A Note on the Effects of Elections Subject to Judicial Review

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The most recent U.S. presidential election raises the question of whether agents who believe that losing candidates will contest the results of close elections perceive their vote as more important, and whether they will be more or less likely to vote. The analysis involves a two-player voting game with one of the players initially in a weaker position because they lose all ties. The key variable is the probability of the "weaker" player winning the post-election judicial review process. The relationship between this probability and the level of voting for the weaker player is non-monotonic. Also, if this probability is sufficiently close to one-half, contesting close elections leads to increased voting, lowering social welfare.

Introduction

A common refrain in the aftermath of the last U.S. presidential election was "If you ever doubted your vote counts, this close election shows that it does." However, the nation did not experience simply a close election but one that one side contested with the outcome ultimately determined by the judicial system. Contrary to the above sentiment, it is not immediately obvious how agents will react if they believe they are now in an environment with a substantial probability that close elections will be contested and placed under judicial review. Given the course of events, one could argue that the above refrain is false and that an individual vote is even less important.

Consider a close election that is not subject to judicial review. Clearly a vote can have a marginal impact since it might be the tiebreaker or "pivotal" vote. The probability that the vote would be tied is negligible in an election involving a large number of people.[1] Nevertheless, given a tie, an individual vote is decisive. Consider a scenario similar to the last presidential election. If the vote is not close, it stands but if the margin of victory is close to zero, the loser contests the results and the outcome is decided in some other arena such as a judicial or legislative body. A tie-breaking vote is not decisive because a one-vote victory is contested. In the first case I know with certainty my vote will give my side victory, but in the second there is only a probability of victory and that probability is the same whether or not I vote. My vote truly makes no difference and is a waste of resources not only during a landslide but also during a close election. A rational voter should be even less likely to go to the polls.

However, there are at least two counter arguments. First, legislators are themselves elected and judges are either elected or appointed by elected officials. A broader view that considers not a single election in isolation but all elections together suggests there might be increased voting. For example, an election for county judge today could affect the outcome of an election for governor or President tomorrow. The President elected now will affect the makeup of the Supreme Court, which could in turn affect the selection of a future President. This argument presents a bundle of votes in a variety of elections, and is not inconsistent with the idea of a single vote in a single election becoming less important. The fact that a vote in a particular election matters less could induce people to vote more in a variety of elections. Another possible outcome is that agents will gravitate toward two extremes, those who never vote and those who vote all the time.

A second counter argument is the possibility of an election that is neither a landslide nor close, but "almost close." In this case an individual vote could alter the outcome by giving the losing side a "fighting chance." There is still the possibility of a pivotal voter when the vote count is on the margin of becoming contestably close. To illustrate this possibility, suppose there is a community of 20 people voting on a proposition, where the proposition fails if a majority votes NO or the vote is tied. If the margin of victory is two votes or less, the loser contests the election and the courts make a ruling which determines the outcome. If 11 have voted YES and eight NO, a voter who wants the proposition to fail would have a strong incentive to vote. If they abstain, the proposition will pass with certainty. But if they vote NO the vote is contested and there is a positive probability that the proposition will fail. The individual's vote has a marginal impact that may lead to more voting.

Suppose the losing side will contest all future elections with a narrow margin of victory and agents know this. A priori, it is unclear what net effect this will have on voting. Depending on the circumstances of the election, some voters may have a stronger incentive to vote but others may have a weaker one. Some of the factors affecting the outcome would include the proportions of the population for and against a candidate or proposition, the expected voter turnout for each side, polling data before the election, and early exit poll results. Incorporating these and all other relevant variables into a formal model would be challenging.

The model presented in the present paper simplifies by assuming that one player represents each side of a proposition and voting takes place simultaneously. The outcome of the election determines which of the two players receives a differential benefit. One of the players starts out in a weaker position because she loses all ties. If the election cannot be contested, then her vote has no impact when the other player votes. Since the election outcome does not affect efficiency and voting is costly, a social planner who cares only about efficiency would prefer less voting to more. In fact, such a social planner would prefer to avoid voting altogether and use a coin flip. From an efficiency perspective, voting is a waste of resources because the outcome is the same regardless of the level of voting. This view of politics as a zero-sum game, although not without controversy, is well established and goes back to at least Riker [1962]. In reality, agents may feel more satisfied, or less dissatisfied, with a policy or law if they feel it is the result of a democratic process. The model below ignores this possible effect of voting on social welfare.

The main goal of the present paper is to formalize the effect of the possibility of judicial review on voting behavior, particularly for an agent whose vote is irrelevant when the other player votes.[2] The analysis starts with a benchmark model of uncontestable elections and proceeds to a model of contestable elections subject to judicial review.[3]

A Simple Model

There are two players Yea (Y) and Nay (N) who are on opposing sides of a proposition, where Y benefits if the proposition passes, and N benefits if it fails. The proposition passes if Y has a majority and fails if N has a majority or there is a tie. The players move simultaneously and choose one of two actions: VOTE or ABSTAIN. Voter Y is at a disadvantage since she wins only if she votes and N abstains from voting: {ABSTAIN, VOTE}. However, N wins in the remaining three cases. Let B be a parameter that represents the importance of the election to the players, victory yielding a differential benefit $B > 0$. In other words, the disutility of losing is normalized to zero and the utility of winning is B. A second parameter, $C > 0$, represents the cost of going to the polls. Assume $B – C > 0$ to allow the possibility of voting. Notice that in this environment Y's vote has no impact on the outcome given that N votes. That is, if N votes, a vote by Y is irrelevant and a waste of resources.[4]

A close election is defined as a tied vote that occurs if they both VOTE or both ABSTAIN. In the first game the results of a close vote stand, and Y's side accepts the results of the election.[5] This game has the following payoff matrix.

		Yea	
		VOTE	*ABSTAIN*
Nay	*VOTE*	B-C, -C	B-C, 0
	ABSTAIN	0, B-C	B, 0

If we only consider pure strategies, there is no Nash or dominant strategy equilibrium. There is a mixed strategy Nash equilibrium analogous to the totally mixed strategy equilibrium of Palfrey and Rosenthal [1983, 1985]. Player N is indifferent between voting and abstaining if

$$P_Y = C/B, \tag{1}$$

where P_Y is the probability that Y votes. Likewise, Y is indifferent between voting and abstaining if $P_N = 1 - C/B$, where P_N is the probability of N voting. A Nash equilibrium exists if Y votes with probability C/B and abstains otherwise, and if N votes with probability $1 - C/B$ and abstains otherwise. In this equilibrium it turns out that the respective probabilities of voting sum to one, with Y's probability of voting equal to N's probability of not voting and vice versa.[6] As the likelihood of N voting decreases, voting becomes more worthwhile from Y's perspective.

The size of B relative to C determines which player has the greater probability of voting. In particular, if $C/B < 1/2$, then $P_Y < P_N$. Otherwise, Y has the greater probability of voting. Assume that the benefits of winning the election are more than twice the cost of voting (B > 2C), which implies $P_Y < 1/2 < P_N$.[7] The differential benefit of victory relative to the cost of voting determines which player has the higher probability of voting. Consider that N only needs to vote if Y votes. Otherwise, he can choose ABSTAIN and still receive B. An increase in C/B makes voting less attractive for N. The only way for N to be indifferent between voting and abstaining is if voting becomes more necessary, i.e. if it is more likely that Y will vote.

Both players vote probabilistically, and the following represents social welfare:

$$W = P_N P_Y (B - C - C) + P_N (1 - P_Y)(B - C) + (1 - P_N) P_Y (B - C) + (1 - P_N)(1 - P_Y) B,$$

which reduces to

$$W = B - C. \tag{2}$$

Social welfare is the benefit of winning minus the cost of voting. Since B is the differential benefit of winning and there is always a winner and a loser, society always enjoys an expected total benefit of B. The probabilities of N and Y voting sum to one and the average probability of voting is one-half, so that the expected voting cost incurred by both players is C.

Elections Subject to Judicial Review

Consider a second game where a tied vote is contested and Y wins the review process with probability λ, the degree to which the judicial review process favors Y. Accordingly, this game has the following payoff matrix:

		Yea	
		VOTE	*ABSTAIN*
Nay	*VOTE*	(1-λ)B-C, λB-C	B-C, 0
	ABSTAIN	0, B-C	(1-λ)B, λB

The equilibrium depends on the value of λ, with a higher λ giving more power or influence to the weaker player Y. It might be expected that Y becomes more likely to vote as λ increases. However, this relationship is non-monotonic, and Y will vote with certainty only if λ is sufficiently close to one-half.

Case One: $\lambda < C / B$

In this case, the probability of Y winning a contested election is too small to alter the best responses. For example, given that N chooses VOTE, Y still prefers to choose ABSTAIN since $\lambda < C / B \Rightarrow \lambda B - C < 0$. Again, there is one mixed strategy Nash equilibrium where

$$P_Y^1 = \frac{C - \lambda B}{B(1 - 2\lambda)} < C / B, \tag{3A}$$

and

$$P_N^1 = \frac{(1-\lambda)B - C}{B(1 - 2\lambda)} > 1 - C / B. \tag{3B}$$

As before, probabilities sum to one, and the average probability of voting is still one-half. There is no change in social welfare but there is a transfer of expected utility from N to Y. In the original game N has expected utility B – C and Y has expected utility of zero. But in this contestable game, Y's expected utility increases to $\frac{\lambda(C-\lambda B)}{(1-2\lambda)} > 0$.

Although the total level of voting does not change, it is interesting to note that Y is always *less* likely to vote compared to the game without judicial review. Furthermore, Y's expected utility increases despite that fact that she is less likely to vote. The introduction of λ increases Y's influence or "vote income" which can improve her expected utility through the probability of winning and the expected cost of voting. It is possible for one of these two variables to remain constant. If Y votes with the same probability as before, her voting costs remain the same but there is a better chance of receiving B. Alternatively, Y can reduce her voting costs by lowering her probability of voting in such a way that she is just as likely to receive B. It is also possible for both the probability of winning and the expected cost of voting to change. In this range of λ , at least some of Y's increased utility comes from a reduction in expected voting costs.

Case Two: $C/B \le \lambda \le 1 - C/B$

In this case, there is a pure (dominant) strategy equilibrium {VOTE, VOTE} and the overall level of voting doubles, where

$$P_Y^2 = P_N^2 = 1. \quad (4)$$

There is no mixed strategy equilibrium.[8] Both players vote with certainty and social welfare is

$$\widetilde{W} = B - 2C > 0. \quad (5)$$

Social welfare is lower than in case one because the social benefit of voting remains the same but the level of voting doubles. The chances of winning after contesting the election are high enough to induce Y to vote with certainty but not so high to revert back to a mixed strategy equilibrium.

Y's expected utility is $\lambda B - C$, and linear in λ throughout the interval $[C/B, 1-C/B]$, ranging from zero to $B-2C$. It is possible for Y's expected utility to be lower in this case than in case one, implying that a higher λ can harm Y and benefit N.

Case Three: $1-C/B<\lambda$

There is a mixed strategy equilibrium similar to case one, with

$$P_Y^3 = \frac{\lambda B - C}{B(2\lambda - 1)} > 1 - C/B \text{, and} \tag{6A}$$

$$P_N^3 = \frac{C - (1-\lambda)B}{B(2\lambda - 1)} < C/B\,. \tag{6B}$$

The chances for victory for Y are so high that there is a role reversal with Y voting with a higher probability than N. In fact, Y votes with a higher probability in this case than does N in the original game, but neither the overall level of voting nor social welfare differs from case one.

The expected utility for Y is $\frac{\lambda(\lambda B - C)}{(2\lambda - 1)}$, and always greater than in case two since $B - 2C < \lambda B - C < \frac{\lambda(\lambda B - C)}{(2\lambda - 1)}$. To see this, notice that the first inequality reduces to $1 - C/B < \lambda$ and the second inequality holds if $\lambda < 1$.

Discussion and Extensions

Overall voting and social welfare does not change in cases one and three relative to the benchmark model. Only in case two, when λ is close to one-half, does total voting increase, causing social welfare to decrease. This result is due to what some may consider cynical assumptions, namely that an election without voting costs is a zero-sum game and the act of voting incurs non-negligible costs greater than any psychic benefit. Some may think that voting costs are negligible [Tullock 2000] or that more voting is a good thing. However, the present model ignores the substantial costs that contesting a vote may involve. For example, Florida had to bear the high cost of the recounts after the U.S. presidential election in 2000.

In regard to the effect on the weaker voter Y, the relationship between λ and the probability of Y voting is non-monotonic. To summarize,

$$P_Y^1 < P_Y < P_Y^3 < P_Y^2 . \tag{7}$$

It is possible for a contestable election to lower Y's probability of voting. If λ is too small, Y's influence has increased just enough to induce N to vote with a higher probability to protect himself, making winning less likely for Y and causing her to vote with lower probability. Y votes most often when the agents are evenly matched and the mutual best response is to always vote. If λ is too large, Y becomes the stronger player but the imbalance in influence is large enough to revert back to a mixed equilibrium.

The expected utility for Y is also non-monotonic in λ, but it is always (weakly) greater than zero, her expected utility in the standard election. When λ = C/B, Y's expected utility is zero and she is no better off with contestable elections. It is also true that Y can be worse off when λ is greater than C/B. The worst case scenario for a social planner who cares about equity as well as efficiency is when $\lambda = C/B + \varepsilon$, where $\varepsilon > 0$, because Y's expected utility is close to zero while social welfare is lower.

The present analysis could be generalized to multiple players on each side of the proposition. Such an extension to the benchmark model would most closely resemble the participation game of Palfrey and Rosenthal [1983] in which there are $F > 1$ and $G > 1$ players on sides Y and N respectively. In the relevant version of their game, voting costs are identical, the payoff of winning is symmetric, ties are broken in favor of one side, and the cost of voting is less than one-half the benefit of winning. The added wrinkle is that there is a free rider problem because all members of the winning side enjoy the benefits of victory, including those who chose not to vote. Also, it may not be clear which side is in the weaker position. Even when ties are broken in favor of the members of N, one also has to consider the size of F relative to G.

Palfrey and Rosenthal [1983] show that multiple player games have no pure strategy equilibria but a multiplicity of mixed strategy equilibria. One class of equilibria involve all members of one side playing the same mixed strategy, a proportion of the other side voting with certainty, and the remainder abstaining with certainty. Another class of equilibria involves all the members of both Y and N using mixed strategies. There is an equilibrium similar to the benchmark model with all members of Y voting with probability P_Y, and all members of N voting with probability $P_N = 1 - P_Y$. Since incorporating judicial review λ changes a basic assumption of their paper, it is not clear whether the present results are robust to the addition of multiple players for each side. A reasonable conjecture is that the present results remain intact if F and G are equal.

If the model were to endogenize the behavior of candidates, there is a possible relationship between candidate behavior and λ. For instance, the median voter model generally assumes all agents vote, at variance with the present model in which voters often use mixed strategies of participation. When λ is far away from one-half, candidates would prefer appealing to the side with greater participation. However, when λ is close to one-half, they would choose positions more consistent with those predicted by the median voter hypothesis.

The model could also be extended to incorporate more features of actual elections and to test robustness of the present results. Instead of giving ties to one of the players, a coin toss could be used with both players starting on equal footing. Heterogeneous voting costs, non symmetric gains from winning, or uncertainty about other players' costs or benefits could be included. Another possibility is to endogenize contestability with a sequential game in which candidates decide whether or not to contest an election after the vote.

Conclusion

The paper used a two-player voting game to examine the effect of judicial review for close elections on overall voting, social welfare, and the voting behavior and expected utility of the disadvantaged player who initially loses all ties.

The possibility of judicial review might be expected to increase voter turnout, particularly for agents unlikely to win a standard election but empowered by a chance to win due to a judicial ruling. The effect on voting behavior is not obvious, however, and depends on the perceived chances of winning the post-election review process. Unless the probability of winning such a contest for the weaker voter is sufficiently close to one-half, total voter turnout and social welfare remain the same as in an election with no possibility of judicial review. If the probability is close to one-half, the overall level of voting doubles and social welfare falls. In this case, the increased voting due to judicial review is a deadweight loss.

One might also expect that as the bias of the judicial review process towards the weaker player increases, her expected utility and level of voting also increase. However, it is possible for an increase in this bias to lower her expected utility, as well as decreasing her likelihood of voting. It is also possible for the weaker player to vote with a lower probability than in an election with no possibility of judicial review.

Footnotes

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[1] Owen and Grofman [1984] formally show how extremely small this probability is when there are many voters. As Tullock [2001] recently stated, “The probability that an individual’s vote will influence the presidential election is lower that the probability that the prospective voter will be killed in an automobile accident on the way to the polls.”

[2] See Ledyard [1981, 1984] and Palfrey and Rosenthal [1983] for early papers with non-cooperative games of voting.

[3] The mechanics of judicial review and the degree to which it may favor one of the players are exogenous to the model. It is beyond the scope of this analysis to discuss the legal reasoning behind a review, or why the election results should or should not stand.

[4] This would not be the case if the tie-breaking rule were a coin toss.

[5] The decision to contest the election is exogenous to the game presented, so that contesting is done by others and is not part of the players’ action sets.

[6] This feature of the model is a result of the homogeneous cost of voting, and disappears if voting costs are heterogeneous.

[7] If B is normalized to one, this condition becomes $C \in (0, ½)$.

[8] When solving for a mixed strategy in this parameter space, one of the two voting probabilities is negative, which is not possible. When λ equals one-half, the probabilities are undefined.