Demonstrating Knowledge and Understanding through Mathematical Writing

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Writing in the Mathematics Classroom

Writing in the mathematics classroom can be beneficial to students and their learning as well as to teachers and their knowledge of each student’s level of understanding throughout the school year. Completing writing assignments in the mathematics classroom asks students to use metacognition, higher order thinking, and self assessment which may benefit both students and teachers. Writing can take many different forms, and the following will be discussed as various implementation techniques: journals, autobiographies, letters, or various informal writing assignments.

While completing any of these writing activities and being asked to write and explain their reasoning or what operations they performed and why; students are often able to clarify and deepen their understanding of mathematics. Having students complete writing assignments in the mathematics classroom requires students to pause and consider what they are learning in the classroom (Meier & Rishel, 1998). This indeed, can be of benefit to the student and their teacher, as well as giving teachers the opportunity to read student’s reasoning and thinking skills. With these benefits available through writing assignments in the classroom, more teachers need to be using these tasks in their curriculum to further the learning of their students. Although it will take some time to incorporate writing into a curriculum as well as read the responses, the benefits of doing so will be discussed in order to convince more teachers into implementing writing activities into their curriculum.

Many teachers throughout the United States have implemented writing in their classrooms and have demonstrated positive results in their students’ knowledge and understanding of mathematics. Writing assignments can also be viewed as an example of formative assessment. The U.S. Department of Education’s National Mathematics Advisory
Carpenter (2008) stated that teachers’ regular use of formative assessment improves their students’ learning at all levels (p. 47). In Principles and Standards for School Mathematics, the National Council of Teachers of Mathematics (NCTM) reviewed over 250 research studies and concluded that student learning is enhanced when teachers use formative assessment in making judgments about teaching and learning (p. 22). In order for teachers to begin implementing writing techniques into their classroom and to further assist their students’ education, several techniques will be discussed, as well as their advantages to both students and teachers in the classroom. There are also disadvantages to using writing in the mathematics classroom since many students are not used to completing writing assignments in a mathematics class. It is important to keep this in mind while further discussing putting writing activities into action along with mathematics.

Metacognition and Higher Order Thinking

Having students use metacognition is to have them think about their thinking: what they know and what they do not know, as well as evaluating their own thinking processes, and using previous knowledge to explain their thinking. Metacognition also refers to higher order thinking and is an integral part of the writing process, specifically through mathematics writing.

In 1956, Bloom developed a classification of educational learning based on different objectives teachers set for their students (Anderson & Krathwohl, 2001). These objectives can be represented in a triangular diagram listed from simple to complex, respectively as: creating, evaluating, analyzing, applying, understanding, and remembering. According to Benjamin Bloom, using higher order thinking benefits students by using the creating, evaluating, and analyzing objectives also known as the three highest levels of intellectual behavior in Bloom’s
Taxonomy. For example, evaluating progress towards the completion of a task is considered a metacognitive activity.

According to NCTM, writing in the math classroom can help students assess themselves while using the evaluation level from Bloom’s research (pg.44). NCTM recommends using self-assessment exercises with students in order for them to judge their own work. Metacognition is key in this process since metacognition gives students the opportunity to manage and assess their own learning (Burke, 1994). Burke (1994) goes on to recommend that teachers introduce metacognitive strategies in order for students to self-reflect regularly and become “adept at monitoring, assessing, and improving their own performances and their own thinking” (p. 97). Striving for use of metacognition and higher order thinking within the mathematics classroom will prove to be of benefit to both students and teachers.

**Mathematical Writing Benefits to Students**

It has been suggested that students can benefit from completing writing activities in the mathematics classroom, with the intent of deepening mathematical understanding. One of the ways to begin students writing in the mathematics classroom is to have them write an explanation for their solutions. While students write about their thinking through a problem, and their solutions to problems, they are interpreting the meaning of their answers (Enyart & Van Zoest, 1998, p. 165). Students cannot simply compute tasks based on a rule or formula, but they must have a deeper understanding of the mathematics they are doing and be able to explain why they got the answer they did and why it makes sense. Enyart and Van Zoest (1998) further state that writing provides an opportunity for students to use higher-order thinking which has been proven beneficial to students as indicated in Bloom’s taxonomy (p. 165).
While completing writing assignments, students are able to verbalize to their teacher what they understand, and what they are having difficulty understanding. Students are then able to go back and review their self assessments in order to focus on what they told themselves they needed to more fully understand. Having a better understanding of how students are approaching mathematical concepts can benefit the student in the long run, as the teacher is now able to assist each student with the specific problem each student is having. In this regard, writing assignments can give students more personalized feedback on their thinking and understanding of the content, from both their own view as well as their teachers.

Another benefit to students is to make them realize that they are all capable of doing mathematics. Some students believe that doing mathematics is simply a skill that some students have and others do not, but having students participate in writing activities can help to address this common misconception. A possible benefit of writing is that it can free students from thinking of mathematics as only finding the correct answers to questions posed by someone else (Countryman, 1992, p. 11). Writing tasks allow students to explain their thinking and how they came about the answer that they did, contrasted by following a rule that their teacher told them to use or following an algorithm from their textbook. Writing assignments are a chance to give students the opportunity the work the problem the way they feel is best, using the knowledge that they already have about the concept or previous concepts.
Mathematical Writing Benefits to Teachers

In addition to the student benefits listed above, writing activities benefit teachers. A benefit for teachers using writing tasks in the mathematics classroom is that it can give the teacher a deeper understanding of the knowledge held by their students. Writing requires students to think, which is what teachers need to encourage in order for students to progress mathematically. Writing activities give students the opportunity to share their thoughts, their confusion, their half-formed ideas, their frustrations, and their triumphs with their teacher (Meier & Rishel, 1998, p. 7).

Writing provides an opportunity for the teacher to determine what students understand, apart from the correctness of their answer. On a correct answer, the writing that is completed along with it, can demonstrate to the teacher whether the student understands the concept or if they mimicked a demonstrated process and do not have an understanding into why the process works. Having students explain their thinking and reasoning throughout a problem can also help the teacher to see where errors occur and if they involve miscalculations or misunderstanding (Enyart & Van Zoest, 1998, p. 167). Because writing can demonstrate a teacher’s sense of student understandings from a lesson or introduction of a new topic, many misconceptions and misunderstandings may go undetected if students are not required to write.

Furthermore, writing activities can illustrate students’ feelings and attitudes toward mathematics class and help the teacher monitor the effects of classroom activities. Often, having students respond to the prompt “What I liked most about this year was…” allows students to reflect upon any of their favorite activities and the material covered over the year can be used to plan future classes (Enyart & Van Zoest, 1998, p. 168).
Implementation Techniques

Recognizing the potential benefits of implementing writing into the mathematics classroom, the next step is to begin writing implementation. Many teachers may have wondered how to start using writing tasks in their classroom and the following collection of techniques shall serve as a useful resource. Teachers may decide to begin the year by having students write a mathematical autobiography or to have students begin a mathematical journal which will continue throughout the year. Keep in mind that not all techniques will work for every classroom, as each teacher and set of students is unique. It is recommended to start small which will help students get used to the idea of writing in the mathematics classroom since this may be the first time students have written in a mathematics class (Meier & Rishel, 1998, p. 7).

To begin incorporating writing tasks into the mathematical classroom, the following is a list of activities that will introduce students to the idea of writing and demonstrating their mathematical knowledge and understanding.

Informal Writing

Quick informal writing can be particularly useful in the mathematics classroom. Freewriting, in which students write continuously for a set amount of time with no consideration for spelling or grammar lets students’ thoughts flow freely. Free writing can be beneficial to students as it can help students discover what they know and what they do not know about mathematics (Countryman, 1992, p. 19). “Using free writing at the beginning of class focuses the class while identifying areas of students’ confusion. The students are preparing for what follows by writing a short answer at the start of class and reminding themselves of what they should be able to explain at this point” (Meier & Rishel, 1998, p. 10). An example from Meier and Rishel
(1998) for a free writing prompt is to have students defend or counter if \((x + 5)^2 = x^2 + 25\) is a correct answer (p. 10).

Countryman (1992) explains another way to have students to begin writing in the math classroom is to have them complete sentence stems such as:

- Math is like…
- Prime numbers are…
- Teachers usually say…
- The trouble with math is…
- I can do word problems when… (p. 19)

Freewriting, sentence stems, as well as having students write a note on the back of homework assignments explaining any difficulties they may have had with the assignment are small, quick ways to begin implementing writing into any mathematics classroom. All of these activities require a small amount of time but can be useful in helping students become active learners, writers, and assessors of their own learning.

*Mathematical Autobiographies*

Every student has a mathematical history, from their mathematics classes throughout their education as well as with experiences outside of the classroom. Students have a history of their own understanding of mathematics, and whether or not they believe they have the ability to do it. Having students share both positive and negative past mathematical experiences as well as their opinions of mathematics is a way to implement writing in the classroom. “Asking students to write a mathematics autobiography at the beginning of the school year gives them permission to talk about what they know best: themselves, what they care about, and what they know” (Countryman, 1992, p. 22). Mathematical autobiographies give students the chance to focus on their own learning and think about what works for them and what does not work for them as
well. Beginning the school year with this writing activity can help to establish a relationship with students and begin conversations with them about their mathematical ideas, opinions, feelings, and learning styles. Teachers who ask their students to write math autobiographies early in the year report that they know considerably more about their students much earlier in the year, and issues like confidence and self-esteem are brought into the open where they can be confronted (Countryman, 1992, p. 24).

**Letters**

Letter writing is another technique that can be introduced into any mathematics course to begin mathematics writing and for students to become accustomed with the concept. There are different types of letters that can be used, such as: writing a letter to another student, writing a letter to the teacher, or writing letters to a college student pen pal.

An example of a letter writing assignment is to have students write a letter to another student (real or fictional) who has been absent from class. The theme of the letter could be to explain a specific concept, review an extended lesson or unit, or tell the student what he or she missed on a certain day (Enyart & Van Zoest, 1998, p. 165). While writing such letters, students reason through information and are willing to ask their teacher questions if they are having trouble understanding the assignment or content (Enyart & Van Zoest, 1998, p. 165). Students can also benefit from writing letters to a friend or other student in a similar class who is behind in learning the same material and explaining what they think is most important from a lesson just presented. While the teacher “learns the enormous variation in what students find important and interesting with the respect to any particular topic,” reflecting on lessons and discussing important information will benefit the student as well (Meier & Rishel, 1998, p. 9).
Students can also write post-exam letters to their teacher “outlining how they prepared for the exam, what their reaction is to their performance on the exam, and how they plan to prepare for the next exam” (Meier & Rishel, 1998, p. 10). These letters can provide an opportunity to discuss effective study strategies for future tests and is a way to incorporate more discussion with each student on their performance throughout the year. Additionally, teachers may also ask students to write them letters explaining various other mathematical concepts introduced during the year.

Teachers may opt to have their students participate in a pen pal letter writing project as another choice for writing implementation. Students may write to other students learning the same material or write to pre-service teachers at a local university is. Pen pal projects can offer students the opportunity to discuss mathematical concepts, answer mathematical problems, and receive personalized feedback while practicing effective mathematics writing. Pen pals are able to learn more about each other’s interests and use that information to create personalized math problems for school aged students to complete and explain in a return letter. Students are able to hone their communication skills and receive feedback about their learning. Hopefully this process will prove motivating for some students (Lampe & Uselmann, 2008, p. 197).

**Journals**

Having students keep a mathematical journal to track their experiences of learning mathematics throughout the school year is another way to implement writing into the classroom. Joan Countryman (1992) described journals to her students as a way for them to discuss their troubles through the problem, and to keep track of where they are going, and where they have been in mathematics (p. 27). Having students write for five to ten minutes at the end of the week
should be a sufficient amount of time for students to reflect over the week and assess their understanding of any new topics that were introduced. Countryman (1992) goes on to explain that she selects one or two questions from the list below for her students to write about in order to stimulate their thoughts and reflections within journal entries:

- What did you learn from a class, activity, discussion, assignment?
- What questions do you have about the work?
- Describe any discoveries you make about mathematics or yourself doing mathematics.
- Describe the process you undertook to solve a problem.
- What confused you? Challenged you? What did you like? What did you not like?
- What are you thinking?
- Why does this step make sense?
- Write this again in a different way.
- What makes this problem difficult?
- What does this formula mean?
- Explain what we are doing so a younger student will understand it.
- Are you stuck? What information do you need to get unstuck? (p. 42)

Reading such journal entries tells the teacher “considerably more about what students grasp and do not understand, like and dislike, care about and reject as they study mathematics than any formal or traditional math assignments” (Countryman, 1992, p. 28). Student journal entries also demonstrate their thinking skills to their teacher and give insight about how each student is approaching the mathematical content of the curriculum, which is a benefit to teachers as previously discussed. Furthermore, journals are useful for incorporating metacognitive strategies into instruction. Burke (1994) explains that “teachers can use journals as metacognitive strategies by assessing the reflectiveness of the students’ responses, the evidence of transfer to other classes or life outside of school, and the students’ ability to plan, monitor, and evaluate their own work” (p. 99). Asking such questions as “Did you do well in this assignment?”, “If you had to do this assignment over, what would you do differently?”, or “What help did you need from your teacher?” can help students reflect on their individual work (Burke, 1994, p. 99).
Conclusion

In conclusion, it seems that the benefits are numerous and the disadvantages few. Writing in the mathematics classroom seems to be mostly beneficial for students in that it deepens their understanding of mathematics. There are also benefits for teachers and since it is suggested to start small with writing integration, grading such assignments does not have to be labor intensive. Overall, putting writing activities into practice can be beneficial in demonstrating the knowledge and understanding of students to their teachers.
References


