FIFTH ANNUAL

CAPSTONE DAY

Department of Mathematics

Georgia College

November 19, 2016

8:15 a.m. - 2:15 p.m.

Health Sciences Building

Department of Mathematics
Georgia College

Earning a college degree is a significant achievement and requires dedication and tremendous effort by each student. Several programs have been developed to help students majoring in mathematics to succeed. The First Year Academic Seminar provides an introduction to department faculty, departmental and university expectations, policies, resources and opportunities following graduation. The department conducts informal social activities and presentations by faculty and guest speakers to encourage faculty and student interaction. The department newsletter, Sum News, serves to inform, acknowledge and encourage student majors to become involved in activities related to the major such as mathematics competitions and professional meetings. The academic honor society Kappa Mu Epsilon has been organized to encourage and provide a supporting network for the student body.

Professional schools, businesses, government and industry recognize that mathematics majors are problem solvers and are highly skilled in the use of logic and reasoning. A degree in mathematics opens many careers that are closed to those without quantitative skills. Actuarial science stands as one major example. Moreover, the demand for mathematics in education-especially in secondary schools-is tremendous. In fact, the chronic nation-wide shortage of mathematics teachers is due in part to the demand in so many other areas for talented mathematics majors.

2016 Capstone Day Schedule

8:15 – 9:15 a.m. Registration and Breakfast **HSB Student Lounge (3rd Floor)**

9:15 – 9:25 a.m. Opening Remarks **HSB 300**

9:25 – 10:10 a.m. Session I **HSB 300**

- 9:25 - 9:45 a.m. ***Analyzing the Weight Lifting Performance on the 2000-2016 Olympics Games*****,** Elyse Renshaw
- 9:50 - 10:10 a.m. ***Investigating Proofs of the Quadratic Reciprocity Law*****,** Cuyler Warnock

10:10 - 10:30 a.m. Break **HSB Student Lounge**

10:30 - 11:40 a.m. Session II **HSB 300**

- 10:30 - 10:50 a.m. ***Unique Visualizations: Exploring Symmetries with Complex-Valued Functions and Group Theory*****,** Kelsey Windham
- 10:55 - 11:15 a.m. ***Lie Algebras of Generalized Quaternion Groups,*** Samantha Clapp
- 11:20 - 11:40 a.m. ***The JAM Derivative: An Exploration of Function-Order Derivatives***, Matthew Pearson

11:40 a.m. - 12:45 p.m. Lunch **HSB 314**

12:45 - 1:55 p.m. Parallel Session III **HSB 300**

- 12:45 - 1:05 p.m. ***The Role of Prime Numbers in RSA Cryptosystems*****,** Henry Rowland
- 1:10 - 1:30 p.m. ***Deterministic and Probabilistic Approaches to Card Shuffling*****,** Cassidy Amason
- 1:35 - 1:55 p.m. ***A Comparative Study of the Mathematical Curricula of France and the State of Georgia*****,** Megan McGurl

12:45 - 1:55 p.m. Parallel Session IV **HSB 304**

- 12:45 - 1:05 p.m. ***The Benefits of Writing in a Mathematics Classroom*****,** Darby Bagwell
- 1:10 - 1:30 p.m. ***The Two Types of Assessment Tasks and What They Tell a Teacher*****,** Savanna Cash
- 1:35 - 1:55 p.m. ***Investigating Spatial Visualization Van Hiele Levels and Early Experiences***, Sarah David

2-2:15 p.m. Closing Remarks **HSB 300**

2016 Capstone Day Abstracts

Cassidy Amason
Deterministic and Probabilistic Approaches to Card Shuffling

In this talk we consider the riffle shuffle and analyze the outcomes of deterministic and probabilistic shuffles. Famous sleight-of-hand experts Persi Diaconis and Martin Gardner have contributed widely to this field of study. We first show how to move an individual card to any given position within the deck. This being a matter of skill, we then move to the more common "clumsy" shuffling. Throughout this talk we analyze the properties associated with the group of all permutations for a deck of 52 cards and ultimately determine how much shuffling is required to randomize a deck.

Darby Bagwell
The Benefits of Writing in a Mathematics Classroom

The National Council of Teachers of Mathematics lists communication as one of the five process standards. They also mention writing as a way to improve communication in the math classroom. Despite being recognized as a helpful tool, writing is not typically associated with math. The purpose of this project is to see the benefits that applying writing into a math class can have on the student, the teacher, and the dialogue between the teacher and the student. In this study I applied writing into a college pre-calculus class in the form of journals, and surveyed the students on their views of writing in mathematics. I will present the findings of my research and provide insight into the affect these journals had on the participating students.

Savanna Cash
The Two Types of Assessment Tasks and What They Tell a Teacher

Educational assessment is one of the most essential aspects in determining a student's knowledge and growth. The NCTM assessment principle states, "Assessment should support the learning of important mathematics and furnish useful information to both teachers and students." This study evaluates two types of assessment tasks, open and closed questions, by means of surveys and a created assessment that was given to a pre-service education class here at Georgia College. The purpose of my research is to answer are content specific open-ended questions: said to be easier or more difficult than comparable closed questions, more likely to elicit higher level responses, more susceptible to see misconceptions, and able to tell teachers more information about their students' knowledge and understanding?

Samantha Clapp
Lie Algebras of Generalized Quaternion Groups

For every finite group, there is an associated Lie algebra. The Lie algebra can be viewed as a subspace of the group algebra with certain bracket conditions imposed on the elements. If one calculates the character tables of these finite groups, the structures of the associated Lie Algebra can be described. In this work, we consider the family of generalized quaternion groups and describe the associated Lie algebra structure completely.

Sarah David
Investigating Spatial Visualization Van Hiele Levels and Early Experiences

Van Hiele (1957) determined that learning geometry is experiential. In other words, it is not strictly developmental; it depends on how one engages with shapes and their relationships. Earlier studies of spatial visualization have posited that males are more spatially visual than females (Fennema 1977). However, the literature does not take into account the different environments in which males and females grow up and their variation in childhood experiences. My capstone investigates the possible relationship between early experiences, such as playing video games or sports, and a person's spatial visualization level.

Megan McGurl
A Comparative Study of the Mathematical Curricula of France and the State of Georgia

The purpose of our study is to make a comparison between the high school math content of the state of Georgia and the high school math content of France, whose students perform better at the international level than U.S high school students. A report by the Pew Research Center (2015) found that U.S students are scoring higher on national math assessments than they did two decades ago. However, U.S students' performance is still behind all the other major industrial countries (Desilver, 2015). We use a cross-sectional analysis to investigate the causes of this low performance of U.S students in math. We aim to read the entire math curricula of France and the State of Georgia to identify similarities or any major differences.

Matthew Pearson

The JAM Derivative: An Exploration of Function-Order Derivatives

In Quantum Mechanics the derivative is an anti-hermitian operator in Hilbert Space. In this space this operator can be represented by an infinite dimensional matrix. Considering this, some questions were raised about what could be done with this matrix representation. Letting D represent the derivative operator, we have that $DD = D^2$ represents the second order derivative operator, and thusly D^n represents the nth order derivative operator. Non-integers orders have been considered since the beginning of Calculus. With these defined, it is interesting to consider non-constant orders of differentiation. Letting $f(x)$ be some function, it is interesting to consider the case $D^{f(x)}$, the case of an operator which acts as a derivative of order $f(x)$. Since Quantum Mechanics deals with state vectors in a complex space, we shall define this operator using Fourier transforms on functions expanded into Fourier series. We thus have an operator from the set of complex functions to itself with utility for any periodic or bounded real or complex function. We notate this new operator $J^{\alpha m}$, since within these parameters the operator acts as a multi-ordered variant of the integral operator.

Elyse Renshaw

Analyzing the Weight Lifting Performance on the 2000-2016 Olympics Games

The goal of the first part of this study was to examine weightlifting performance individually between male and female lifters. The goal was to best fit a model that would predict total weight lifted using body weight as well as height. These models can help us compare different size lifters. Our model with height as well as weight best fits the data for women and men. Adding height as a predictor improves the predictive power of our model because it provides an additional index of body size while we previously only considered weight. Height is currently not measured in weightlifting performances, but perhaps in the future, as the popularity of the sport increases, competitions could be split into weight classes as well as height classes.

The second part of the study was to examine the differences in weightlifting performance based on gender differences, differences in weight classes, and differences between countries. The data showed that there are considerable differences in competition level between male and female lifters with male lifters outperforming females for every criteria examined. We show that there is a negative correlation between weight class and percentage difference from 1st place and we show that China is by far, outperforming all other countries.

Henry Rowland

The Role of Prime Numbers in RSA Cryptosystems

Prime numbers play an essential role in the security of many cryptosystems that are currently being implemented. One such cryptosystem, the RSA cryptosystem, is today's most popular public-key cryptosystem used for securing small amounts of sensitive information. The security of the RSA cryptosystem lies in the difficulty of factoring an integer that is the product of two large prime numbers. Several businesses rely on the RSA cryptosystem for making sure that sensitive information does not end up in the wrong hands. This paper highlights the importance of prime numbers in modern day implementations of RSA cryptosystems. We will discuss how an RSA cryptosystem can be successfully implemented, some of the methods used to find primes considered appropriate for RSA cryptosystems, as well as cryptanalysist's techniques for factoring a product consisting of such primes.

Cuyler Warnock

Investigating Proofs of the Quadratic Reciprocity Law

For over 300 years, number theorists have investigated quadratic residues and their properties. A quadratic residue modulo p is an integer a such that $x^2 \equiv a \pmod{p}$ for some $x \in Z_p$. Number theorists such as Fermat, Euler, Legendre, and Gauss were interested in finding conditions for distinct primes p and q so that p would be a quadratic residue modulo q and q would be a quadratic residue modulo p. Through the contributions of these mathematicians, a surprisingly eloquent relationship was discovered: the quadratic nature of p modulo q is the same as the quadratic nature of q modulo p if and only if $p \equiv 1 \pmod{4}$ or $q \equiv 1 \pmod{4}$. This result, first proved by Gauss in the late 1790's, has become known as the Quadratic Reciprocity Law. Since that time, there have been hundreds of different proofs published. These proofs exhibit an astounding variety of methods derived from various branches of mathematics. We will investigate three different proofs, each varying in proof technique and complexity.

Kelsey Windham

Unique Visualizations: Exploring Symmetries with Complex-Valued Functions and Group Theory

Symmetrical patterns are present in many areas such as: architecture, art, music, and mathematics. The connection between math and art has been known for thousands of years. Using Fourier analysis, we construct rosette, frieze, wallpaper, color-reversing wallpaper and color-turning wallpaper functions to generate symmetry groups. In addition, we create unique visualizations of these functions with the help of the domain-coloring algorithm and a software.

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