Comparing Virtual and Concrete Manipulatives Effect on the Conceptual Understanding of the FOIL Method

Kristen N. Hartley

Georgia College and State University

ABSTRACT

Defined as “an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered,” (Swan & Marshall, 2010) manipulatives are used in mathematic classrooms all across the United States and other countries. Stein and Bovalino state that manipulatives provide a concrete way to link abstract information to already established knowledge thus giving new concepts a deeper meaning (2001). The purpose of this study is to compare concrete and virtual manipulatives to examine if one fosters a deeper conceptual understanding of the FOIL Method. Students in a Middle Grades Cohort at Georgia College were given both a pre and post assessment to assess their level of understanding of the FOIL Method after a lesson using either virtual or concrete manipulatives. They then were taught using the other type of manipulative to assess whether students preferred virtual or concrete manipulatives.

INTRODUCTION

Due to the fact that I am a mathematics major with an education concentration, I take pure mathematics classes as well as mathematic education classes in which we learn to understand mathematical concepts through various techniques, one of those being manipulatives. These classes were very interesting because I was given the opportunity to develop conceptual understanding on topics that I had previously learned without the use of manipulatives. Concrete manipulatives were used in these classes as well as virtual manipulatives. Taking these classes led me to wonder if concrete or virtual manipulatives serve as better tools to facilitate a class to reach conceptual understanding of a topic. I also was interested to understand which manipulative was easier for students to use, which manipulative students preferred to use, as well as some of the positive and negatives that come from using both types of manipulative.

Students are no longer making the connections between mathematical concepts and the real world thus resulting in less and less interest in learning mathematics. Mathematics classrooms are becoming less of a place to come and learn and more of a place to go through the motions of the recurrent steps and rules of mathematics. Manipulatives can give students a chance to learn a concept in a way that they have not seen or thought about before and make those connections that will make mathematics more enjoyable. Stein and Bovalino (2001) state that manipulatives provide a concrete way to link abstract information to already established knowledge thus giving new concepts a deeper meaning. Making these connections will lead to a better conceptual understanding and in turn a better appreciation for the subject of mathematics. Through these connections made with manipulatives and the mathematical abstraction process, conceptual understanding of mathematical concepts can be gained (Durmus & Karakirik, 2006). While both concrete and virtual manipulatives fall into this category of increasing conceptual understanding, does one increase conceptual understanding more?

My research questions are:

- Do students gain a better conceptual understanding with virtual or concrete manipulatives?
- After experiencing both concrete and virtual manipulatives, do students prefer one type over the other?

LITERATURE REVIEW AND THEORECTICAL FRAMEWORK

Manipulatives "provide a concrete way for students to link new, often abstract information to already solidified and personally meaningful networks of knowledge, thereby allowing students to take in the new information and give it meaning" (Allen, 2007). There are numerous studies that support the use of manipulatives and their usefulness in gaining conceptual understanding. Johnson found that from one year to the next, her students increased their test scores anywhere from 4% to 42% through the use of hands on manipulatives and the more the students used them, the higher their percent increase was. Of Johnson's twenty students, 68% of them increased their test scores, and of the 8 students that she had both the current and previous year, half of them increased their scores (Johnson, 2015). A similar study shows that, while a few students may have decreased in scores, a group of 22 fifth grade students as a whole increased their understanding of geometry through the use of pattern blocks showing a mean score change of 13.182 (Allen, 2007). While many studies show that manipulatives are beneficial in showing better test scores, and thus conceptual understanding, there are also studies that show a differing viewpoint. A study conducted on a first-grade class over an eight-week period studying addition, subtraction, and measurement showed working with a work book produced higher test scores on a traditional paper and pencil test and no statistical difference on a teacher designed test that used manipulatives. This study also shows that whether students were taught traditionally or with the use of manipulatives, they still learned the concepts. The difference was the enjoyment that students experienced in using manipulatives as opposed to book work (Rust, 1999). Although the study was not created to measure enjoyability of manipulatives, students tend to retain more information when something is more enjoyable.

Manipulatives have been utilized since ancient times when the civilizations of Southeast Asia used clay boards covered in sand to make tallies to keep track of things. Even early Americans had their version of manipulatives such as corn kernels strung onto string used to count. Eventually, Friedrich Froebel saw the potential for use of such objects in a mathematics classroom and created the first true manipulative in the late 1800s (“Benefits of Manipulatives,” n.d.). A manipulative is defined as “an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered” (Swan & Marshall. 2010, p. 14) This definition gives way to a physical object that a student can pick up and move around to do what they want with it; this is a concrete manipulative. However, this definition can be a bit restricting in the current technology-driven age that we live in today. Virtual manipulatives are "an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Durmus & Karakirik, 2006). There are many websites made to house applets of virtual manipulatives such as the National Library of Virtual Manipulatives, Didax, Math Playground, and, the one used for this study, Braining Camp. Manipulatives, both physical and virtual, are meant to make abstract ideas easier to understand and give them more meaning than if manipulatives were not used (Durmus & Karakirik, 2006). Manipulatives can be used to teach a wide variety of topics some of which include measurement, probability and statistics, and number relations (“Research on the Benefits of Manipulatives,” n.d.). Manipulatives give students a way to connect abstract mathematical ideas with something easier to understand to promote a deeper understanding of the concept at hand whether it be through a concrete manipulative or a computer-based virtual manipulative, but does one work better than the other?

Just like any other learning tool, there are advantages and disadvantages to the uses of manipulatives in a mathematics classroom. As shown in research, manipulatives help develop a deeper conceptual understanding of mathematical concepts. Long term research shows that manipulatives help develop key aspects in students’ success in mathematics such as their ability to work together, relating mathematical symbols to the real world, and making their learning experience a priority. The use of manipulatives "is supported by both learning theory and educational research in the classroom." Due to this, some states mandate their use in mathematics classrooms ("Research on the Benefits of Manipulatives," n.d.). They can be used at any stage of the learning process to ensure that students are making the connections necessary to have deeper understanding instead of just learning the steps and repeating them. Stein and Bavalino state that manipulatives are important tools that help students think and reason in a more meaningful way (2001). A study of 820 teacher responses to a survey from 250 schools in Australia shows that some of the top reasons that teachers choose to use manipulatives are to heighten interest, provide a visual aid, provide hands-on learning, introduce concepts, and encourage and promote language skills (Swan & Marshall, 2010). While all of these are good reasons to include manipulatives, well intentions do not always correlate to good lessons and conceptual understanding.

Even though manipulatives have shown to be useful in a mathematics classroom, there are challenges that come with their use. The Australian teacher responses mentioned previously also included responses such as "sometimes students miss the point of the lesson if it is always explained using the same manipulative" and "kids will pick up the 'wrong' concept from a manipulative." A student may look at the manipulative given to them and not see the connection that the teacher does since a teacher already possesses the mathematical knowledge needed to make those connections (Swan & Marshall, 2010). Thus, a teacher must think in a way like their students, of seeing the content for the first time, in order to anticipate what their students may think. In other words, the object being presented does not translate to the mathematical symbols that are used and the desired conceptual understanding; that correlation must be made clear by the teacher and the lesson. Anticipating these misconceptions ahead of time and preparing for them can take some time; however, if the proper preparation is taken, the results will be positive (Stein and Bovalino 2001). If the preparation is lacking, the students will most likely not grasp the connections that the teacher wishes for them to. Swan and Marshall touch on the issue of availability and organization of manipulatives and suggest that better organization of manipulatives within a school could lead to better results (2010). School budgets may not allow for many concrete manipulatives and computers may not always be readily available for students to use virtual manipulatives on.

Even within each type of manipulative, there are advantages and disadvantages that are exclusive to each type as well as the advantages and disadvantages that they share. Below are two tables that include some of the advantages and disadvantages of each type, concrete and virtual manipulatives. All of these advantages and disadvantages were considered when picking a concept and manipulative to use for this study.

Concrete Manipulatives	
Advantages	Disadvantages
• Develop Problem Solving Skills • Useful in connecting the three stages of learning • Help students feel competent • Student has more control • Can simulate real-life situations easily • Less expensive than computers • Allows for better teacher student interaction • Gives a way for students to receive information both visually and kinesthetically	• Overuse can lead to the ignoring of the connections to mathematical symbols • Limitations on what can be done with them • No feedback

Virtual Manipulatives	
Advantages	Disadvantages
• Adaptability • Many are on free to use websites	• Overuse can lead to the ignoring of the connections to mathematical symbols

• Develop Problem Solving Skills • Give immediate feedback • Useful in connecting the three stages of learning • Help students feel competent • Have a larger variety of available experiences • Easier to move around • Allows for more complex operations • More accessible at home (good right now during COVID) • Many provide step by step instructions	• Each student, or a small group, must have a computer to access them which can be expensive • Can't touch them • Some content has yet to be developed this way • Forces more abstract thinking leading to some students to miss the concept • Less teacher insight into student thinking, less opportunity to correct misconceptions • Can sometimes be too leading

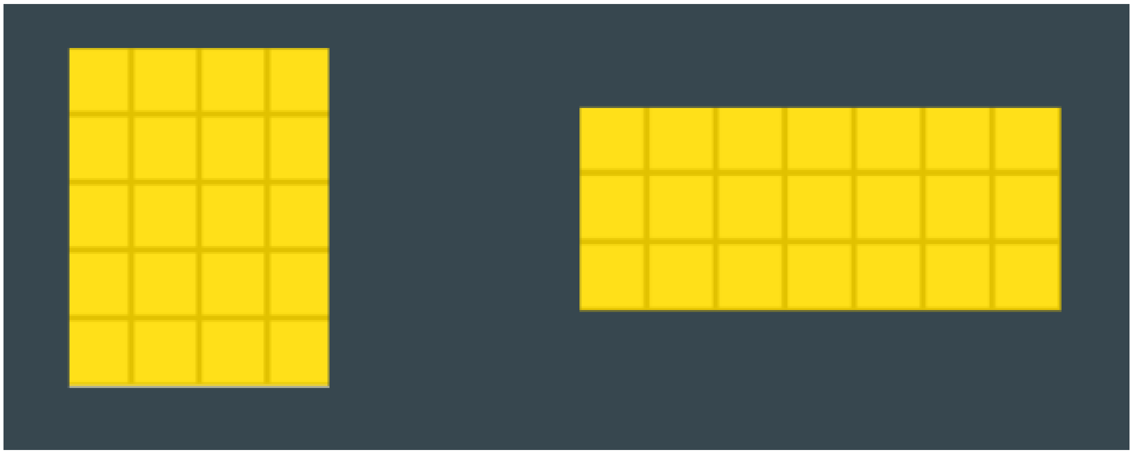
("Pros and Cons of," n.d.), (" Benefits of Manipulatives," n.d.), ("Advantages and Disadvantages," n.d.).

In order to successfully learn mathematics, we must achieve mathematical proficiency. Mathematical proficiency consists of five components: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. While all of these are important and influence each other, for the purpose of this study we will focus on conceptual understanding which will be used to gauge how effective each type of manipulative is. Conceptual understanding is "an integrated and functional grasp of mathematical ideas." A student that possesses conceptual understanding knows more than just facts and the process of how to solve something; they understand why concepts are important and where it would be useful in the world. Conceptual understanding supports learning new concepts, making connections between concepts, and concept retention. The connections that are made between different representations of a mathematical concept is an important indicator of conceptual understanding. A student should be able to show and explain a concept in different ways and understand how the different representations are useful in different situations. If there are no connections made, there is little to no conceptual understanding present; if there are many connections made, conceptual understanding is greater. In other words, "the degree of students' conceptual understanding is related to the richness and extent of the connections they have made.” Fractional addition is one example; fractions can be expressed as themselves, using concrete materials, through a story, or on a number line. Ultimately, through one of the representations, the student will be able to find a common denominator and solve the problem. When presented with new material or a problem that a student may not understand, it will be easier to learn or figure out if they have conceptual understanding of the concepts used in it. For example, students that understand single-digit addition will find multidigit addition easier to master. Conceptual understanding can also help with avoiding making errors and producing very useful knowledge clusters. Seeing the connections between concepts can lead to students picking up on a hierarchical system of knowledge that lends itself to the comprehensive of new concepts easily and the need for less effort in some areas. For example, a student may see 8+9 and see that it is 1 more that 8+8. The student has made the connection between whole numbers and addition that will prevent him from having to strictly memorize addition facts, which will save time when the numbers are larger. As we can see, conceptual understanding is useful on all fronts of mathematics and will lead to being more successful in mathematical endeavors. (Kilpatrick et al., 2001). In my research, the pre and post assessment will include multiple representations of multiplying binomials as well as asking students to explain in words how they came to their solution. This will give me insight into their conceptual understanding of the FOIL Method both before and after the administration of a lesson involving concrete or virtual manipulatives.

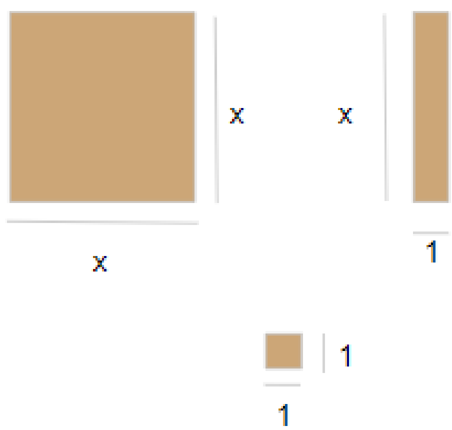
METHODOLOGY

I conducted my research in Dr. Santarone’s Middle Grades Cohort Math Education class. This semester was interesting in that half of the class comes to class on Tuesday while the other half is done virtually, then on Thursday, the two groups switch. This is a situation that is a result of COVID-19, and being that it was structured this was, it provided an easy way for me to conduct my research being that I wanted to compare virtual and concrete manipulatives. This was the reason we chose this class as our subjects. One group was given the concrete manipulative lesson on Tuesday and the virtual manipulative on Thursday and vice versa with the other group of students. The groups were pretty evenly split with 11 students in one and 13 students in the other. For the virtual lesson, I met with students on a Zoom meeting. For this lesson, the Algebra Tiles app on the BrainingCamp website was used; students were asked before class began to sign up for the free trial. The website can be found here: https://www.brainingcamp.com/. The physical lesson was taught by Dr. Santarone; however, I created both lessons. This was done for time efficiency so that students results from the pre and post assessments would be accurate. Before any lesson was taught, the students took a pre-assessment, found in Appendix A. This assessment consisted of six questions. The first three, a series, asked students to multiply a binomial, explain their steps to get to their answers, and draw a picture of the problem and explain how their picture represents it. The last three questions ask if the students used manipulatives growing up, if they have used algebra tiles specifically, and which type of manipulative, physical or concrete, do they anticipate liking better.

Both the concrete and the virtual lessons were very similar in structure and deliveries. The lesson began with a simple refresh problem. Displayed were two rectangles that were filled with a grid. Students were asked to identify the side lengths and the areas of the rectangles then prompted with the question “What if one of the side lengths were replaced with x?” Shown are the two rectangles included in the virtual manipulative lesson. The rectangles have side lengths of 5 and 4 on one rectangle and 3 and 7 on the other. The rectangles shown in the concrete manipulative lesson were very similar.

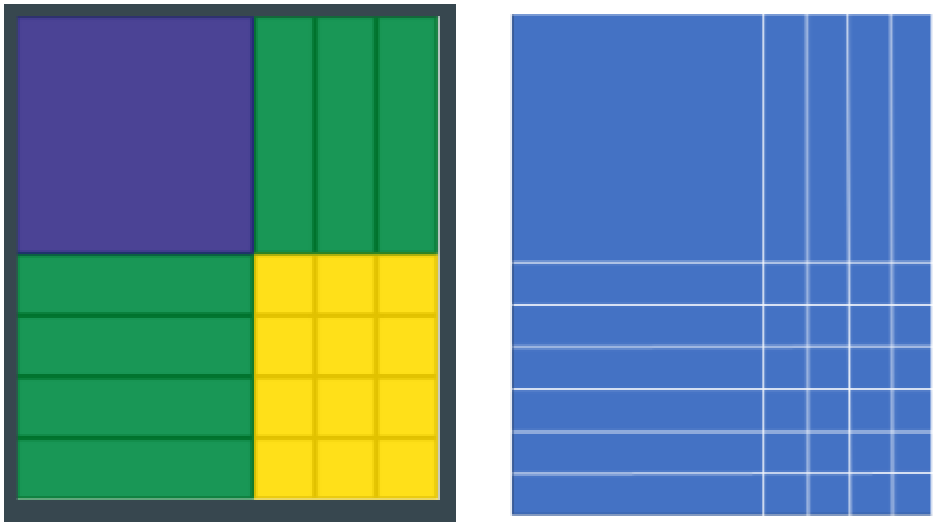


Before moving on to more complicated examples, I defined what Algebra Tiles are and reviewed the three tiles, their side lengths, and their areas. Below is an example of what the students were shown when defining each Algebra Tile.



The square shown on the bottom has two side lengths of 1 unit yielding an area of 1 unit^2. The rectangle on the upper right has a side length of 1 and a side length of x. Here, I made sure to emphasize that the side length x was not a length that could be generated by the side length 1. Students could use the smaller 1 by 1 square to see that the smaller side length is equal to 1. Students could use their manipulatives, both concrete and virtual to see that this is true. This rectangle gives an area of x units^2. The upper left rectangle has two side lengths of x. The area of this rectangle is x^2 units. After explaining this to the students, I asked them to find the side lengths and area of two more rectangles. There rectangles included side lengths such as x+2, x, and 7. The side lengths were a mixture of the forms x+2 and x or x+5 and x. None had two side lengths of the form x+2. An example of one of these rectangles is included below.

After discussing the two rectangles side lengths and area, I asked the students to construct their own rectangle given two side lengths. For example, from the concrete manipulative lesson, students were asked to build a rectangle with side lengths x+2 and x+5. For this type of problem in the virtual manipulative lesson, I asked if a student would like to share their screen with the group so that we could see the rectangle that they formed and explain their reasoning that led them to this rectangle. With the concrete manipulatives class, Dr. Santarone asked students to bring their tiles to the front and demonstrate their answers for their classmates. I then asked the students to find the side lengths and area of one last rectangle, this time where both side lengths were of the form x plus a number. The rectangles that students were asked to find the quantities for are included below.



After a student gave and explained their answer for this example, they were given a few problems, such as (x+6)(x+2), and were asked to solve using their manipulatives. Students could think of x+6 as one side length and x+2 as the other, form the rectangle produced by these side lengths, and see that their area is the solution to the problem. Students were then given time to work on three On Your Own problems. The time given was gauged on how well students grasped the concepts and how quickly they used the manipulatives. With the virtual manipulative lesson, I asked students to send a message in the Zoom chat that they were finished. Then, I asked students to volunteer to show their answers and explain to their classmates how they came to that answer. This was once again done through screen sharing for the virtual manipulative class. With the physical manipulative lesson, Dr. Santarone has students bring their algebra tiles to the projector, show their classmates their answer, and explain how they came to that conclusion. If time permitted, which for both the physical and virtual lessons, students were asked to try the challenge problem. This problem included a coefficient other than one on one of the x’s. For example, the concrete lesson’s example was (2x+3)(x+4). Students were given a few minutes to work on this problem and were then asked to share with the class.

At the conclusion of Tuesday’s class, both concrete and virtual groups were asked to complete the post-assessment which can be found in Appendix B. This assessment was structured very similar to the pre-assessment. The same series of three questions were included where students were to multiply a binomial, explain their steps to get to their answers, and draw a picture of the problem and explain how their picture represents it. Also included on the post assessment were questions of whether this manipulative helped their understanding of the FOIL method and a rating of the manipulative on a scale of 0 to 5 with 0 being “I do not like this at all” and 5 being “I loved this.” The pre and post assessments were graded based on the same rubric and would be used to gauge if the students gained a conceptual understanding of multiplying binomials. This will be done by grading the first three questions, the series of working out a problem, explaining it, and drawing a picture that represents it, on a scale of zero to five. On this scale, a zero represents that there was no attempt made to answer the question and five being there are no errors present meaning that students used the FOIL Method correctly, gave an explanation that correctly matched the method, and the rectangle drawn has correct dimensions and all the FOIL method components. The rubric can be found in Appendix C.

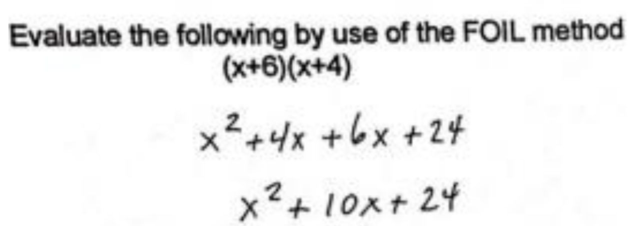
After Thursday’s class and both class groups had been taught both lessons using concrete and virtual manipulatives, students were asked to complete the Ending Questionnaire, found in Appendix D. This questionnaire included questions about how much they liked each manipulative and which they liked better. Also asked was which manipulative helped their understanding of the FOIL method more and what are some of the advantages and disadvantages of both physical and concrete manipulatives. Lastly, students were asked if they would use manipulatives in their future classrooms and why.

FINDINGS

Overall Averages

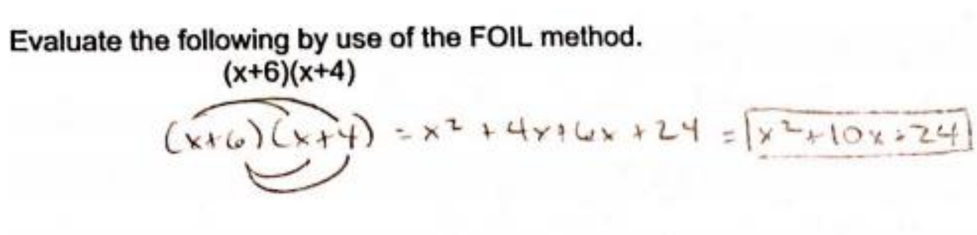
After analyzing the data, I found that students increased their conceptual understanding of the FOIL method through the use of manipulatives. Following are two examples of typical responses from the pre and post-assessments for each question of the three question series; one of the shown examples is from the group taught using virtual manipulative first and the other is from the group taught using the concrete manipulatives first. Each question had a maximum of 5 possible points.

For the first question on the pre-assessment, students were asked to evaluate (x+6)(x+4) using the FOIL Method. Shown first is the example from the virtual manipulatives class followed by the example from the concrete manipulatives class. Both students were given a score of 5 out of 5 on this question since they correctly multiplied the binomials using the FOIL Method without any errors. There were only five students, out of twenty-five, that did not earn a 5 for this question. This shows that students have a procedural fluency for this concept, most likely from their previous mathematical experiences. The class average for this question on the pre-assessment was a 4.44.


Evaluate the following by use of the FOIL method
(x+6)(x+4)

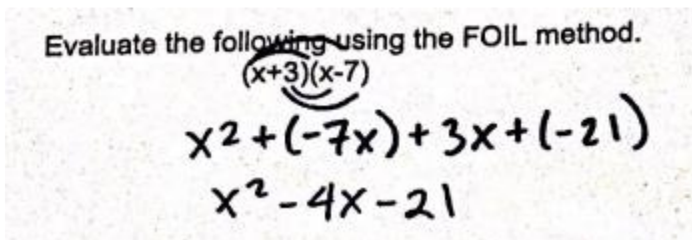
$x^2+4x+6x+24$

$x^2+10x+24$


Evaluate the following by use of the FOIL method.
(x+6)(x+4)

$(x+6)(x+4) = x^2+4x+6x+24 = \boxed{x^2+10x+24}$

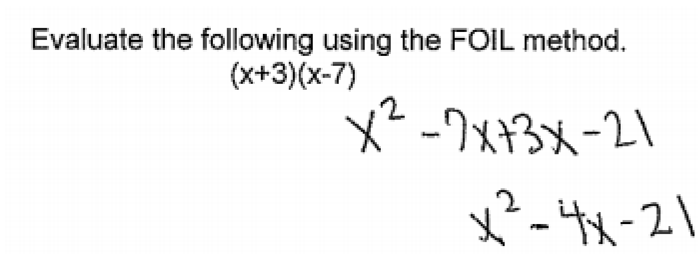
The same series of three questions on the pre-assessment were included on the post-assessment, but students were asked to evaluate (x+3)(x-7) after being taught a lesson using virtual or concrete manipulatives. Below are two examples that show the first question responses on the post-assessment; the first is from the class that used virtual manipulatives, and the second is from the class that used concrete manipulatives. Both students were given a 5 out of 5 for this question. The class average for this question on the post-assessment was a 4.88 showing that the procedural fluency present before the lesson is still intact.



Evaluate the following using the FOIL method.
(x+3)(x-7)

$x^2+(-7x)+3x+(-21)$

$x^2-4x-21$

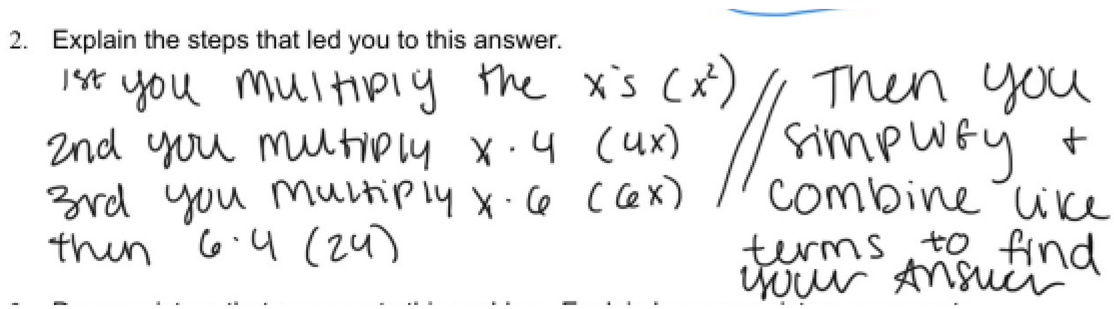


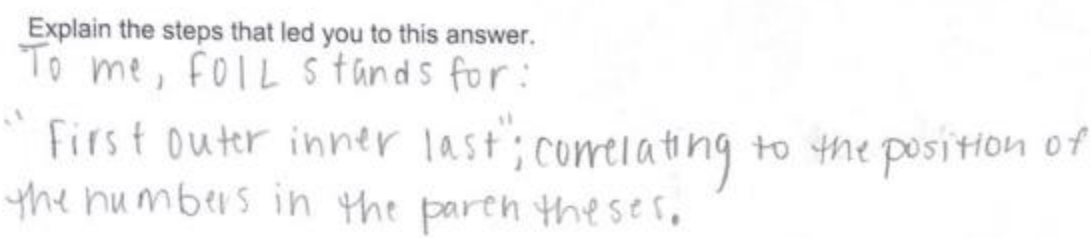
Evaluate the following using the FOIL method.
(x+3)(x-7)

$x^2-7x+3x-21$

$x^2-4x-21$

Next students were asked to explain the steps that lead them to this answer. For this question, I was looking for students to use words that show they understand the method being used. Being able to correctly communicate mathematical understanding is a key sign of conceptual understanding. Again, first is the example from the virtual manipulatives class followed by the example on from the concrete manipulatives class. Both students received a three out of five for this question. This was the score that most students received for this question in both classes on the pre-assessment. These students were given a three because they understand that each variable and number in the first term must be multiplied by each variable and number in the second term; however, the explanation used is simply something that they have memorized over the years of being in mathematics classrooms. The distributive property is not mentioned, and there was not much explanation other than the meaning of the acronym FOIL and what those components are in this particular question. The class average for this question on the pre-assessment was 2.76.


2. Explain the steps that led you to this answer.
1st you multiply the x's (x^2) // Then you simplify + combine like terms to find your answer
2nd you multiply x · 4 (4x)
3rd you multiply x · 6 (6x)
then 6 · 4 (24)


Explain the steps that led you to this answer.
To me, FOIL stands for:
"First outer inner last"; correlating to the position of the numbers in the parentheses.

On the post-assessment, students were once again asked to explain the steps that they used to achieve their answer in the first question. An example of the second question from the post-assessment is shown below. Although there were a few students that mentioned the distributive property and the FOIL Method in the way I was looking for, most students' answers for this question became more concise and the response was mostly the same but in different words. For both examples shown below, the students were given a 3 out of 5 for reasons similar to the pre-assessment. The class average for this question on the post-assessment increased to a 3.36 showing that students can communicate their mathematical understanding of the FOIL Method through words better and on a deeper level than the students could before they were taught a lesson using manipulatives.

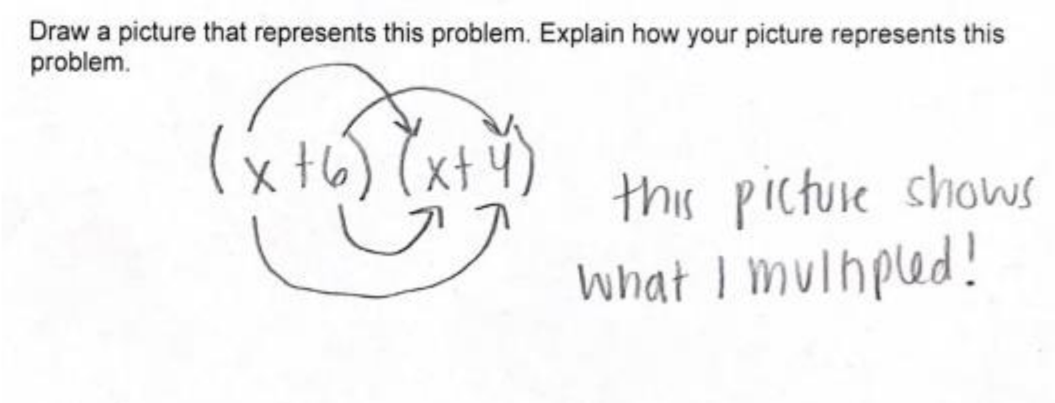
Explain the steps that led you to this answer.

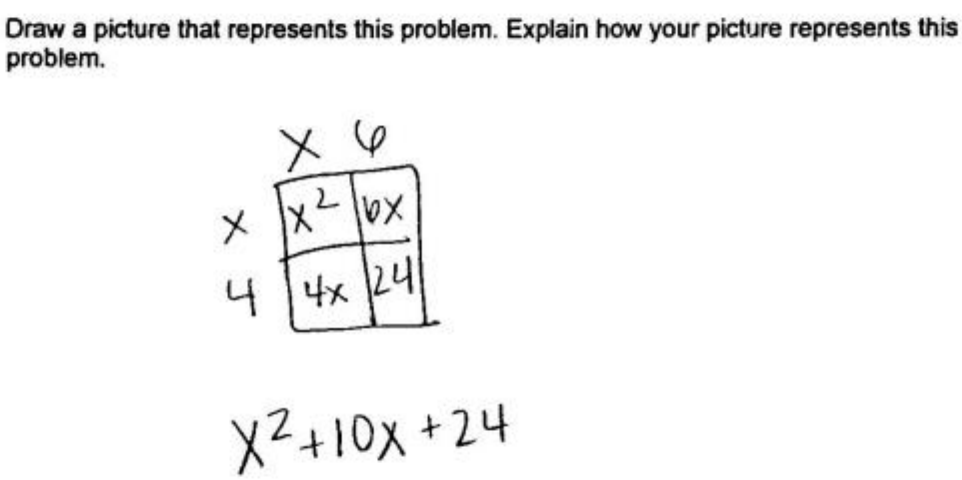
I multiplied the 1st, outer, inner, and last and added pos 3x to neg 7x to get neg 4x

Explain the steps that led you to this answer.

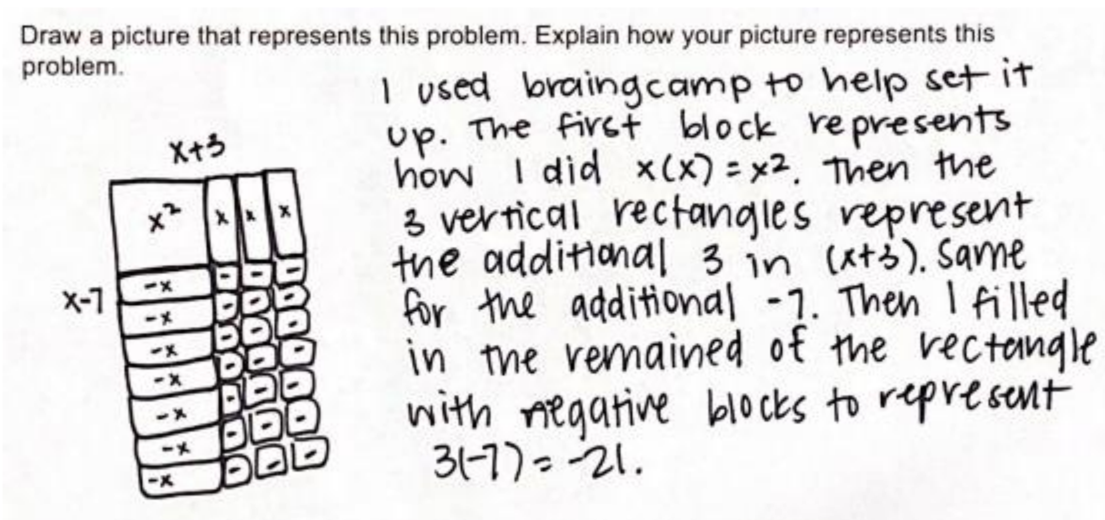
1. FOIL = First, Outside, Inside, Last
2. Simplify

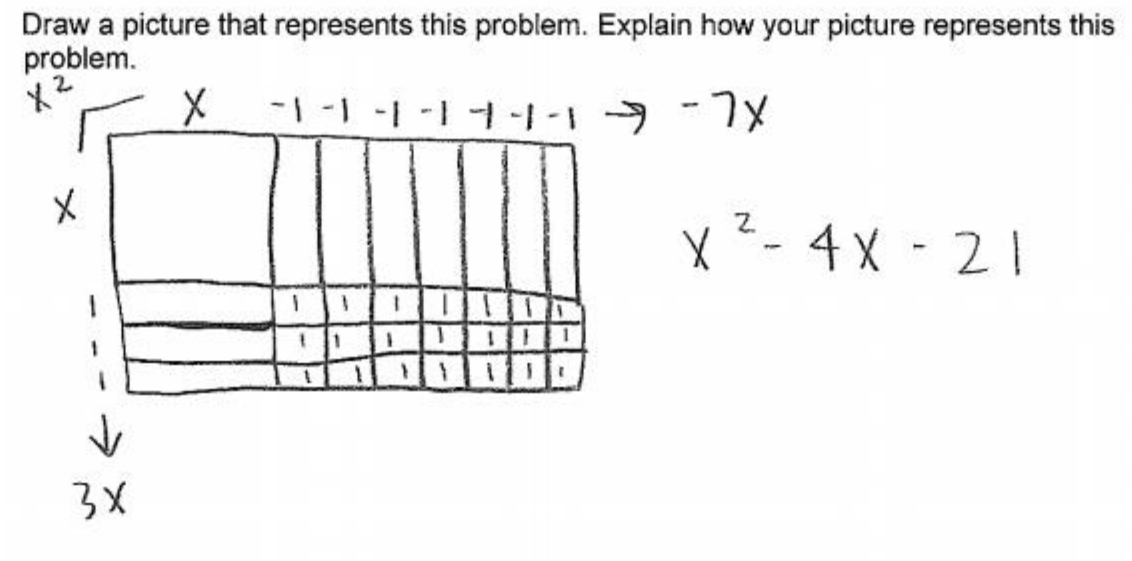
The last question of the three-question series asked students to draw a picture that represented the problem they were given and explain how it represents it. Another key ability that a person with conceptual understanding must have is the ability to make connections between representations of a concept. Here, students were asked to draw a picture that represents the multiplication problem and to explain the connection between their visual and the FOIL Method used in the first question. A virtual class example is pictured first followed by a concrete class example. Here, the student from the virtual class was given a 1 out of 5 for this answer. This score was given here because there was an attempt to answer the question; however, this is not a mathematical picture, there is no rectangle present in the representation, and the explanation given is inadequate and does not explain anything. This answer is something that I also saw in the first question. Students drew arrows to show what was being multiplied by what which can be helpful to ensure that all terms in the first term are distributed to all terms in the second term. Nevertheless, this is not a proper mathematical picture or explanation. The example from a student in the concrete class shown here was given a score of 3 out of 5. Here the student uses another method similar to the first example shown to ensure all terms are distributed properly. The components of the FOIL Method are present, however the areas of the squares that contain x^2, 6x, 4x, and 24 are the same size and there is no explanation present. The square containing 6x, 4x, and 24 do not properly represent themselves. There should be 24 individual ones units, 6 x units, and 4 x units. This example is another method that could be used to achieve procedural fluency, but this does not represent the mathematical picture of the why behind the FOIL Method. The class average for this question on the pre-assessment was a 1.4 demonstrating that students did not have a deep understanding of why the FOIL Method works; they had a shallow conceptual understanding.



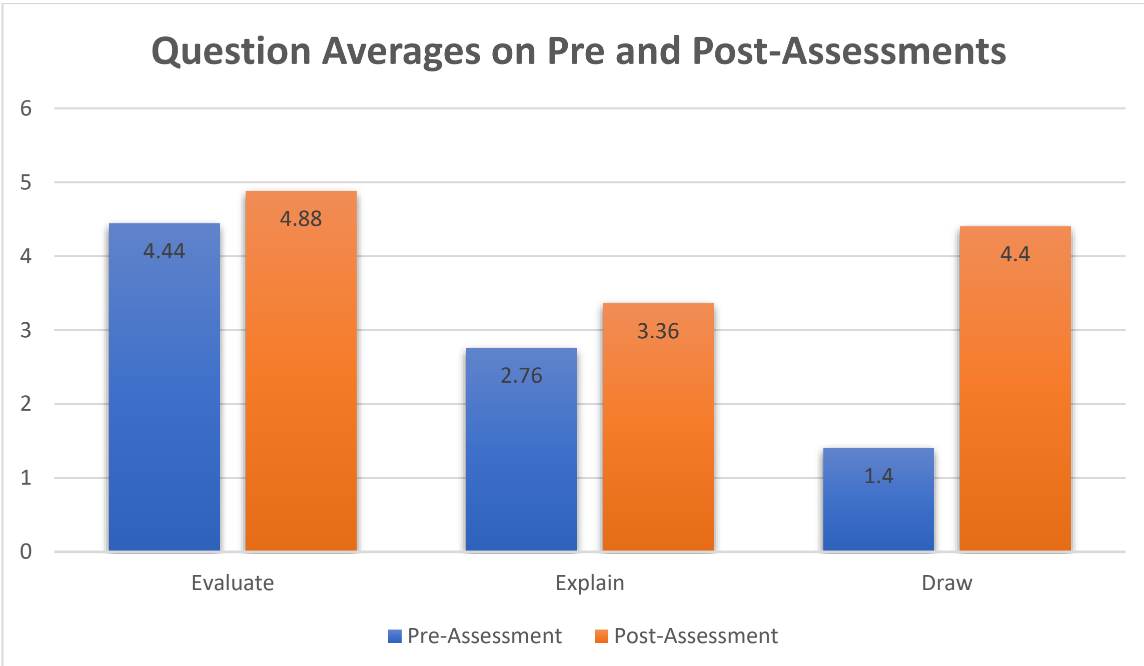


Below are two examples, one from each class, that show the third question of the series on the post-assessment. The first example is a student from the class that used virtual manipulatives. While the lines here may not be precise, this is an excellent example that shows each term as side lengths of a larger rectangle then the subsections that are formed by those. The student used the BraingCamp website to set up the rectangle with the proper side lengths and find the area, the solution to the problem. The student then explains what each size of the smaller rectangles within the larger rectangle represents that correspond to what is multiplied using the FOIL Method. This student received a 5 out of 5 for this question. The second example is from a student in the concrete manipulatives class. The student sets up the rectangle and labels the side lengths properly. The rectangle lacks labeling on the inner pieces and there is no explanation provided, only the picture. For these reasons, the student was given a 4 out of 5. The average for the post-assessment on this question was 4.4 showing tremendous growth in the conceptual understanding of the students.

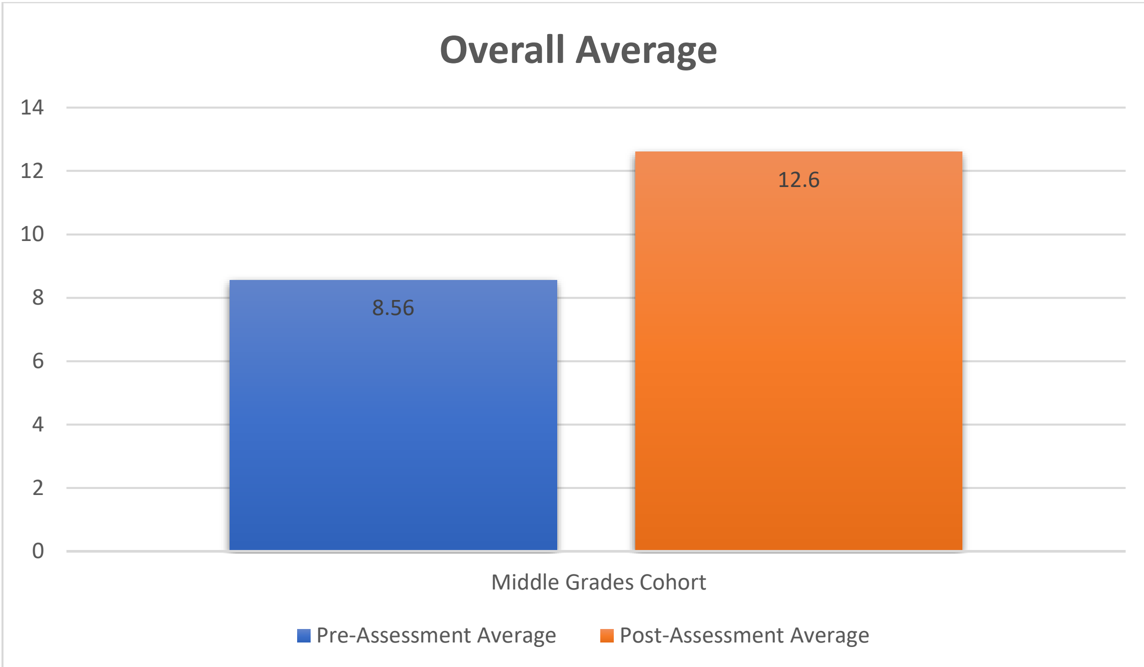




Overall, the students increased their scores from the pre-assessment to the post-assessment. There was only one student whose score did not increase but instead remained constant. Both the pre and post-assessments have a maximum score of 15 consisting of three questions with possible five points each. As a whole, the class average for the pre-assessment was 8.56 with question averages of 4.44, 2.76, and 1.4 respectively on the three series questions of evaluating the problem, explaining the steps, and drawing a picture. This presents that the students have procedural fluency already from their past mathematical experiences, but their conceptual understanding is weaker. On the post test, the class average was 12.6 with question averages of 4.88, 3.36, and 4.4 respectively. This shows that students increased both their overall scores and their scores on each individual question using both the virtual and the concrete manipulatives. This leads to the conclusion that the conceptual understanding of the class as a whole was increased through the use of manipulatives. Below is a graph that shows the pre and post-assessment scores of the whole class. Here, we can see that the students had procedural fluency both before and after the lesson; however, students increased their scores for the explain and evaluate questions demonstrating that they have made connections that were not there before they were taught using manipulatives thus increasing their conceptual understanding.

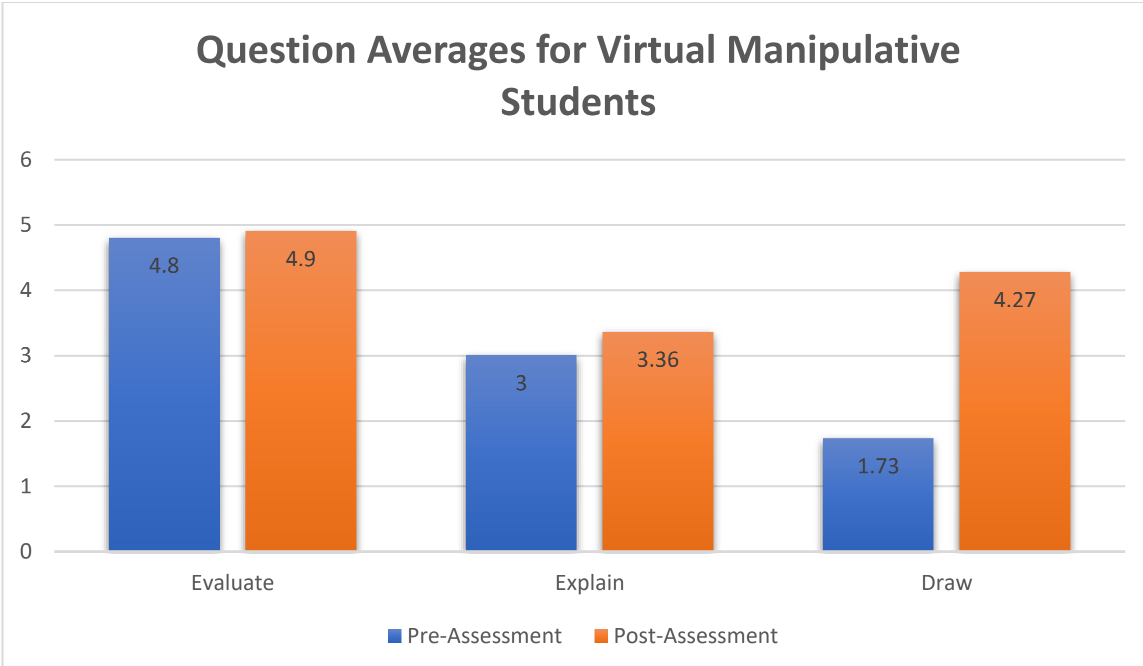


The second and third questions are where conceptual understanding comes into play. In the literature review, conceptual understanding was reviewed and defined as "an integrated and functional grasp of mathematical ideas" where connections between various representations of a mathematical concept show conceptual understanding (Kilpatrick et al., 2001). The scores for both the questions asking for students to explain their steps and the question asking students to draw a pictorial representation of their solution increased from the pre-assessment to the post-assessment. Thus the connections between pictural representation and being able to compute numbers is present more in the post-assessment than in the pre-assessment; therefore, the conclusion can be made that the conceptual understanding of the students has increased. The graph below represents the average scores of the Middle Grades Cohort on their pre-assessment and post-assessment.

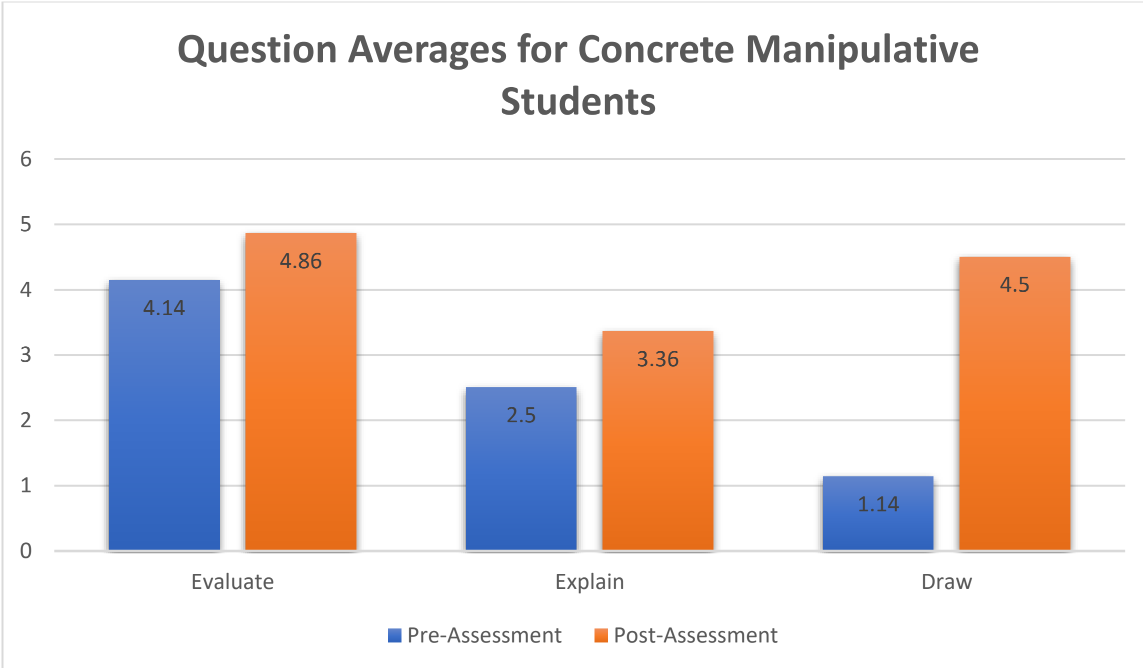


Virtual versus Concrete

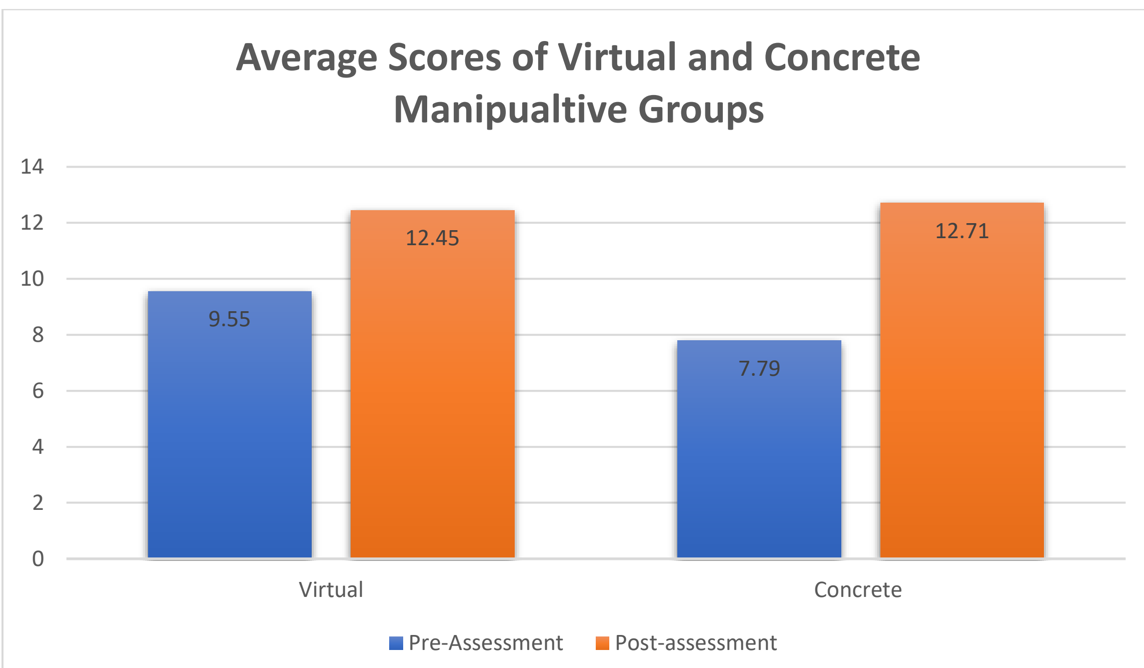
After analyzing the growth of the class as a whole, I wanted to investigate which type of manipulative resulted in more growth; therefore, I looked at each of the two groups individually: students who used virtual manipulatives first and students who used concrete manipulatives first. The pre-assessment average score for the virtual manipulative first group was 9.55 with question averages of 4.8, 3, and 1.73 respectively on the three-question series. This group of students increased their scores to an average score of 12.45 on the post test with question averages of 4.9, 3.36, and 4.27 respectively. One can see that this group increased overall from the pre-assessment to the post-assessment as well as on each individual question. Students increased their individual scores anywhere from staying the same to an increase of 7 points. Below is a graph that shows the averages for each question for the pre and post-assessment for the groups of students that used virtual manipulatives.



The pre-assessment average of the group that used concrete manipulatives first was 7.79 with question averages of 4.14, 2.5, and 1.14 respectively. Each student in this group increased their scores by at least two points on the post-assessment; a couple even increasing by 7. The post-assessment average for the group using concrete manipulatives was 12.71 with question averages of 4.86, 3.36, and 4.5 respectively. Below is a graph that shows the individual question averages for students that used concrete manipulatives first.



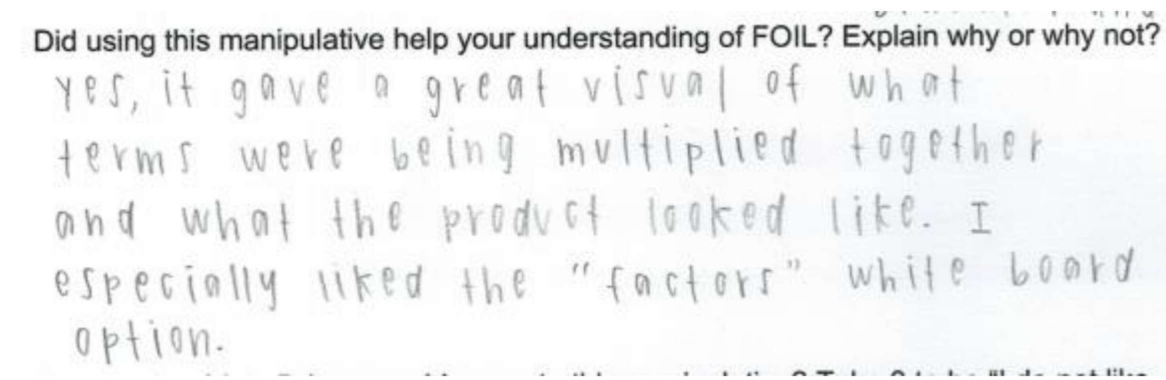
Below is a graph that shows the average scores of the pre and post-assessments of each group of students where virtual represents the students that used virtual manipulatives and concrete represents the students that used concrete manipulatives.



Both groups of students increased their average scores as well as average scores on each question. Between both the concrete and virtual manipulatives groups, there were only two students that did not receive a 5 out of 5 on the first question on the post assessment. This is because the students in the Middle Grades Cohort have already been taught how to multiple binomials and now had a refresh throughout the lesson; therefore, they already have procedural fluency to achieve this kind of an average for this question. To gauge the increase of their conceptual understanding, we look at the scores for the second and third questions. Both groups had an increase in the average for the second question: 0.36 for the virtual group and 0.86 for the concrete group. Both groups also increased their average score for the third question: 2.54 for the virtual group and 3.36 for the concrete group. This shows that on both the second and third questions, the questions we look to for conceptual understanding growth, the students that used concrete manipulatives first increased their averages more although not significantly more. This larger increase is partially due to the fact that the group of concrete manipulative students started out with a lower average score than those that used virtual manipulatives.

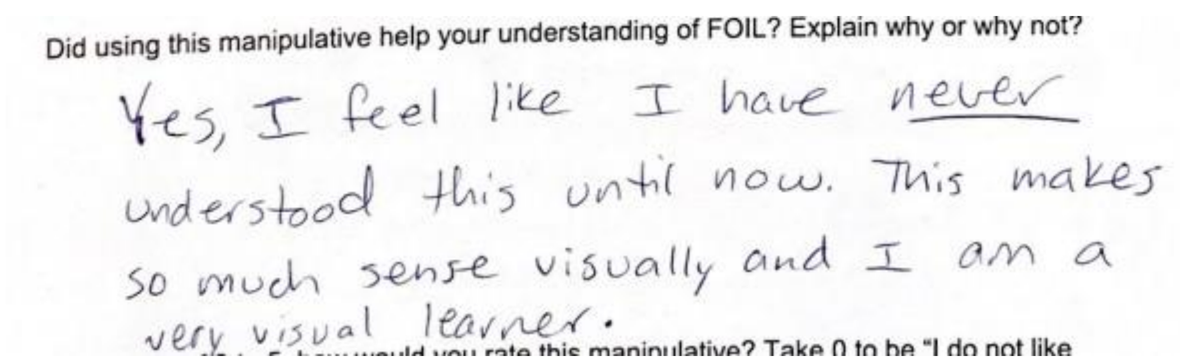
Students were also asked within the post assessment if they thought that the manipulative helped their understanding of the FOIL Method. Of the eleven students in the class that used virtual manipulatives first, eight said that the virtual manipulative did help their understanding of the FOIL Method while three said that they did not. One student's response for the answer no was: "Not really. It honestly confused me more and I like the first, outer, inner, last method better. I see the potential of it but it is not right for me." All thirteen of the students that used concrete manipulatives first thought that they helped their understanding of the FOIL Method. Below are two student responses that thought manipulatives helped their understanding of the

FOIL Method; the first response is from a student that used virtual manipulatives and the second response is from a student that used concrete manipulatives.



Did using this manipulative help your understanding of FOIL? Explain why or why not?

yes, it gave a great visual of what terms were being multiplied together and what the product looked like. I especially liked the "factors" white board option.



Did using this manipulative help your understanding of FOIL? Explain why or why not?

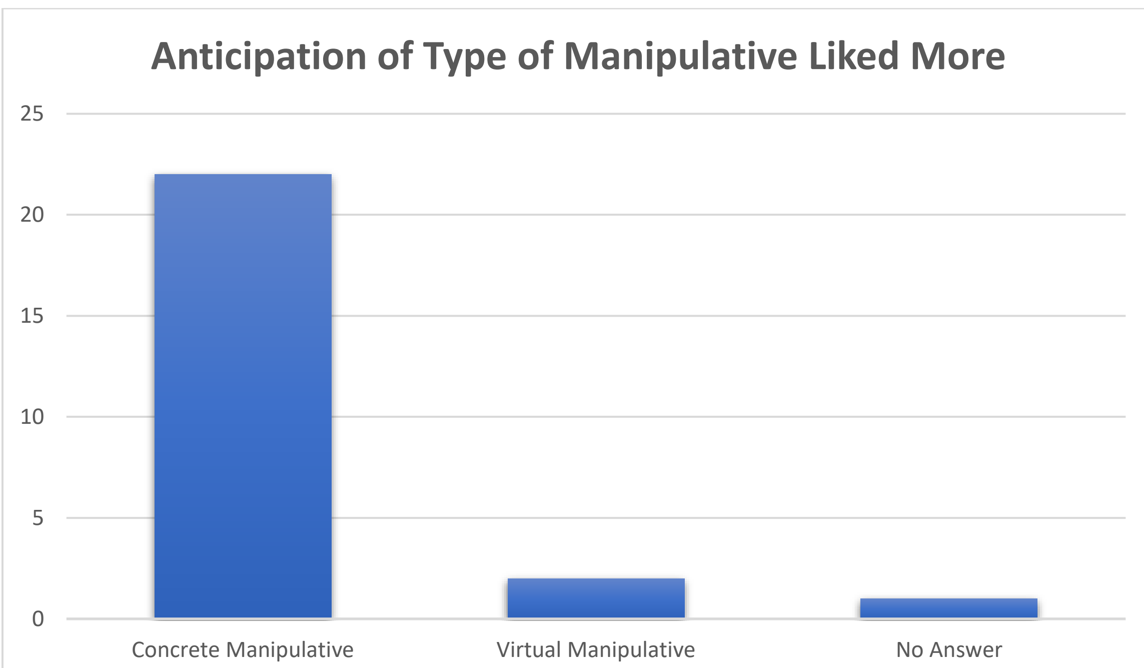
Yes, I feel like I have never understood this until now. This makes so much sense visually and I am a very visual learner.

would you rate this manipulative? Take 0 to be "I do not like

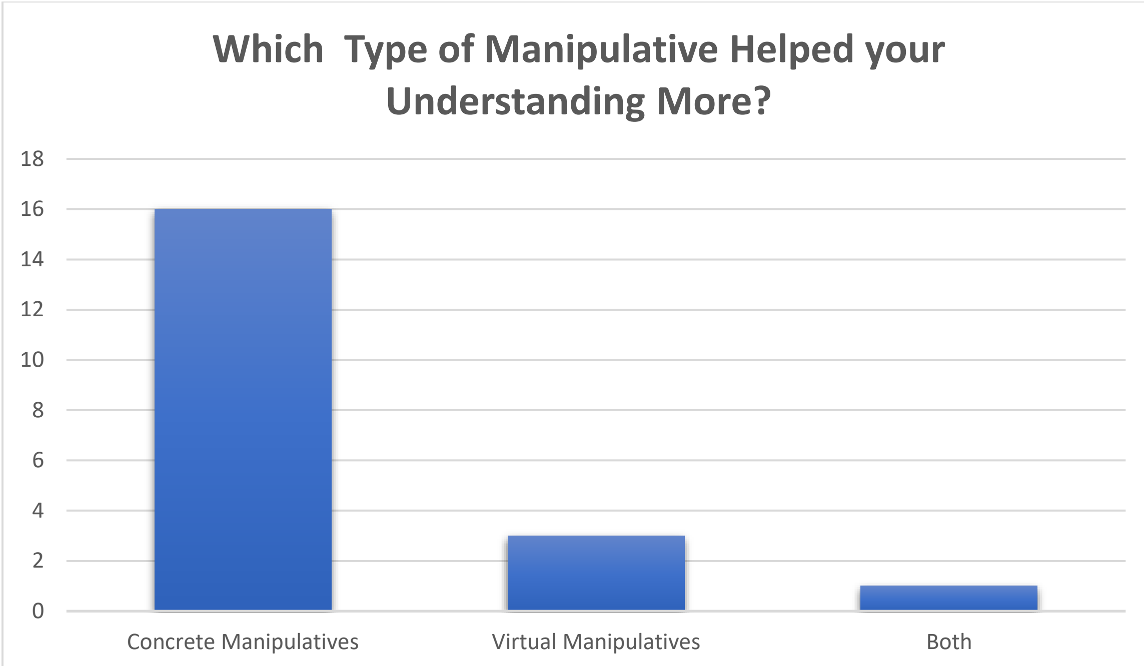
From this, we can see that the students felt that, overall, they gained understanding from the use of manipulatives; however, there was a larger percentage of the concrete manipulative students that felt they helped with understanding versus the percentage of virtual students that felt they helped with understanding. This is consistent with the results of the post-assessment in that the concrete manipulatives increased conceptual understanding slightly more than virtual manipulatives. There are many factors that could be reasons for that such as meeting with the virtual students over a zoom call. Learning from a distance can pose a challenge since the teacher cannot access the students screens to be able to efficiently correct student misconceptions.

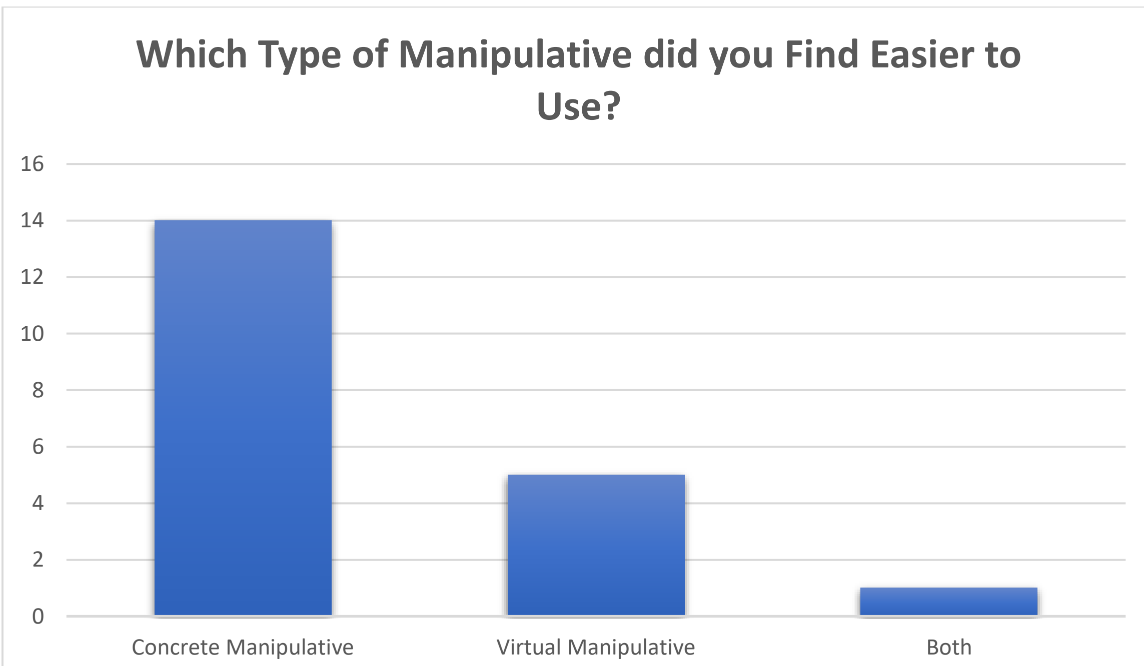
<u>Which type is preferred?</u>

Within the pre-assessment, students were asked which manipulative they anticipated liking more at the conclusion of the classes as well as if they have ever used Algebra tiles before. Eight students answered that they have indeed used Algebra Tiles before while fifteen said that they have not. There was one response that indicated that the student was unsure if they have or have not due to not knowing what Algebra Tiles are, and one that just was not sure. A majority having not used Algebra Tiles before ensures that there will be little bias in answering which they anticipate liking more. There were 22 out of 24 students that answered that they anticipate liking concrete Algebra Tiles more than virtual Algebra Tiles with one student leaving the question blank. Below is a graph of which type of manipulative students anticipate liking more before they have been taught using either manipulative.



In the ending questionnaire, the students were asked which type of manipulative, concrete or virtual, they thought helped their understanding of the FOIL Method more and which they found easier to use. The response to both of these questions was overwhelmingly in favor of concrete manipulatives. Sixteen students found that concrete manipulatives helped their understanding of the FOIL Method compared to three that said virtual manipulatives helped their understanding more. There was one student that responded that both help their understanding of the FOIL Method equally. Students were also asked to give reasons for their answers. Reasons that students gave to concrete manipulatives helping understanding more were that they are physical learners and this type of manipulative was more hands-on and that they struggled to use the virtual manipulatives but not the concrete. On the contrary, students reasoned that the virtual manipulatives displaying the equations and the pieces being labeled helped further their understanding of the FOIL Method more than the concrete manipulatives. The student that responded that both helped their understanding equally also responded that they found both equally easy to use. There were five students that found virtual manipulatives easier to use compared to fourteen that found concrete manipulatives easier to use. Some reasons that were given in favor of concrete manipulatives were that they took less time, were more hands-on, could see the FOIL Method better, were easier to move the pieces around, and that it took less time to figure out than the virtual Algebra Tiles. A few reasons that were given in favor of virtual manipulatives being easier to use were that they create more precise, straighter lines between tiles, they don’t have to be picked up and moved, and that it is easier to check yourself with them since they show the equations. Below are two graphs that show how many students said that concrete, virtual, or both helped their understanding more and which was easier to use.





The majority of students predicted that they would like concrete manipulatives better. Based on the student responses from the Ending Questionnaire, after each group of students had experienced both concrete and virtual manipulatives, students did in fact favor concrete manipulatives. I was surprised that students had such a clear preference for the concrete manipulatives considering the world that we live in is very technology oriented. In order to gain insight into their reasoning, I asked students to list the advantages and disadvantages of each type of manipulative.

Pros and Cons

Within the Ending Questionnaire students were asked to list some of the advantages and disadvantages of each type of manipulative. Within the literature review, some of the advantages and disadvantages of manipulatives in general and each type of manipulative were examined. There were many answers provided by the students that match what I found in my literature review. Some of those include that it is easier for teachers to access student work and student thought through concrete manipulatives, but it is harder to visualize what is happening with the concrete because it is more difficult to keep up with what represents what. This struggle can be alleviated through the use of dry erase boards used in conjunction with concrete manipulatives. Students also identified the problem that there could be a shortage of concrete manipulatives depending on the situation at various schools. They also identified the contrary that virtual manipulatives may not always be accessible since they are used on a computer or other electronic device that a school may not have and both manipulatives have the ability to not be accessible for students at home. Students also pointed out that there may be technical difficulties with virtual manipulatives, but stated that they are better for the COVID situation that is currently happening in that they are good for distance learning and carry less germs. One point that I found in my literature review is that virtual manipulatives can force abstract thinking more, and if a student is not ready for that, they can miss the concept. The students responded similarly when they said that they can be more difficult to understand and that learning can feel shallow with virtual manipulatives. Below is a chart of the responses given by the students for the advantages and disadvantages of both concrete and virtual manipulatives. An asterisk indicates that the advantage or disadvantage was found in both the student response and the literature review.

Concrete Manipulatives	
Advantages	Disadvantages
• More hands-on • Easier to move around • Easier to work in groups • Better teacher accessibility * • Fun and easy to manipulate • Better retention • Easier to make adjustments and correct mistakes • Can try multiple methods • Can use classmates as resources	• May not be enough * • Easy to lose • Expensive • Hard to visualize what they represent * • Irregularity in piece size or missing pieces • Germs • Don't show the values * • Can't save the results* • No immediate feedback * • Negatives and positives are harder to keep up with *

Virtual Manipulatives	
Advantages	Disadvantages
• Better organization • Gives the equation and labels components * • Easy to use for practice • Can be used for any class size • Unlimited materials within the app (never run out of tiles, blocks, etc.) • No germs • Everything is in one place • Can save your results (by screenshot) • No time spent dispensing materials • Can't lose them • Gives guidance *	• Takes longer to use/more time consuming * • Takes time to figure out how the program works * • Not as hands-on- Not as great for kinesthetic learners * • Difficulty moving things around- Less fluid • Less personal and engaging • Learning feel shallower * • Less fun • Technical difficulties • Issue of availability of technology * • Harder to stay focused * • Only one problem at a time- Must clear them after each • Can be harder to understand *

CONCLUSION

Through my research, I found that conceptual understanding of multiplying binomials using the FOIL Method was enhanced through the use of manipulatives whether virtual or concrete. Furthermore, I found that concrete manipulatives increased the students' conceptual understanding of the FOIL Method slightly more than virtual manipulatives did as well as being preferred more to students. Students were able to make connections between the algebraic and pictorial representations of the FOIL Method thus seeing abstract mathematical ideas on a deeper level through visual representations. The students already possessed procedural fluency of this method of multiplying binomials before any lesson was taught; therefore, the use of manipulatives is beneficial to the conceptual understanding of students on all levels of learning. I also found that most of the advantages and disadvantages of previous research to be consistent and applicable to my research including that it is easier for a teacher to access student thought through concrete manipulatives. The fact that virtual manipulatives require time for students to learn the program was also relevant throughout the process of my research. When teaching this lesson to both groups of students, I let students play around with the program and get a feel for it before moving on with the lesson.

I hope that future mathematics teachers will take my research as informational and use manipulatives in their classrooms to deepen their students conceptual understanding of various mathematical concepts. Although each type of manipulative has their own challenges of incorporating into the classroom, students can benefit greatly from their use. Manipulative use at all grade levels should be encouraged by schools, and mathematics teachers should be educated on how to properly use them in their classrooms in order to helps students develop their mathematical skills and understanding.

Appendix A
Pre Assessment

1. Evaluate the following by use of the FOIL method.
   (x+6)(x+4)

2. Explain the steps that led you to this answer.

3. Draw a picture that represents this problem. Explain how your picture represents this
   problem.

4. How often did you use manipulatives in a mathematics class growing up on a scale of 0
   to 5? Take 0 being never and 5 being very often.

   0 1 2 3 4 5

5. Have you ever used algebra tiles?

Yes No

6. Which type of manipulative do you think you will like better?
Physical Virtual

Appendix B

Post Assessment

1. Evaluate the following using the FOIL method.
(x+3)(x-7)

2. Explain the steps that led you to this answer.

4. Draw a picture that represents this problem. Explain how your picture represents this
problem.

4. Did using this manipulative help your understanding of FOIL? Explain why or why not?

5. On a scale of 0 to 5, how would you rate this manipulative? Take 0 to be “I do not like this at all” and 5 to be “I loved this.”

0 1 2 4 5

Appendix C

Grading Rubric

	0	1	2	3	4	5
Evaluate	No attempt made to answer the question	No mathematical logic in the attempt to solve the equation	Mathematical logic is used but not related to the FOIL method	Uses the distributive property but not the FOIL method	Partial use of the FOIL method, but with some errors	No errors; correct use of the FOIL method

Explain	No attempt made to answer the question	Only used words to explain. Explanation has no mathematical logic	Connections to algebra were made, but no use of words	Used words to explain but the connection with algebra was weak	Used words to explain with algebra connections with some errors	No errors; explanation given correctly matches the FOIL method
Draw	No attempt made to answer the question	Attempted to draw but was either not a picture or had no mathematical connection	A rectangle was drawn but has no connection to the dimensions or area in the problem	A rectangle with correct dimensions was drawn; components of FOIL not included	A rectangle with correct dimensions and FOIL components, but has incorrect labeling or no explanation	No errors; A rectangle with correct dimensions and all FOIL components

Appendix D

Ending Questionnaire

Please explain the reasoning behind your answers.

1. Which manipulative (physical or virtual) helped your understanding of the FOIL method more?

2. Which manipulative did you find easier to use?

3. What are some pros and cons that you experienced for each type of manipulative?

	Physical	Virtual
Pros		
Cons		

4. Will you use manipulatives in your classroom? If so, which type do you anticipate using more and why? If not, why not?

RESOURCES

Advantages and Disadvantages. (n.d.). Retrieved from http://manipulativelearning.blogspot.com/p/advantages-and-disadvantages.html

Allen, C. (2007). An action based research study on how using manipulatives will increase students' achievement in mathematics. (). Retrieved from https://files.eric.ed.gov/fulltext/ED499956.pdf.

Benefits of Manipulatives. (n.d.). Retrieved from https://www.hand2mind.com/resources/benefits-of-manipulatives.

Durmus, S., & Karakirik, E. (2006). Virtual Manipulatives in Mathematics Education: A Theoretical Framework. The Turkish Online Journal of Educational Technology. 5.

Johnson, A. (2015). Manipulatives in the Math Classroom. Retrieved from https://minds.wisconsin.edu/bitstream/handle/1793/72373/AnnaJohnson.pdf?sequence=1&isAllowed=y.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). The Strands of Mathematical Proficiency, *Adding It Up: Helping Children Learn Mathematics* (pp. 137-142). Washington, DC: National Academy Press.

Pros and Cons of Virtual Manipualtives. (n.d.). Retrieved from https://bubbl.us/MzAzMDE0MC81NTgyOTgvODIyYWQ1NjM5NjA1YTBhZjM1NjY1ZjEwNTI5ZTVmNzY=@X.

Research on the Benefits of Manipulatives. (n.d.). Retrieved from file:///C:/Users/noelh/Downloads/Research%20on%20the%20Benefits%20of%20Manipulatives.pdf

Rust, A. L. (1999). A Study of the Benefits of Math Manipulatives versus Standard Curriculum in the Comprehension of Mathematical Concepts. Retrieved from https://files.eric.ed.gov/fulltext/ED436395.pdf.

Stein, Mary Kay, Bovalino, Jane W. (2001). Manipulatives: One Piece of the Puzzle. *Mathematics Teaching in the Middle School*. 6 (6), 356-359.

Swan, Paul, Marshall, Linda. (2010). Revisiting Mathematics Manipulative Materials. *Australian Primary Mathematics Classroom*. 15 (2), 13-19.