics of an equitable learning environment in mathematics classrooms, and it is our goal to see to what extent teachers are implementing equitable practices in their classrooms daily. We also hope to see how equity can positively impact how individuals' study and learn mathematical concepts. Keywords: equity, high expectations, supportThe Importance of Equity in Mathematics Education
Creating equitable learning environments for many educators involves different approaches. In other words, what one educator may deem important for equitable learning environments may not be as important to another educator. Furthermore, there is no precise recipe for equity in mathematics. However, many researchers have made advances to define how we as mathematics educators can create equitable learning environments. For our research study, we chose to focus on five themes that represent equity in mathematics education. The idea is that if these themes are implemented, the classroom environment will attain equity. The themes we are studying in relation to equity are as follows:

1. Strong Support for all students;
2. High Expectations;
3. Collaborative Learning;
4. Accommodating Differences;
5. Open ended teaching/learning strategies.

Once we set forth these themes, we designed our procedure primarily based on observation. My role as the researcher was to observe two different classrooms in differing areas and make note of any equitable themes that appeared throughout the classroom instruction.

The goal of this study was to see what equitable themes the chosen educators implement in their classrooms, and to also analyze what is missing as well. As an outside observer the goal of the one-on-one interview process with the educator and myself was to see the perspective of the teacher; that is, to see how they define equity in mathematics education. The research questions posed are:
> How do educators define equity?
> What aspects of teaching, as it relates to equity, encourage or discourage student participation and confidence?
> How and to what extent do educators implement the above-mentioned themes in their classrooms?

Through the course of observing the two different classrooms, we were able to identify several circumstances where the teachers were implementing equitable practices. The interview process allowed me to gain a better understanding of how the individual educators enact equity in their classrooms and how they argue the practices they choose to implement; equitable or not.

## Literature Review

The idea of equity in mathematics education is expressed by the National Council of Teachers of Mathematics as having high expectations and worthwhile opportunities for all, accommodating differences to help everyone learn mathematics, and resources and support for all classrooms and students (NCTM, 2000). The NCTM defines equity in a way that aligns with many of the themes we have chosen to study in relation to creating equitable learning environments. Furthermore, the NCTM expresses,

Educational equity is a core element of this vision. All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study-and support to learn-mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students (NCTM 2000).

The idea suggested by the NCTM that every student should not receive identical instruction is an indispensable aspect in developing an equitable learning environment. This statement further
iterates that equity and equality should be thought about as two separate entities in relation to education.

In the paper entitled Making mathematics meaningful in multicultural context, Gloria Ladson-Billings voices, "The notion of an equity pedagogy refers to the opportunities that all children have to benefit from classroom instruction" (Ladson-Billings, 1995). Additionally, Ladson-Billings expresses that sorting and grouping students into lower levels where they receive minimal instruction from the teacher, while the students in the higher levels are presented with a more challenging and intellectually stimulating curriculum is an example of where some of these inequities occur. It is important to note where these inequities exist so that we can recognize them and work to diminish them.

It should be noted that the NCTM's statement on equity states that it is important for students to not receive identical instruction. However, Ladson-Billings goes against this in a way by stating that giving students different levels of instruction may create some inequities when some students are not being challenged enough. It may be the case that an educator might think they know the level of instruction that the student can handle. However, that student may be capable of performing at a higher level, and they are left with an instruction that is not challenging them enough.

Another theme that is important in creating equity in the classroom is collaborative learning. Ladson-Billings notes that many classrooms are working to implement cooperative and collaborative learning, where all the students that participate are expected to meet more rigorous educational challenges. This type of collaborative learning often occurs through an open-ended approach, which allows students to analyze the topic at hand and come to their own conceptual understanding. However, collaborative learning can occur in different ways throughout the
classroom as well. For example, Ladson-Billings discusses the experience of an educator, Mrs. Rossi, and how she handles her classroom instruction. Ladson-Billings expresses that she observed Ms. Rossi constantly telling students they were capable of mastering different mathematics problems. Additionally, the students in the classroom were cheering each other on and often celebrated the times when students were able to explain and elaborate on how they arrived at a specific solution. Ms. Rossi also used probing questions to help her students further understand the mathematics concepts. She would ask questions such as "How do you know?", and "Who knows? Who can help him out here?" This allowed her students to organize their thoughts on certain problems and helped the students develop their own problem-solving strategies. Ladson-Billings elaborates, "Instead of attempting to maintain the students at low levels of academic performance, Ms. Rossi provided challenging mathematics content for all of the students."

This paper closes by relating equity back to multicultural education. Ladson Billings elaborates, "Real education is about extending students' thinking and ability beyond what they already know." Part of creating an equitable learning environment often involves being responsible for your students' past experiences with mathematics. Ladson-Billings expresses that teaching mathematics not only requires knowledge on the subject matter, but additionally indepth knowledge of students.

In the paper entitled Learning from Teaching: Exploring the Relationship Between Reform Curriculum and Equity by Jo Boaler discusses the importance and previous research that has been conducted regarding the benefits of an open-ended teaching approach. In this paper, Boaler discusses the QUASAR project (Brown, Stein, \& Forman, 1996; Lane \& Silver, 1999; Silver, Smith, \& Nelson, 1995) which focused on the fact that mathematical proficiency could be
attained by all students if educators placed a greater emphasis on problem solving, communication, and conceptual understanding. The project involved six educators in urban middle schools who had students who were socially and culturally diverse. These educators spent five years working to develop and implement a more open and discursive mathematics curriculum (Boaler, 2002).

One of the aspects of this project that benefitted the students the most and is arguably one of the main contributions to achieving equity is the fact that throughout this project, "students learned about facts and algorithms, but they also learned when, how, and why to apply procedures, which they used to solve high-level problems" (Boaler, 2002). The conclusion that the above-mentioned researchers took away from this was that the QUASAR students' achievement had a positive outcome, meaning the students performed at significantly higher levels than comparable groups of students on a range of different assessments (Boaler, 2002). This project also showed that the gains of achievement were distributed equally among different racial, ethnic, and linguistic groups of students. Most importantly, this paper helped us to see how implementing an open-ended teaching approach can create equity in the classroom. It is important to note that in the paper Making Mathematics Meaningful by Gloria Ladson-Billings, Ms. Rossi's classroom was open and discursive as described in the QUASAR project.

## Method

## Participants

In this study, we analyzed four different classrooms at the high school level. The study involved two different schools, one school was in the suburbs around Metro-Atlanta. Through the course of this paper I will refer to this school as 'School A'. The other was a private school
located in South-Eastern Georgia. Similarly, I will refer to this school as 'School B' throughout the rest of this paper.

Figure One:


Figure Three:


Figure Two:

School A, Class A


Figure Four:

Figures one and two represent the racial make-up of School A, while figures three and four represent the make-up of School B. As shown by the figures, School A had a more diverse makeup in comparison to School B. The goal was to study two different classes taught by the same teacher at each school. We conducted our research by intently observing the classrooms, while making notes of certain aspects that are important in equitable learning environments. The research also involved a one-on-one interview process with the educators.

## Procedure

The purpose of this research is to analyze multiple classroom environments to see if they contain any of the themes of equitable learning in which we are interested. The goal was to see if the classroom had strong support for all students, high expectations, collaborative learning, accommodation of differences, and open-ended teaching/learning.

After observing the classroom environment in relation to our equity in mathematics principles, our next goal was to interview one-on-one with the educator. The purpose of this part of the study was to have an open conversation about equity outside, but mostly inside the classroom. The questions I asked the teachers one-on-one are as follows:

1) How would you define equity in your own words?
2) How would you define equity in terms of math education?
3) How do we as educators create equitable learning environments?
4) Do you find groupwork to be effective or ineffective in your classroom? If effective, what have you found that works? If ineffective, what does not work?
5) How would you characterize your teaching methods?
6) Do you think that the way desks are arranged in a classroom matters?
7) Do students in your classroom feel comfortable asking one of their peers for help?
8) Do the students in your classroom ask questions/seek help when they do not understand something?
9) From my experience, most questions in a classroom are not specific questions but rather "I do not get it. Can you show me how to do this?" Is there a way to fix this way of learning? How can we get students to become more responsible for their own learning?
10) How often do you ask for feedback from your students regarding your instruction? If you do, does the feedback help you?
11) In your opinion, what is the most important characteristic of an equitable learning environment?
12) What is the best and/or hardest part about teaching?

The arrival upon these questions did not come immediately but rather through previous research, and additionally my own experiences observing in classrooms prior to this research. The first few questions were to gauge how and to what extent the educators were able to define equity in their classroom, and what possible techniques they use to implement it. Questions four through seven were discussed to see how the educators run their daily instruction. These questions were asked to see if the teachers could expand on their use of group work and open-ended learning or lack there-of. The most interesting of these questions in my opinion was when I asked how the educators define their teaching methods. This question was made open-ended and broad, so that the teachers could analyze their teaching techniques. Question nine expands on the idea of how students will ask questions and learn in the classroom when an open-ended approach is utilized. The idea is that the students will start to become more responsible for their own learning if they are exposed to teaching methods that allow them to come to their own conceptual understanding. It was interesting to see how the educators view and interpret equity in their classrooms throughout the interview process, because it is a topic that can be interpreted in a variety of ways.

## Findings

## Theme One: Strong support for all students

In relation to theme one, in the classroom I looked for moments where the teacher was supplying strong support for the students. For example, I observed when the teacher supplied individual help to the students. We also thought it was important to analyze the confidence of the students' mathematical abilities. The idea is that if students are receiving the support they need to succeed in a mathematics classroom, they are more likely to have an increased confidence in relation to mathematics. Furthermore, we analyzed whether the classrooms had resources. This included the funding that the schools received and any additional trainings that the were made available to the educators. Additionally, we looked at if teachers were knowledgeable in relation to understanding how to implement equitable practices in their classrooms.

## School A

Throughout my observations, I concluded that the school had the resources and funding required for students to be successful. I came across this conclusion by analyzing my interview with the educator and additionally by just looking around the classroom at the resources that were available to the students. For example, the students in this classroom had access to graphing calculators, which were helpful in the lesson they were taught because the students were analyzing the graphs of various functions in vertex form. The students also had access to textbooks and the teacher utilized a Smart Board.

The teacher had mentioned that they had the opportunity to receive support from outside the classroom in developing creative lesson plans for the students. For example, the educator discussed a time where a specialist came in and taught the students and the teacher how to implement and learn a lesson centered around modeling one-step and two-step equations using algebra tiles. Using algebra tiles to learn about and solve one-step and two-step equations is an example of an open-ended learning approach. Creating and delivering a lesson plan with a
manipulative such as algebra tiles is a great way for students to discover their own understanding through a hands-on activity. Boaler says, "I open these and other teaching practices for closer examination and contend that the differences between equitable and inequitable teaching approaches lie within the different methods commonly discussed, and that greater insights into equity will require an understanding of the ways teachers work to enact different approaches" (Boaler, 2002). The approach of teaching with a manipulative such as algebra tiles helps create unique teaching methods in which educators may implement in their classrooms to create equity. Currently, many educators may not feel comfortable delivering a lesson centered around manipulatives. It is necessary to inform educators how to develop these helpful lesson plans that take a less traditional approach to teaching.

The teacher was consistently walking around the classroom to supply individual help to the students on an as-needed basis. Additionally, the teacher worked hard to keep every student engaged. Throughout the course of instruction, the teacher called randomly upon students to supply next steps to different problems they were working through. The educator took time to reflect on the content they covered by allowing the students time to ask questions on any misconceptions they may have developed. This is a great way to supply support to students because it makes the students see that questions are not only a way to gain a clearer perspective but additionally that their questions are important.

In these classrooms, I saw a lack of confidence regarding mathematical ability. Students were expressing concerns frequently such as, "Can you just tell us what to do? I am confused, and I do not know anything," and "What is this for? I cannot do it." Many of the students were highly dependent on the teacher and were consistently needing help. Students that receive the strong support necessary to be successful in a mathematics classroom are more willing to exhibit
confidence in their problem-solving. However, from what I observed in these classrooms it seemed that this willingness to try and attempt problems without losing motivation was absent.

## School B

The geometry and pre-calculus classrooms I observed at this school was like School A in the sense that the students in these classes seemed to have the resources needed to succeed in a mathematics classroom. The classroom had resources such as calculators, and even rulers and compasses for the geometry classes. The teacher had mentioned that they make themselves available before and after school for the students to receive extra help on an as-needed basis.

While working on practice problems throughout instruction, the students in these classrooms were less dependent on asking the teacher for help. These students were more willing to work through these problems individually, so questions directed at the teacher were few and far between. This led me to believe that the students are receiving the support necessary from the educator since they are exhibiting an increased confidence in their problem-solving.

The teacher had not mentioned receiving help with implementing creative lesson plans with a more open-approach, but the teacher at this school was also a bit more experienced in the sense that they had been teaching longer than the other teacher. The idea is that teachers, experienced or not, should be receiving support in developing lesson plans outside of their ideas alone. The reason it is important for experienced teachers to receive this training as well is because education is always evolving and there will always be new ways to develop and implement better, open-ended lesson plans for future students. I know that as a possible future educator, I would appreciate the extra help and guidance from a specialist that can perhaps show me different teaching methods and implementations of lessons that will result in a better learning experience for the students.

Additionally, while I observed the pre-calculus class, the teacher spent class time helping the students prepare for the SAT. This falls under the theme for strong support for all students because the teacher took time to help the students with something that does not directly relate to pre-calculus itself, but instead invests in the students' future college careers.

## Theme Two: High Expectation

The most important thing I looked for in relation to high expectations was whether the students were being challenged by the curriculum. The idea is that the educator should create a rigorous and stimulating instruction that works to keep each student engaged by emphasizing their weaknesses and expanding on their strengths. Additionally, I looked for students justifying their work and thought processes. Having expectations for students to implement this type of communication in the classroom can increase the confidence of the students' mathematical abilities. Furthermore, I looked to analyze any minor expectations that the teacher put into effect. An example of this might be if the teacher creates a warm-up to review topics from previous class periods or executing homework checks.

## School A

There were many instances where students were justifying their thought processes aloud, which was a positive aspect of the classrooms regarding high expectations. An example of this occurred when the teacher wrote the expression $-x^{2}+8 x-12$ on the board and asked the students what the leading coefficient of the expression was. Many of the students said that the leading coefficient was -1 . Next, the teacher wrote the equivalent expression to the previous one $-1\left(x^{2}-8 x+12\right)$ on the board and asked the same question once again. One of the students expressed "It is one because the number next to the $x^{2}$ is one!" It is important to note the positivity in this interaction because the student was able to justify in their own words how they
define the leading coefficient in an expression. It follows that by the distributive property, the leading coefficient of the second expression would still be -1 . Despite the incorrect response, the student was still able to justify and speak their thought processes aloud. The following is an example of a student interaction that I heard:

Student A: "It is pointing downward because of the negative, right?"
Student B: "Yes, we have to see whether it's negative or positive to see which direction the graph points."

More explicitly, the students were clarifying that the sign of the leading coefficient will determine whether the graph opens up or down. This pair of students worked efficiently and often clarified questions and their thought processes with one another.

Throughout observation, I also made note of how often the students were being challenged. In both classrooms I observed, the classroom instruction involved the teacher showing the students how to convert different functions between the three different forms, i.e. standard, vertex, and point-slope. After doing examples together as a class on the board, the teacher issued an activity that had comparable examples to the ones that they did together. In other words, this instruction was repetitive in the way that the students were not being given examples that would challenge or push them to understand the applications better. In these classrooms there was a handful of students that were grasping the material and finishing the activity quickly, so in particular these students were not challenged. The remaining students in the classroom were struggling to work through the examples. It seemed that the knowledge the students had on the topic was surface level and that the repetition of similar problems did not engage the students. Creating a more creative and open instruction might have allowed the
students to learn the topic more in-depth and additionally would allow the students to be more engaged and challenged by the material.

Some minor expectations in the classrooms at this school involved the educator expecting the students to complete a warm-up activity at the beginning of every class. In addition, the teacher implemented homework checks which suggested that the students were to have the homework they were assigned from the previous class completed for a grade. Expectations like the ones mentioned above work together to create a routine for the students.

## School B

Several of the students in these classes were willing to explain and justify their work on the board in front of their peers. In the geometry class, they were working on solving for arc length and angle measures. Most of the students in the class were wanting to write their work on the board for class discussion. The teacher did a good job of facilitating this because the students were asked to tell the class the justification for each step that they wrote on the board. One student expressed, "How do you want me to explain this? We have been solving two-step equations since middle school. Why do we have to explain what we are doing?" I looked at this questioning as a positive thing since the student who expressed these concerns seemed to have a good understanding of important math concepts, such as being able to solve one-step and twostep equations. In the end the student found a way to justify their work that they put up on the board. Here is an interaction between the teacher and a student:

Teacher: "Class, why should the sum of opposite angles be 180?"
Student: "Because it is... the arcs are divided into two and 180 degrees is half of 360 degrees."

This student started out by giving a somewhat lazy answer and then proceeded to follow up with a better and more elaborate explanation to the question. The justification of the students thought processes is increasing the confidence of the students in the class.

There are some instances where it seems that the students are not being challenged. For example, as previously discussed when the student stated that they thought solving two-step equations was easy. However, the teacher did provide example problems outside of what they were shown. For example, instead of just having the students solve for arc length using one circle and one angle, the teacher extended their understanding by providing examples that used two circles which required setting up two-step equations instead of one-step. This way of teaching required the students to extend what they were already taught by the teacher and apply it to a more difficult example. An increased amount of class time was spent on these examples, so the students were challenged.

Additionally, at the end of each class, the teacher expected each student to state something that they learned in class today or something that they were able to improve on. The teacher expressed at the end of each class, "How many of you have learned something today?" Students gave a variety of responses:

Student A: "How to find the length of an arc"
Student B: "How to find the measure of outside and inside angles."
Student C: "When the x value is in a different place, it makes solving the problem slightly different."

This was a way for the teacher to see how their students were feeling in relation to the content they were learning. It was nice that at the end of each class, every student had the opportunity to be heard.

## Theme Three: Open-ended Learning

The idea behind open-ended teaching $\backslash l e a r n i n g$ is that the students can learn in unique ways, such as through engaging activities in which the students are able to come to their own conceptual understanding. Typically, an open approach involves group discussions so that the students can discuss the topics among themselves. Instead of a more lecture-based learning where the students are given the information directly, an open-ended approach allows the students to discover the new topics they are learning individually or in groups. Jo Boaler states,
"But whilst opponents and proponents of different curriculum have argued about the importance of open-ended work or structured questions, they have rarely considered the ways in which teachers may manage such approaches, the teacher knowledge they may require (Ball \& Bass, 2000) or the changed student roles that may need to be learned in order for different approaches to be successful (Corbett \& Wilson, 1995).

It is important for us as future educators to figure out ways in which open-approaches to learning can be implemented successfully in the classroom. In an open-ended environment the students' understanding of new topics should not be rushed. Furthermore, the students should have the opportunity to develop their own complete understanding.

## School A

The teacher at this school used a more typical approach to teaching in the way that the educator used a lecture-based learning style. The students were taught how to convert between point-slope, standard, and vertex form. In a way, this topic could be considered a skill that is procedural. The emphasis is that if the classroom is more open the students will not look at the topics they are learning as procedural or repetitive. Another idea behind an open-ended approach is that the teachers are creating opportunities in the classroom where the students work through
difficult problems that cause them to think more critically. Creating lessons and activities where the students are more engaged, which leads to an increased in-depth knowledge about the topics they are learning. At this school, this part of an open approach was lacking. As discussed previously, the students were shown how to do examples by the instructor and then they were given similar examples that required the same steps to solve. The teacher had an opportunity to open the classroom up by teaching the students how to complete the square. The students in this classroom had only previously looked at a traditional approach to factoring and they had looked at factoring using the quadratic formula. However, they had not been exposed to factoring using completing the square. The specific example they were working on was an expression that would have been easier to factor using completing the square. Instead, the teacher chose to have the students use the quadratic formula, a technique that the students were already familiar with, to solve for the factors. Allowing the students to learn a new technique such as completing the square would have been a good way to open the classroom up. This would have allowed the teacher to bring in other related concepts, such as zero-pairs, which are used in the completing the square approach.

Although there was not specific evidence of an open-ended teaching approach, the educator did a good job of keeping the discussions open. For example, when the teacher was going over example problems with the class, the students were offering lots of answers and solutions to the problems they were working through. This allowed for discussion between students and the teacher. In regard to an open-ended approach, a greater emphasis is put on the characteristics of the classroom instruction. However, it is also important that the students and teacher are comfortable with making mistakes and the idea of productive struggle. In this
classroom, many mistakes were made by the students and every time either the teacher would help the student understand their mistakes or their peers would help them.

## School B

There was a similar approach at this school in the sense that I did not see many techniques of an open-ended approach to teaching. Like the first school, the teacher had a traditional teaching approach because the students were shown how to do examples by the teacher and then the students were expected to work on similar examples individually. However, the teacher did make attempts to open the classroom. The educator attempted this by giving the students a more challenging problem that extended what they were already taught and had already practiced. More specifically, the teacher took what the students knew about finding arc length using a given angle and gave them another problem that involved finding the arc length but also extending it to another circle which led the students into setting up a two-step equation to solve for the arc length. The problem given to the students was beneficial because it was not only extended to what they had already learned, but the problem also caused the students to think more critically about how the ratio of arc lengths compare between inner and outer circles.

The reasoning behind why the instruction was not open was that the teacher told the students they had ten minutes to attempt the problem and, at the end of the ten minutes, they would go over the solution as a class. This led to a decrease in motivation in discovering the solution because the students knew that they would end up seeing the correct solution, whether they figured the problem out or not. The idea behind an open learning environment is that the students would be given the challenging problem, along with a sufficient amount of time and necessary resources to develop their own understanding, which did not occur in this classroom. It should be noted that the attempt by the educator to open up the classroom was a positive thing
because the students were encouraged to think beyond what they already learned However, some big components of an open-ended teaching approach were still lacking.

## Theme Four: Collaborative Learning

The main component that I was searching for in relation to collaborative learning was the use of structured group work. This would involve the students being placed into groups so that they can work through different activities and problems that help further their understanding of the material that they are learning. The use of structured group work can be beneficial because it allows students to hear different perspectives that come from their peers. Additionally, in a classroom you often have students that have different thought processes or students that approach problems in varying ways. Seeing that sometimes there is more than one way to solve a problem in mathematics is important. Mathematics has the stigma of being extremely procedural, but when student see different approaches it somehow seems less procedural.

I also looked for whether problem solving through communication occurred. This communication falls under the use of structured group work but can also happen when students are communicating with the peers around them. Furthermore, we looked at how the students interacted with each other. For example, we analyzed the explanation of concepts between students and how comfortable the students seemed asking each other for help. Even though there is a greater emphasis put on structured group work in the classroom, it is also important that the students are encouraged to discuss amongst one another through the problem-solving process.

## School A

I did not observe any structured group work at this school. The students experienced a more traditional teaching style in the way that they were guided through examples step-by-step and then asked to solve problems on their own or in pairs. Even though there was no structured
group work, a decent amount of the students in the classroom were communicating with the peers that were seated around them.

There were many instances where students were communicating and problem solving together. One exchange I observed was the following:

Student A: "How do we find the x -value of the vertex?"
Student B: "You can use $\frac{-b}{2 a}$ to find the axis of symmetry!"
This communication was positive because the first student reached out to one of their peers for help when they were stuck. Additionally, the second student heard the first student ask how to find the x -value of the vertex, but in their response the proceeded to call it the axis of symmetry. This specific student had an impressive grasp on the content they were learning. For this example, one of the students was eagerly attempting to get the attention of the teacher, but they were helping another student. Instead, the student turned to one of their classmates for help:

Student A: "How do we get the y-intercepts?" (When graphing vertex form.)
Student B: "You use ( $0, \mathrm{c}$ ), where you plug in 0 for x and c for y. "
The student at first was set on getting help from the teacher but found asking one of their peers for help was useful. Hearing these exchanges between students helped me see the handful of students who were learning the material well enough to help one of their peers to understand.

The teacher did a great job at bringing the classroom together with questions that extended the material they were learning and even addressed some common misconceptions when working through some of the problems. For example, the teacher asked the students to think about how the graph would change if the vertex was $(-6,6)$ instead of $(6,6)$. Many of the students were quick to respond that the axis of symmetry would change so the graph would be shifted to the left twelve units. Throughout the process of working through the activity, the
teacher was consistently walking around the classroom helping students. It seemed that whenever the educator noticed a bunch of students making the same mistake, the educator would bring the class together for collaboration and clarification. For example, the teacher wrote the following on the board, $y=4(x-1)(x-1)+9=4\left(x^{2}-2 x+1+9\right)$, and asked the students if the statements were equivalent. There were mixed responses at this point, but after some discussion as a class the students were able to figure out that the four should not distribute to the nine in the equation. One student then elaborated that the reasoning was due to the order of operations. The student expressed, "You have to do parentheses, exponents, and multiplication before you add."

Furthermore, in this classroom there was encouragement to justify thought process. The justification that occurred was a positive aspect because the students were frequently justifying their thought process to their peers without being prompted to do so. The desks in the classroom were arranged in rows. There was a middle section of rows that faced the white board at the front of the room and then there were rows on the sides that faced each other. Throughout the interview process, the teacher made note that it was important for the classroom to flow so that when walking around the room for individual help, each student is easily accessible. The teacher also mentioned that in the past when they had put the desks in clusters, the students were more likely to become more off-task and distracted.

## School B

The second school was like the first school because no structured group work was implemented in these classrooms either. This school was different mainly because there was not a lot of communication between the students. Many of the students in these classrooms chose to work individually on the problems they were given to attempt. It was also interesting to see that
the students did not depend on assistance from the teacher when working through problems. Communication between students and teachers still occurred at this school but happened far and few between. For example, the students were given a problem while doing SAT review that required the use of the quadratic formula. In this case, the $b$-value in the quadratic formula was -5 but the students discussed with one another that the negative value does not technically matter since $(-5)^{2}$ and $(5)^{2}$ give you the same result, which in this case is twenty-five.

One slightly negative instance I observed occurred when one of the students was eager to show their work when finding the solutions to the equation $2 x^{2}-5 x-20=0$. The student at the time was helping one of their peers with the problem, but the teacher had told them to sit down so they can go over it as a class. The student had hinted at possibly wanting to explain their work to the whole class, but the teacher still insisted on teaching the example themselves. It can be quite difficult to encourage discussion between students, and even to get students to discuss their thought process in front of the class. This specific student was more than willing to create this type of discussion, but the teacher shut it down. It is important to embrace these teaching moments because often justification and discussion helps substantially regarding understanding certain mathematical concepts.

The teacher elaborated more in the interview on why they do not like structured group work and students collaborating in pairs. In this case, the educator expressed that collaboration often leads to more off task behavior. The teacher also stated that they like students to work individually because that is how it is when students are in testing environments. According to the teacher, the more the students work on problems alone, the better they do on the tests because they are used to figuring out how to solve problems without assistance. It is important that
students do well on tests, but there are so many benefits that group work and collaboration create that the students are missing out on.

## Theme Five: Accommodating differences

The main aspect I looked for in relation to accommodating differences involved analyzing the different ability levels in the classroom and whether the teacher was utilizing a differentiated instruction to reach the different levels. Another aspect of accommodating differences in the classroom entails accepting different approaches to problems. The idea is that mathematics is not always procedural, and that there can be multiple approaches to one problem. Accommodating differences also involves teachers molding and adapting to how students learn. Often in a classroom, teachers are going to have students that learn differently. For example, one student might understand a concept better if you present it visually, while another student might gain a better understanding by just listening. The National Council of Teachers of Mathematics expresses, "All students should have access to an excellent and equitable mathematics program that provides solid support for their learning and is responsive to their prior knowledge, intellectual strengths, and personal interests." (NCTM, 2000) As stated by the NCTM, accommodating differences is about understanding your students' backgrounds in mathematics and their strengths to provide them with an equitable environment that caters to these differences.

## School A

While observing at this first school, I observed that many of the students were at different ability levels. In other words, while working through the exercise sets they were given, a handful of the students finished quickly while some of the students struggled with the exercises. In this case, the students that finished the work quickly were not challenged or engaged by the
assignments they were given. It should be noted that a differentiated instruction may have been beneficial here. Instead of giving the students all the same activity, perhaps the teacher could have produced a more challenging exercise for those that were getting the material they were already presented. As discussed previously, the teacher chose not to teach completing the square because of the level of misunderstanding regarding the concepts. However, the kids that had a grasp on the material would have benefitted from learning a different way to factor. Going back to high expectations, as educators we must create an environment where we push each student to be the best they can be. The bar of success that we set for each student may be different, but we should always push to develop techniques in the classroom where the students are able to meet and exceed their individual bars. Additionally, the techniques we develop should be dependent on what level of mathematical success the student is currently achieving.

A positive aspect of these classrooms was that the teacher was accepting different approaches to problems. For example, I observed the following interaction between the teacher and one of the students:

Teacher: "How do we evaluate $(x-2)^{2}$ ?"
Student: "We can factor backwards!"
Frequently as educators, we look for a specific answer to the questions we ask. In this scenario the teacher might have expected the student to say "foil" or "distribute". However, the student said something slightly different. This response caused the educator to be slightly taken aback, but the teacher responded, "Yes, that is a correct way to think about this concept!" Instead of immediately correcting the student by guiding them towards the more typical responses stated previously, the teacher allowed the student to express and use their own interpretation of foiling and distributing. The teacher also went over different ways to approach problems. The students
were first given the expression $2 x^{2}-32$, and then preceded to factor out a two to get the expression $2\left(x^{2}-16\right)$. One student exclaimed that they can use a difference of squares to factor the expression. The teacher said that was good, and then showed the students that they can also write the expression as $2\left(x^{2}+0 x-16\right)$, which allows the students to factor using the way they utilize more frequently. This was positive because if the students did not recognize the difference of squares right away, the students were shown a way to factor the expression based on an approach to factoring that the students were familiar with.

## School B

At this school, there did not seem to be as much differing ability levels; that is, the students all seemed to be moving at the same pace, with no students achieving at a significantly higher level compared to the other students. I suspect that the problems that the teacher was asking the students to solve were challenging to the point that none of the students were finishing too quickly or expressing how easy they were.

The teacher was also presenting different ways to think about problems. During preparation for the SAT, the students in the precalculus class came across a problem that essentially asked for what value of x will make the following equation true $18-3 x=15$. Most of the students went through the problem by solving it like a normal two-step equation and got that $x=1$. After the discussion on how to solve the problem as a two-step equation, the teacher posed the following question: "What would the ' 3 x ' part of the equation have to be to make the equation true?" Many of the students responded that if $3 x=3$, then the equation would be true. The teacher explained that solving for x in this equation also yields one. The teacher then stated that sometimes going through all the steps in a problem is not the best way, and that thinking outside of a more traditional way can lead to positive and more efficient results.

## Interview Process

The interview process of my research allowed me to ask the educators questions about equitable learning environments and how this topic relates to them as an educator. The one on one aspect of the interview allowed me to learn more about what I observed in the teacher's classrooms. For example, both teachers did not implement structured group work in their classrooms when I observed, so in the interview I got to learn why there was no group work. I asked the educators a total of twelve questions. As previously discussed, the development of these questions was one of my main focuses. These questions were designed in a way to possibly elaborate on some of the above-mentioned themes and how they relate to equity in the educator's classrooms. Throughout this section I will discuss some of the questions I asked and comment on the responses that were given.

I started the interview process by asking the following questions: How would you define equity in your own words? And How would you define equity in terms of math education? These were intended to be asked as two separate questions, but both teachers' responses to the first question naturally flowed into the second question. The responses were as follows:

Teacher $\boldsymbol{A}$ : "I feel like equity is, I guess, again, it's kind of hard to define, but it's more about being fair instead of being equal. It is more about making sure that all the students have enough resources or enough help and what not to get them all to the same level. So, you may help Susie with one problem and you may scaffold or help with a couple of more steps, but Bobby may need a lot more help or a lot less help to get to the same level. The kids may see something as oh they are not being fair, but it's that everyone needs different things to get them to that same level."

In this response, the educator defines equity as more about being fair instead of equal. This part of the response is great because it touches on the difference between equality and equity. If every student received the same exact instruction and help, things would be considered equal, but equity is about being fair. The NCTM states that an unproductive way of thinking about equality versus equity is, "Equity is the same as equality. All students need to receive the same learning opportunities so that they can achieve the same academic outcomes" (NCTM 2014), while a more productive way to think about equity and equality would be, "Equity is attained when students receive the differentiated supports necessary to ensure that all students are mathematically successful" (NCTM 2014). The educator also elaborates that we should be working to get students all to the same level, however, that is not how to be successful in creating equity. The goal is that as educators, we should focus on supplying all the resources that a specific student may need to succeed in order to reach their own bar of success.

Teacher B: "I think what defines a classroom is you need to have a differentiated instruction because everyone learns differently, and we need to cater to the needs of every student in the classroom. Sometimes some students learn better when they hear, some learn better when you write, some learn better when you do both, and some may have to see it visually and then maybe must sing sometimes. It has to be a balanced classroom environment for everyone to learn what they want to because students do learn differently."

Throughout the response of this educator, they touch on an important characteristic of an equitable learning environment, which is accommodating to differences within the classroom. Carey, Fennema, Carpenter, and Franke express, "the development of a curriculum designed to serve all students has perpetuated inequities" (Carey et al, 1995). As educators, we should try to
create a differentiated instruction that caters to the needs of every student. This teacher takes pride in their use of differentiated instruction to help students who learn differently understand the material well.

The next question is as follows, "How would you characterize your teaching methods?", these were the responses given:

Teacher $\boldsymbol{A}$ : "If you are going for how I teach in class, it is probably more direct instruction. I mostly do direct instruction with time for them to practice or we'll do an extra day every now and then to try and do some activity. They seem to respond the best in my experience so far with direct instruction, but I do know that is something I need to pull away from more and do more task-oriented things. Everyone always has something they need to improve on, and that is something I need to work on."

The teacher discusses how they use a more direct approach to teaching, which is more lecturebased followed by practice. This practice was evident while observing. The teaching practices that this specific educator implemented were not in line with the practices that were put into effect by the QUASAR project (Brown, Stein, \& Forman, 1996; Lane \& Silver, 1999; Silver, Smith, \& Nelson, 1995) which implemented a more open and discursive learning environment. The educator additionally recognized that this is an aspect of their teaching that could be improved upon. As human beings and teachers we should all be looking to identify our weaknesses and figure out ways they can be improved upon.

Teacher B: "So I guess the question is asking, what part of my teaching do I take pride in? I am into music and I play a lot of sports and I am also actively involved with all the other clubs and all. For me when I am looking at a typical classroom I already know that a student athlete is in there, students who can learn visually, students who can learn just
by writing on the board. I already know the students learn differently, so I keep it all in my mind to make sure that everyone is receiving the right instruction. Most of the time what I do is suppose I am teaching a topic. I always try to connect the dots from the previous lesson or where they have seen this before. It is important for them to know that it is not always an abstract thing but that it's already there, but we are doing it in a different format. I try to bring in some fun elements into the classroom, I have been doing that for the last twenty-five years, that is my personality. I try to bring a fun element into the classroom because math can be boring sometimes. I do not want them to get glued to the board, but instead want to get them interested in learning. If something does not work I have back up ways to approach problems. When the students ask you a question and you can answer, then you have the self-confidence which always makes you a better teacher. I teach very confident, have good preparation, and connect to the students. All of these elements are important, and I think that is why I have personally enjoyed teaching because I know that I can develop a good relationship with the students, and at the same time get their respect and then move on to the teaching."

To begin with, I thought it was great that the educator thought of their teaching methods as something the take pride in. The teacher also talks about the importance of getting to know your students so that you can understand how they learn. The educator makes note of the importance of making connections to previous topics to make the mathematics seem less abstract to the students. Gloria Ladson-Billings elaborates, "Real education is about extending what they already know" (Ladson-Billings, 1995). This is a great method since the abstract part of mathematics often discourages students. Gloria Ladson-Billings agrees that making mathematics meaningful and engaging is important to creating equity,

Another question I will be discussing is: Do you find groupwork to be effective or ineffective in your classroom? If effective, what have you found that works? If ineffective, what does not work? The teachers responded as follows:

Teacher A: "I mean I think it depends on how you use it, a little bit of both. Depending again on which groups. So, I have some classes where it can work really well in but some of the other classes depending on how it is used it can work well, but again it can also lead to a lot more off-task behavior. It's kind of a balancing act because then you have kids who talk a lot more with their friends they don't get as much work done, but then you also have kids who would never do work otherwise who are now starting to do some work. So it is kind of balancing the two of those. For most of my classes at least this year, because the ability level is a bit lower than previous years, I have not been doing as much groupwork. Just because again with this year having a more varied ability level I feel like then it gets more kids that will have other people do the work and them not doing anything, so this year I have done it a lot less."

This educator mentions that the lack of groupwork in their classroom is highly dependent on the ability levels of the students in the classroom. The teacher had also mentioned that they had not implemented groupwork at all this year, due to that reason. It seems that the educator is assuming that it will be difficult to get students who are at varied ability levels to communicate effectively with one another. I think that it would be worth the effort to try and get these students to communicate with one another, even though the educator is slightly timid to introduce this type of collaboration. There may be sometimes where trial and error with the groups might occur, but it could be possible to create an effective set of groups despite the mixed-ability levels.

Teacher B: "It depends on the topic that I am doing like today I was doing an honors stat class and we were talking about Venn diagrams and how many ways they can create them. Those kind of classes I want to have a group discussion for them to see what they can come up with. It depends on the topic, like topics I prefer that they try on their own. Like today we were doing a practice quiz. I would rather ask them to try it on their own. Depending on the topic I do like groupwork. It is like a collaboration. Suppose you know something, and I know something and then you are waiting for that one thing to click so that you can get the problem done. Sometimes students learn better through their friends and when one of the students explains, they kind of get faster because they are the same age group. Sometimes students pick it up from other students quite easily and then they can collaborate and share their knowledge across and then they see act where they are not understanding, or they can ask a question more freely than with adults. I feel that is important that we as teachers let students have collaboration among themselves and try to figure it out for themselves.

I agree that learning to work individually is important, but it is also important for the students to learn how to communicate their thoughts regarding mathematics. This also ties back to the QUASAR project because there was a constant encouragement of communication between students and justification of work. This is what the educator discusses in the second half of the response when they discuss the importance of students learning from their peers and students being able to freely ask their peers questions.

Lastly, I will talk about the following question: From my experience, most questions in a classroom are not specific questions but rather "I do not get it. Can you show me how to do
this?" Is there a way to fix this way of learning? How can we get students to become more responsible for their own learning? Here are the responses from the educators I observed:

Teacher A: "I don't like that. I'm like okay what do you need help on? And the students say all of it, and I always say "No, bring me a specific question." Then they will bring a question and I ask them where they are getting stuck, and they are like I don't know, can you do it? And I still respond no. If they ask something I will keep questioning them as to what part they are confused on or I'll be like let's start from the beginning and ask them where do we start? We will then walk through the steps to get to where they are confused. This is to get them to help realize they can talk through the process." This teacher utilizes furthering questions to stop this type of behavior from students. If the educator senses laziness in the students, they will ask them questions that cause them to think more in depth about where they are confused specifically. Ladson-Billings states, "When students asked questions, Ms. Rossi was quick to say, "Who knows? Who can help him out here?" By recycling the questions (and consequently, the knowledge), Ms. Rossi helped her students understand that they were knowledgeable and capable of answering questions posed by themselves and others" (Ladson-Billings, 1995). It is important to help students feel confident enough to approach problems head-on in mathematics.

Teacher B: "What I feel is that students should definitely take responsibility of their own learning. They should know what they understand, and what they do not. It is their duty to go to a teacher and ask for help. It is a two-way process, we should know what is going on with the students, but the students should also be responsible for their own work. Most of the students are very honest like if they know it they know it, if they don't know it they
will tell you they don't know it, and they will ask me to explain it which is just fine. It is not like they are trying to act like they don't know it and just want to waste time." At this school, it does not seem to be the problem that students are lazy when problem-solving. This is most certainly a positive aspect because it seems that the educator has instilled the confidence in their students that they are capable of solving challenging mathematics problems.

## Conclusion/Implications

While analyzing and observing at the first school, there some inequities that we should make note of. For example, when the teacher chose not to teach the students how to complete the square because the teacher thought it would be too difficult. One of the main principles of equity is that students should have access to challenging mathematics content that pushes them inside and outside of the classroom. The idea is that an educator should not assume that some topics may be too difficult, but instead give the students an opportunity to learn and at least attempt to apply topics that may better their understanding.

Moreover, the students at both schools did not have the opportunity to participate in collaborative learning in the form of structured group work. At school A, I did not witness collaborative learning occurring in the classroom and later discovered in the interview that there had not been implementation of group work at all this year. I suspect that this stemmed from the teacher being a bit fearful of implementing group work due to the mixed ability levels in the classrooms. However, there has been research conducted that shows that collaboration in the classroom positively influences student understanding. Boaler elaborates, "When the mathematics reform movements were developed, they were based upon the idea that open-ended problems that encourage students to choose and combine different methods, and discuss different solution methods with their peers, would provide productive learning experiences" (Boaler,
2000). Educators should be working together to figure out ways in which they can successfully implement group work and collaboration in mathematics classrooms.

Furthermore, at both schools there was little to no attempt of an open-ended learning approach. It was interesting to read and learn about the QUASAR project (Brown, Stein, \& Forman, 1996; Lane \& Silver, 1999; Silver, Smith, \& Nelson, 1995) and how the use of an openended learning approach led to a higher level of achievement. In a side discussion with the teacher at school A, they mentioned that the idea of implementing an open approach and more collaboration sounds ideal. However, the teacher feels that rushing to cover a set list of topics before the end of the year tests is what the teacher feels to be more important. The idea is that this teacher feels like they cannot implement these techniques due to the set list of curricula they have to cover before the year is done. While in the classroom, this teacher chose to teach topics very traditionally, through lecture and worksheets. There was a lot of repetition in relation to the problems that they were solving which in turn seemed to create a lack of motivation in the students. I imagine that if the students were taught these topics in a more creative, open approach where the students can come to their own conceptual understanding in groups, there would be a higher motivation for learning and in turn a higher level of achievement, as shown by the QUASAR project.

Perhaps one way to create an educational system where open-ended teaching and collaborative learning are embraced rather than ignored would be to focus more on student learning and not on test scores. It is crucial for the future of education that we find a balance between testing and learning. Students should first and foremost have the opportunity to receive an open and discursive mathematics education, where they gain in-depth knowledge on several
mathematics content. Sometimes rushing through the concepts will not always give the best results, so trying to find a balance is going to be important.

When talking with both educators one-on-one, both did not know a lot about equity in relation to mathematics education. The questions in the interview part of the process prompted the educators to discuss topics in relation to equity. Moving forward, if we are going to create an educational system where classrooms and teachers strive for equity we might consider teaching future educators about equity in the classroom and how implementing it might create positive changes in education. In this case, knowledge is power. If educators know how to create equity in their classroom and are taught how to implement it, then there might be positive change that starts to occur.

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