Content Knowledge vs. Pedagogical Knowledge

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Senior Research Capstone

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December 15, 2017

INTRODUCTION

For my study, I am examining the role of an instructors' background on the impact of a student's mathematical learning. Specifically comparing an instructor with substantial content knowledge with an instructor with more pedagogical knowledge. I will attempt to uncover how these instructors prepare a lesson and enact their lesson in the context of a modified teaching experiment. I will also examine student knowledge gains in the topic of Rate of Change for each teaching experiment. The content knowledge instructor will be an undergraduate mathematics tutor and will have taken a variety upper-level mathematics courses, such as MATH 3000 and above. The Pedagogical instructor will be an undergrad in the middle grades cohort and have taken courses from the College of Education, core mathematics courses, and three courses in mathematics education.

A teaching experiment consists of a series of teaching episodes. There are four basic elements within teaching experiment methodology: A teaching episode includes an instructor, one or more students, a witness of the teaching episodes and a method of recording the entire teaching episode. The experiment will be analyzed using the Mathematical Knowledge for Teaching (MKT) framework. This framework consists of two categories, which should illustrate the differences between the math major instructor and the middle grades instructor. One category covers subject matter knowledge, which focuses on knowledge of mathematics and the ability to connect topics throughout the curriculum. The other category covers pedagogical content knowledge, which puts its focus on the knowledge of students and the ability to prepare and teach material. I am interested in researching the impact of an instructor's educational background on their lesson plans, enacted lessons, and student performance.

This is an important topic to me because I am truly interested in its conclusions. I want to become the best teacher I can be and I feel this study will help not only better myself, but better those who came before me as well as those who come after me. This includes the structure of the lesson plan and questioning. This topic has been a big interest of mine throughout the research I completed to find a topic. I have spent many years in the education system and have begun to notice a difference in teachers involving their educational background. Each participant will benefit from this study in different ways. The pre-service instructor will gain experience developing and carrying out a lesson plan, which is a valuable and practical experience for those considering the teaching profession. The mathematics undergraduate instructor will be able to learn the topic at a deeper level, and develop her tutoring skills. Student subjects will review content from the middle grades standards and likely learn the material more deeply or conceptually.

The larger mathematics education community would benefit from this study because it will further knowledge on the impact of strict content knowledge versus pedagogical knowledge development in the preparation of teachers. We will have further implications for teacher education and how we prepare teachers to most meaningfully teach their student.

METHODS

My goal for this experiment is to attempt to uncover the role that an instructors background plays on the students' learning and what they each have to offer. My research will include an instructor with a strong mathematical background and an instructor with a predominantly pedagogical background. My methods will involve the instructors and two student participants involved in a teaching experiment. A teaching experiment consists of a series of teaching episodes. There are four basic elements within teaching experiment methodology: A teaching episode includes an instructor, one or more students, a witness of the teaching episodes and a method of recording the entire teaching episode. The recording can then be used to analyze the data. The instructors will include a pre-service teacher in the middle grades cohort and a mathematics major, who is also a tutor. The procedures are as follows: I will speak to a eighth grade teacher whom I know through an education placement spring 2017. I will speak to the instructor to find two students who come from a similar background who I feel will provide minimal differences in the experiment. I will find my instructors through a university in the southeast. One of whom will be majoring in mathematics and an active tutor. The mathematics major would have taken (6) 3000 - 4000+ level mathematics courses with no pedagogical courses. I will find my pedagogical instructor through the mathematics education undergrad program. They will be a middle grades major with a concentration on mathematics. The middle grades majors will have taken four courses specific for mathematical knowledge for teaching and many pedagogical courses.

	Pedagogical Content Instructor	Mathematics Content Instructor
Mathematical Core Classes	➢ Intro to Mathematical Modeling (MATH 1101) ➢ Probability and Statistics (MATH 2600)	➢ College Algebra (MATH 1111) ➢ Pre-Calculus (MATH 1113) ➢ Calculus 1 (MATH 1261) ➢ Calculus 2 (MATH 1262)
Mathematics Upper Level Classes	➢ Linear Algebra (MATH 2150)	➢ Linear Algebra (MATH 2150) ➢ Calculus 3 (MATH 2263) ➢ Foundations of Mathematics (MATH 3030) ➢ Abstract Algebra (MATH 4081) ➢ Mathematical Analysis (MATH 4261) ➢ Complex Variables (MATH 4300) ➢ Differential Equations (MATH 4340) ➢ Probability (MATH 4600) ➢ Mathematical Statistics (MATH 4620) ➢ Women in Mathematics (MATH 4950)
Pedagogical Classes	➢ Invest Critical Content Issues in Ed (EDUC 2110) ➢ Exploring Socio-Cultural Perspectives (EDUC 2120) ➢ Exploring Learning & Teaching (EDUC 2130) ➢ Writing About Literature (EDUC 2200)	➢ None
Mathematical Pedagogy Classes	➢ Mathematical Investigating (MAED 3100) ➢ Concepts in Algebra (MAED 4080) ➢ Concepts in Geometry (MAED 4510)	➢ None

Each instructor will be provided with the same topic from the middle grades standards, to cover in their lesson. The instructors will be allowed time to create a lesson plan. The experiment will be supervised and observed by note taking, video recording, along with a pre/post quiz. The pre/post test will help me understand the student better. I will be able to test their previous knowledge as well as discover their mathematical disposition. Video recording will be used so that further analysis can be done after the teaching session. The session will be observed to see how the students learn as well as their mathematical disposition. I have also attached a survey I will give to the student after the lesson has taken place to get their personal opinion on the lesson. To analyze the data, I will use a rubric during each instructor's lesson to analyze his or her work in real time. A copy of the rubric is located at the end of this section. This rubric contains the elements of the Mathematical Knowledge for Teaching (MKT), framework. This framework consists of two categories, which will separate the math major instructor from the middle grades instructor. One category covers subject matter knowledge, which focuses on knowledge of mathematics and the ability to connect topics throughout the curriculum. The other category covers pedagogical content knowledge, which puts its focus on the knowledge of students and the ability to prepare and teach material. This survey will not include the student's name to help keep this experiment confidential. There will also be a type of pre/post quiz. The topic of this quiz will depend on the teacher's order of topics since I do not want to cover something that has already been taught. There will be two types of questions covered in the quiz, procedural questions and conceptual questions. An example of a procedural question would be: Find the equation of the line that passes through the points (0,1) and (2,3). An example of a conceptual question would be: The equation at which a potted plant grows is $H = 3.5t + 2$. Explain what each value in the equation means in this situation. Lastly, I will want to interview the instructors after their sessions. I have included some examples of questions that will be asked. The quiz and survey will help me to better understand the impact the instructors had on their students. I will use this information as a reference when I am determining what each instructor made possible during their teaching session with their students.

LITERATURE REVIEW

MATHEMATICAL KNOWLEDGE FOR TEACHING

Mathematical Knowledge for Teaching is about knowing mathematics from the standpoint of helping others learn. To help others learn one must be mathematically ready to teach an idea or method (Ball, 2011).

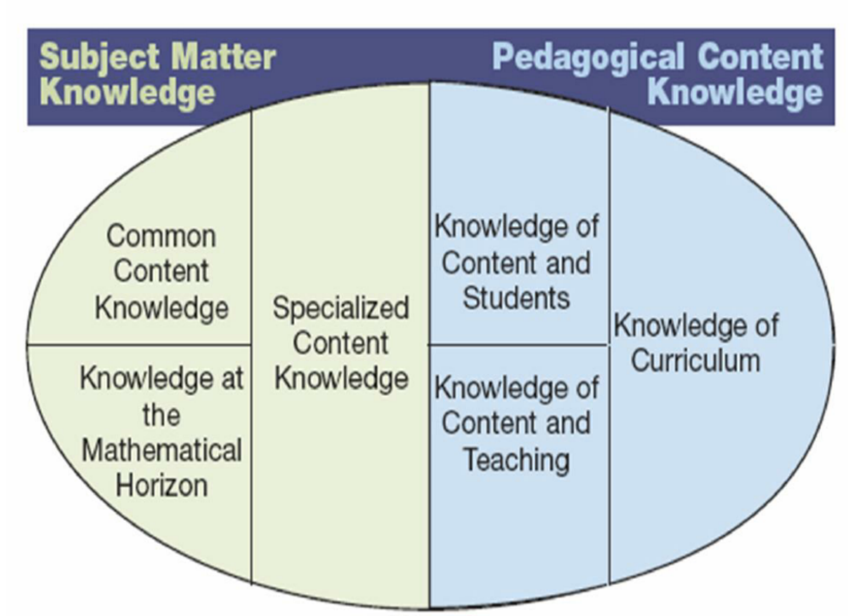

Figure 1: Mathematics Teaching and learning to Teach: MKT Framework (2010)

The diagram above shows the two categories into which the MKT framework is broken down. For example, Common Content knowledge refers to the knowledge needed to solve problems, Knowledge at the Mathematical Horizon measures the awareness of how mathematical topics are related over the span of mathematics included in the curriculum, and Specialized Content Knowledge is the mathematical knowledge and skill unique to teaching. Knowledge of Content and Students is the knowledge that combines knowing about students and knowing about mathematics, Knowledge of Content and Teaching combines knowing about teaching and knowing about mathematics, and Knowledge of Curriculum refers to the knowledge and preparation of materials. This tool is important for examining the impact of each instructor's educational background on their teaching style. The ideas from this framework were also incorporated into a rubric that was used in analyzing the instructors during their lesson. These tasks include providing accurate mathematical explanations and representing mathematical ideas.

MATHEMATICAL STRANDS OF PROFECIENCY

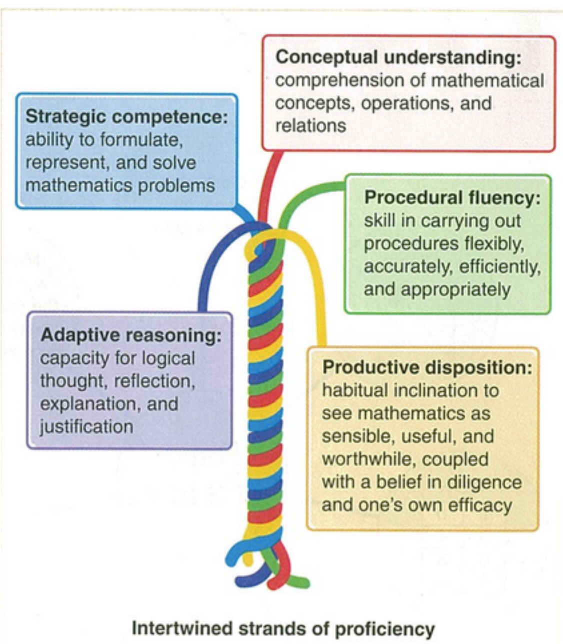

Figure 2: Mathematical Strands of proficiency (2000)

These strands represent the strands of mathematical proficiency. In my study, I focus on the concepts of Conceptual Understanding and Procedural Fluency. With Conceptual Understanding "Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge". Procedural Fluency is "more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation." (NCTM, 2000, p. 20). These concepts were imperative for the analysis of the student's mathematical knowledge gains. The other three strands of mathematical proficiency are more likely to be observed in classroom settings but are more difficult to observe and make assumptions about in a session where the teacher is working one on one with a student.

QUESTIONING

Driscoll states that “Productive instruction includes making it a habit to ask a variety of questions aimed at helping students organize their thinking and respond to algebraic prompts.” and "Giving well-timed pointers to students that help them shift or expand their thinking, or that help them pay attention to what is important.” (1999, p. 3) According to Driscoll, there are five types of questions an instructor asked during a lesson. These are Managing, Clarifying, Orienting, Prompting Mathematical Reflection, and Eliciting Algebraic Thinking. Clarifying questions are intended to help the student clarify their responses. Orienting questions are used to help the student get the student started. Questions that prompt mathematical reflection are intended to ask the students to reflect. Questions that elicit algebraic thinking asked students to describe functional relationships. The table below shows more about each what each type of question is and provides some examples for each.

Question Type	Examples
• Managing: Intended to help set students on task, get their work organized, etc.	Who's in charge of writing it down? What are you doing now?
• Clarifying: Intended to request information from the student when the teacher is not clear about what the student means or intends	Do you know what perimeter is? How did you get 2? Who went first?
• Orienting: Intended to get students started or keep them thinking about the particular problem they are solving	What's the problem asking you to find? Have you thought about trying a table? How did you get 18?
• Prompting Mathematical Reflection: Intended to ask students to reflect their thinking.	How do you explain that? Can you explain how you got the values in the table?
• Eliciting Algebraic Thinking: Intended to ask the students to undo, to build rules for describing functional relationships.	What could it (the value in the equation) represent? What does -2 mean? Can you look for a pattern?

Driscoll, 1999

COGNITIVE DEMAND

Stein, Silver, Smith, and Henningsen (2009) describe low cognitive demand as something that requires memorization and following procedures that are given to the students. Low cognitive demand questions ask students to recall material verbatim that has been presented earlier. An example of this would be if the students were given a formula sheet and asked to complete certain exercises by using the formulas. Tasks with high cognitive demand, on the other hand, "require students to make connections between and among mathematical ideas in new ways". In other words, questions and activities that require a higher cognitive demand ask students to work with bits of information learned earlier to generate an answer with well-structured evidence. A good example of this, for middle grades students, would be a hands-on activity in which they can learn more in-depth and make a larger amount of connections.

STUDENT CENTERED VS. INSTRUCTOR CENTERED

Van de Walle, Bay-Williams, Lovin, Karp (2014) describe two types of interactive approaches within a classroom. They include a student-centered approach and an instructor-centered approach. A student-centered lesson is one where teachers begin where the students are with the students' ideas. The focus is placed more on what the student knows and how to build on that information. This type of approach requires a more flexible lesson. The teacher would go off on tangents related to the student's questions but still ultimately reach the goal of the lesson.

An instructor centered lesson is one in which the student has to do the problem the same way every time. Students do not have the opportunity to apply their own ideas or to see that there are numerous ways to solve the problem. This type of approach has a more structured lesson. The goal will be stated and there would be no variation. The instructor may acknowledge a student's question but they would not explore it much if it does not follow their plan.

PROCESS STRANDS FOR MATHEMATICS

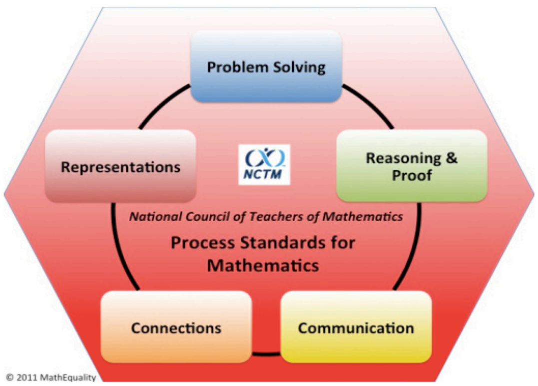

Figure 3: Process Standards for Mathematics (2011)

Figure 3 represents the NCTM's five process strands for doing mathematics: Problem Solving, Reasoning and Proof, Communication, Connections, and Representations. Problem solving involves the process of applying a variety of appropriate strategies based on the information provided, references, recalled, or developed. It is important to involve problem solving so that the student can apply their own thoughts and build from what they know. By giving a student a problem and allowing them to solve it with minimal instruction allows the student the opportunity to learn it the way that works best for them. Reasoning and Proof is about making and investigating mathematical conjectures and developing arguments and proofs. Making conjectures allows the student to work through multiple examples to find a pattern. Once this pattern is assumed they can then begin to work through logical arguments and properties to prove that this assumption would actually work for every case they can come into contact with. The Connections Strand involves recognizing and using connections among mathematical ideas as well as with other subjects. This helps the students see how other topics relate to one another. Communication focuses on organizing mathematical thinking coherently and clearly to peers, teachers and others. Communication is important for the instructor to know what the students are thinking. This allows for the instructor to adapt their lesson to the way the student will learn best. The opportunity for communication is usually most prevalent within a student-centered lesson. Recall that a student centered lesson focuses on what the student knows and how to build on that information. A lesson that involves good communication between the student and instructor is usually prevalent in a student centered lesson. Lastly, Representations are created using multiple representations to organize, record, and communicate mathematical ideas. By incorporating multiple representations within a lesson, the students are provided with multiple ways to learn a new topic or idea.

ANALYSIS

I investigated the impact of an instructor's educational background on their lesson plans, enacted lessons, and student performance. This includes the structure of the lesson plan and questioning. I analyzed the lesson plans regarding their structure, the opportunity for process strands, intended level of cognitive demand, and whether they anticipated types of questions asked. Figure 1 shows a comparison of the instructors' lesson plans. The pedagogical instructor's lesson followed a student-centered approach, allowing the opportunity for the student to communicate their mathematical ideas, while the content instructor follows a more instructor-centered approach. Questioning techniques were not explicitly addressed within either of the instructor's lesson plans. The pedagogical instructor's lesson included a battleship activity which seemed to have potential for high cognitive demand. The intent was for the students to make

Pedagogical Instructor	Content Instructor
• Mostly consists of student centered work • Questioning: o Questioning techniques are not really addressed within the lesson plan	• Mostly instructor centered work • Questioning: o Questioning techniques are not really addressed within the lesson plan

Figure 1

conclusions based on the information created by playing battleship. This activity used procedures with connections to help the student gain a better understanding. The content instructor's lesson seemed to require a low cognitive demand. The intent was for the student to work with equations and known procedures to solve for the slope of a line. This uses procedures without connections.

I also analyzed the lesson plans in regards to the NCTM's five process strands. The following refer to figure 2 above. The pedagogical instructors lesson seemed to allow problem solving from the students through the production of graphs and tables while the content instructors lesson involved giving the students the equations needed to solve the problems. Neither instructor appeared to incorporate reasoning and proof in their lesson. Having a one-on-one lesson would provide ample opportunity for communication between each student to their instructor. Although, communication was not explicitly addressed within the lesson plans. The pedagogical instructor plans make at least one connection to the real world while the content instructor does not appear to make any. The representations the pedagogical instructor plans to use involves pictures and graphs to help the students visualize what they are doing. The content instructor will provide equations and graphs for the student to use. Figure 3 comes from the pedagogical instructor's lesson plan. This is the

Pedagogical Instructor	Content Instructor
• The battleship activity seems to have potential for high cognitive demand from the students o The intent is for the student to be able to make conclusions based on the information created when playing battleship. The goal is to use procedures with connections to gain a better understanding. • NCTM Process Strands o Problem Solving – Lesson plan appears to revolve around looking at graphs and tables. o Reasoning and proof – Does not appear that this will be incorporated. o Communication – Having a one-on-one lesson will provide ample opportunity for communication between the student and the instructor. o Connections – Instructor will make at least one connection to the real world, i.e. the battleship activity. o Representation – Instructor will use pictures of graphs to help the student visualize what they are talking about.	• The lesson seems to require a low cognitive demand from the students o The intent is for the student to work with equations and known procedures to solve for the slope of a line. This falls under the category of procedures without connections. • NCTM Process Strands o Problem Solving – Students are given the equations needed to solve the problems. o Reasoning and proof – Does not appear this will be incorporated. o Communication – Having a one-on-one lesson will provide ample opportunity for communication between the student and the instructor. o Connections – There does not appear to be any connections to the real world or to other mathematical topics. o Representation – Instructor will provide the student with graphs as a visual for what they are talking about.

Figure 2

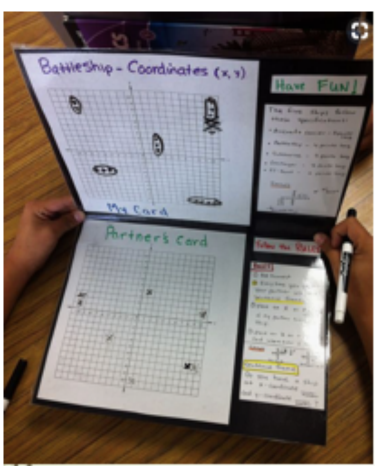

Figure 3

battleship activity the instructor included in her lesson. This type of activity tends to be more conceptual since the students are able to learn and connect ideas through a hands-on process. The content instructor planned on giving the students a worksheet with vocabulary and formulas for the student to solve.

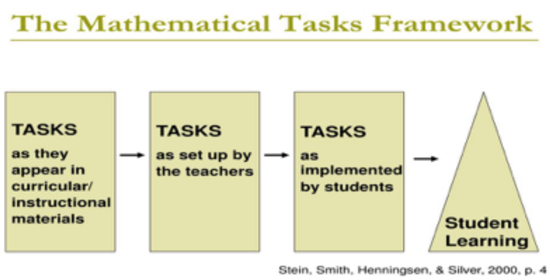

Figure 4

Because lesson plans do not always go the way they are intended I also chose to analyze the enacted lesson plans. By observation, each area of knowledge, pedagogical or content, gives a more in-depth description of the background of each instructor. Having either of these mathematical knowledges allow the instructors to implement certain teaching tasks. Figure 4 shows a framework used for analyzing mathematical tasks. The first box addresses how the task was intended in the curricular materials; since I did not have access to these materials I only dealt with the last three. I viewed the tasks as set up by the instructor, observed how they were implemented by the students, and analyzed the students' learning. Figure 5 shows the comparison of the questioning techniques within the enacted lesson and whether the lesson was student-centered or instructor-centered. The enacted lesson from the pedagogical instructor seemed to be more student-centered. The lesson revolved around what the student knew and how to build on that

Pedagogical Instructor	Content Instructor
• The work seemed to be more student-centered rather than instructor centered. • Questioning: o clarifying questions ➢ Ex: How did you get the slope of this graph? o orienting questions ➢ Ex: If this is what the function looks like, would you say that I am moving anywhere? o Questions that prompt mathematical reflection ➢ Ex: What if the graph looked like this instead? Do you think these would look different? Show me. o Questions that elicit algebraic thinking ➢ Do you know what this means?	• The enacted lesson was a mixture of instructor centered work and student-centered work. • Questioning: o orienting questions ➢ Ex: Do you know what the y-intercept of a line is? Do you know what the negative means? o Questions that elicit algebraic thinking ➢ Ex: What does the -1 mean? What does the 2 mean? Do you know what the negative means?

Figure 5

knowledge. The content instructors lesson was a mixture of both. The beginning of the lesson was more instructor-centered due to a lack of participation from the student. But as the lesson went on and the student grew more comfortable, the lesson became a little more student-centered. The pedagogical instructor asked many different types of questions including: Clarifying, Orienting, prompting mathematical reflection, and eliciting algebraic thinking. The content instructor revolved around two types of questioning: Orienting and Eliciting algebraic thinking. Examples of the instructors' questions asked are located in Figure 5. Within the enacted lesson, the pedagogical instructor was unable to include the battleship activity due to time constraints. However, the lesson still involved a high cognitive demand based on the work they did complete. The student seemed to gain a better understanding using procedures with connections. The content instructor's lesson required low cognitive demand due to the fact that the student was required to state known facts and use equations to solve their problems. The student gained a better understanding using procedures without connections. The pedagogical instructor's student was able to problem solve through the process of constructing graphs whereas the content instructor's student was given formulas to construct graphs. The instructor provided the student with a lot of answers which

Pedagogical Instructor	Content Instructor
• The battleship activity was not introduced to the student. The lesson still involved a high cognitive demand from the students based on the work they did together ○ The student seemed to gain a better understanding using procedures with connections. • NCTM Process Strands ○ **Problem Solving** – Student was able to learn about slope through the process of constructing graphs. ○ **Reasoning and Proof** – The idea of reasoning and proof was not directly incorporated in the lesson. Questions were answered and explained, but only on a minimal level. ○ **Communication** – Lesson was organized and allowed for good communication between the instructor and the student. There was opportunity for the student to clarify mathematical thinking. ○ **Connections** – Instructor provided the student with multiple real world connections. There were connections between representations. ○ **Representations** – They began showing the student graphs which allowed them to visualize what they were trying to accomplish with the equation. **Graph -> Equation**	• The enacted lesson required low cognitive demand from the students ○ The student was required to state known facts and use equations to solve their problems. The student gained a better understanding using procedures without connections. • NCTM Process Strands ○ **Problem Solving** – Student used formulas to construct their graphs. Instructor provided the student with a lot of answers which hindered the student's opportunity to problem solve. ○ **Reasoning and proof** – The idea of reasoning and proof was not directly incorporated in the lesson. ○ **Communication** - The amount of communication between student and instructor grew as time went on. Instructor used more precise language when defining slope. There was opportunity for the student to clarify mathematical thinking. ○ **Connections** - Instructor provided the student with at least one real world connections but did not provide any connections to other mathematical topics outside of slope. There were connections between representations. ○ **Representation** – They began showing the students how to figure out slope and y-intercept using the slope-intercept equation. From this they instructed their student to create a graph to see what they had done with the equation. **Equation -> Graph**

Figure 6

hindered the students' opportunity to problem solve. The idea of reasoning and proof was not directly incorporated in either lesson but when questions were asked to the pedagogical instructor the questions were answered and explained but on a minimal level. The pedagogical instructor's lesson was well organized and allowed opportunity for the student to clarify mathematical thinking. Communication between the content instructor and their student grew as the time went on. Opportunity for the student to clarify mathematical thinking was present. The pedagogical instructor provided the student with multiple real-world connections while the content instructor only provided a few. There were connections between representations with both instructors. Both instructors used equations and graphs as representations in their lessons. One interesting aspect of this was the fact that the pedagogical instructor started with graphs and moved to equations while the content instructor started with equations and moved to connect them with graphs.

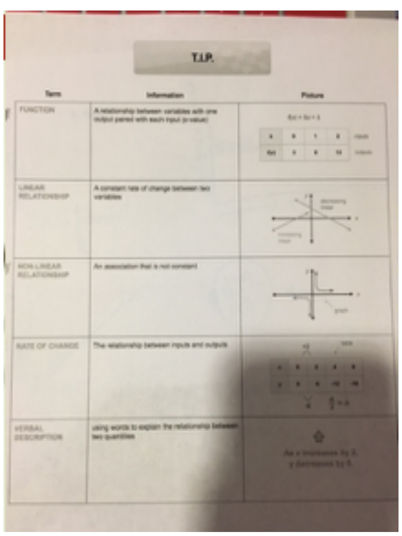

Figure 7

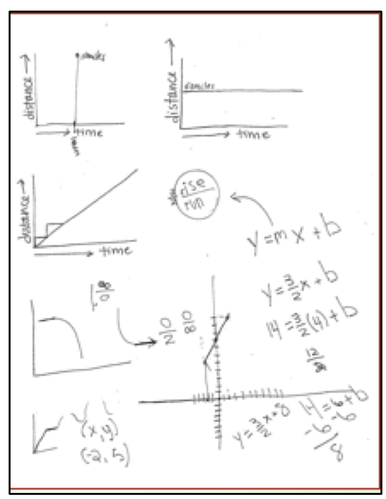

Figure 8

Figures 7 and 8 are from the pedagogical instructors enacted lesson. Figure 7 is a handout that was given to the student to refer. T.I.P. stands for Term, Information, and Picture. The student is given the opportunity to see the term and how it relates to functions. They are also able to see what it would look like when applied to a graph or chart. Figure 8 is the sheet the instructor used to describe the concept of slope. During the lesson, the instructor would ask the student questions and would write down the student's thoughts and responses.

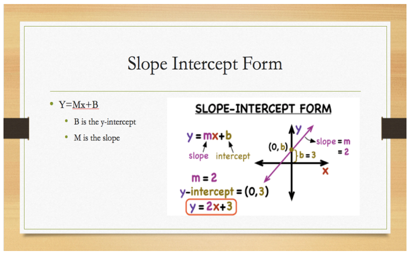

Figure 9

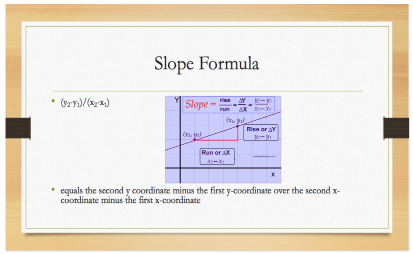

Figure 10

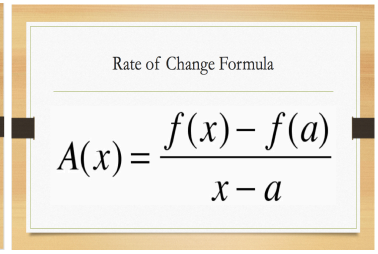

Figure 11

Figures 9, 10, and 11 were pulled from the content instructors lesson. Notice how the slides focus on equations, formal mathematics, and a procedural approach.

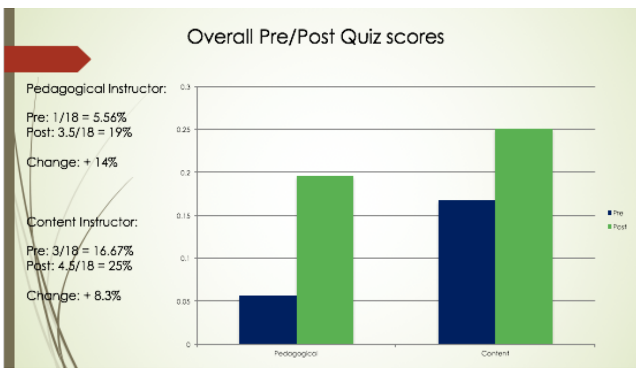

Figure 12

Figure 12 represents the overall scores of the pre and post quizzes given to each student before and after the lesson. The students were given the exact same questions before and after the lesson. The pedagogical instructor's student scored a 1/18 or 5.56% on the pre quiz and a 3.5/18 or 19% on the post quiz. This was about a 14% additive increase. The content instructor's student scored a 3/18 or 16.67% on the pre quiz and a 4.5/18 or 25% on the post quiz. This was an 8.3% additive increase. Notice how the content instructor's student scored better overall on the post quiz but the pedagogical instructors student saw a larger improvement from the pre to post quiz. I believe this means that the content instructor's student had a better understanding of the material coming into the lesson and learned a little bit but the pedagogical instructor's student saw a greater increase in learning the material overall.

The questions on the pre and post quizzes were broken up into two categories: conceptual and procedural. Figure 13 shows the results of each question type from the quizzes. For the conceptual questions, the pedagogical instructor's student scored a 1/16 or 6.25% on the pre quiz and a 2.5 or 15.6% on the post quiz. This is a 9.3% additive increase. The content instructor's student scored a 3/16 or 18.75% on the pre quiz and the same on the post quiz. This indicates that there was no change in the conceptual knowledge of the content instructor's student. For the procedural questions, the pedagogical instructor's student scored 0/2 or 0% on the pre quiz and a 1/2 or 50% on the post quiz. This was a 50% additive increase. The content instructor's student scored a 0/2 or 0% as well on the pre quiz but a 1.5/2 or 75% on the post quiz for a 75% additive increase. One important aspect to take into account is that there were many more points for the conceptual problems than there were for the procedural ones. Also, notice there is no blue bar under procedural for either instructor. This is because both students scored a 0% on the pre quizzes. This information shows that each instructor had a different impact on their student's learning.

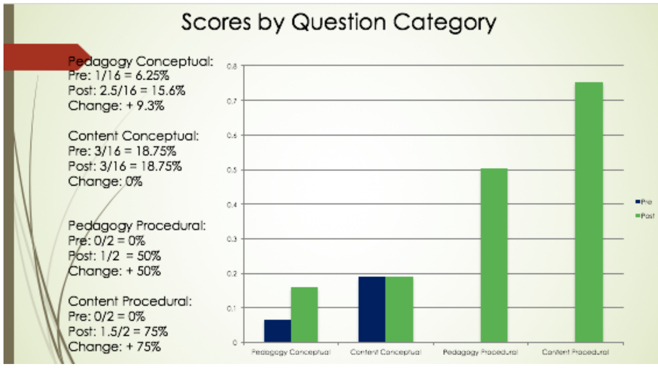

Figure 13

CONCLUSIONS & IMPLICATIONS

Each instructor had an impact on a certain part of their student's learning. The pedagogical instructor had a larger impact on the student's conceptual knowledge, taking advantage of the one-on-one atmosphere to build upon the student's prior knowledge. By providing the opportunity for the student to communicate their mathematical ideas the instructor was able help the student in the best way possible for them to learn. The content instructor had a larger impact on the student's procedural knowledge, providing the student with formulas and multiple opportunities to practice doing examples using the formulas. Each instructor spent different amounts of time preparing for the lesson. It became obvious that each instructor focused on different aspects of learning when preparing. The pedagogical instructor followed the student's responses whereas the content instructor stayed more within their plan.

LIMITATIONS

Time became an issue since the students were given pre and post quizzes they only received about 35-40 minutes of instruction time. Having more instruction time would have allowed more time for the students to process the information. Since there was only one session for each instructor the results may not have been as reliable. With more sessions, we may have seen a greater impact on lessons and student performance. In future research, I would like to hold a sequence of sessions for the teaching experiment. Ideally, I would have liked to hold at least one more session about a week later in order to assess how much the students truly learned. Having this knowledge would benefit the education of future instructors by providing a more direct path for the preparation of teachers.

IMPLICATIONS

I feel as though strictly pedagogical courses prepare teachers well for early childhood and early middle grades. Younger students do not always require the instructors to go deep into a topic. Receiving general pedagogical knowledge will allow for the instructors to teach their students simpler topics. An instructor would not need to explain to the students why something is at a high level. As students get older and begin to ask more questions, strictly pedagogical courses would begin to hinder the instructions ability to delve deep into certain topics. As this becomes an issue, mathematics education would become very beneficial. Not only will the instructors be able to teach the topic, but they themselves will understand the material at a deeper level. Their preparation would consist of learning how to teach as well as learning how to teach mathematics. During their time in a mathematics education path, they will learn about MKT and how to implement it into their classrooms.

I believe the best path for future instructors involves a combination of math education and upper level math courses. Following on a mathematics degree with a focus on teaching would be beneficial for future educators. Having the deep understanding of mathematics will help the instructor to be able to explain topics at a more in depth level while the math education courses would help them to know how to teach them. Having a deep understanding of the content will help the instructors be able to explain topics in multiple ways. Taking upper level math courses will allow the future educators the opportunity to see many topics in new ways while learning how to prove many important ideas in each of these topics. Having the pedagogical knowledge would help the instructors know the students and understand the best way to instruct them so they have the greatest opportunity to learn.

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