

Darby Bagwell

CAPSTONE

Dr. Santarone

November 19,2016

The Benefits of Writing in a Mathematics Classroom

INTRODUCTION

In the standard K-12 classroom, writing and mathematics are not typically associated with one another. It was not until my sophomore year of college that I began to realize how strongly math and writing are related. This immediately stood out to me. Why are writing and math separated for 13 years of our education and then all of sudden thrown together in every upper-level course? As I continued through my classes, I began to realize how truly helpful writing can be. This thought led me to wonder, if writing was always a part of mathematics would students have a better understanding of the concepts behind the computations they are taught. My purpose behind this research is to explore the benefits writing could have for students and teachers, and to see if applying writing in these earlier classes can further students' conceptual understanding of mathematics.

RESEARCH

In the Principles and Standards for School Mathematics, the National Council of Teachers of Mathematics lists communication as one of the five process standards. They state "Instructional programs from prekindergarten through grade 12 should enable all students to organize and consolidate their mathematical thinking through communication, communicate their mathematical thinking coherently and clearly to peers, teachers, and others, analyze and evaluate the mathematical thinking and strategies of others, and use the language of mathematics

to express mathematical ideas precisely” (NCTM, 2000). When expanding on why communication is essential they state “Students who have the opportunities, encouragement, and support for speaking, writing, reading, and listening in mathematics classes reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically” (NCTM, 2000). Despite this being a standard for mathematical teaching, writing and mathematics are not typically associated with one another. However, writing can be an extremely helpful tool when learning mathematics.

Before approaching why writing is helpful, we must first look at what writing in a math class could look like. There are many different types of writing in the math classroom. Vicki Urquhart divides the types of writing into the purposes they serve for the students. She says, students write to keep on-going records about what they’re doing and learning, they write in order to solve math problems, they write to explain mathematical ideas, and they write to describe the learning process. (Urquhart)

1) Students write to keep records about what they’re doing, feeling, and learning

This type of writing is reflective. Reflective writing is just what it sounds like. It is having the students reflect on the aspects of the class. This could include expressing something new they learned that day, forming their opinion on the difficulty of a concept, asking questions about topics that confuse them, or just simply stating their feelings about math on that particular day. Basically, reflective writing is a chance for students to get their thoughts onto paper, which provides an emotional outlet for them to use in their math class (Urquhart).

2) Students write in order to solve math problems

Writing to solve math problems isn’t necessarily having the students write out entire sentences. Instead an example of this would be to have your students write the information they

know about the problem and the information they need to complete the problem. After completing the problem, they would go back and check to make sure their original thoughts were correct. This is a good type of writing to start out with, in order to get the students acclimated to writing in math class (Urquhart).

3) Students write to explain mathematical concepts

Writing to explain mathematics is asking the students to elaborate on a concept they have gone over in class. It could be to have them describe vocabulary in their own words, and provide an example, or given a problem to complete, they could be asked to explain their thought process and how they achieved their result. This is also a great tool for teachers to use as assessment, without the intimidation of calling it a test. Teachers are able to evaluate the level of their students' understanding by looking at the students' responses (Urquhart).

4) Students write to describe learning processes

This type of writing allows students to see the problem solving strategies they use and the processes they select. This can be used when students do not know how to solve a problem, but they can write out the processes they do know, and reflect on what they think they can do. After all the students have adequate time, the teacher can have the students explain their own thinking to a group or the entire class. This allows some collaboration and gives the students a chance to discover the concept as oppose to just being told about it (Urquhart).

Many different strategies could be used to implement these types of writing activities. Math journals are one way to implement all of the ideas listed above. Using a journal is a way for the teacher to provide prompts, which fit into each of these categories. Applying journals can have many benefits in a math classroom. These benefits apply to the student, the teacher, and the relationship between the student and the teacher.

Journal Writing and Mathematics Instruction is an article written by Rafaella Borasi and Barbara Rose. The article talks about the benefits of applying journals in a college level math classroom. The benefits are sectioned into different categories: potential benefits as the students write their journals, potential benefits as the teacher reads the journals, and potential benefits as students and teachers dialogue in the journals. Students will benefit from writing and thinking through their journal entries, while teachers can use the journals to assess the progress of each student (Borasi, 1989).

Potential Benefits of Journals for Students

- **Therapeutic Value**

They found that the students were comfortable expressing their feelings towards math even if the feelings were negative. This expression led to students associating their fear of math with their negative attitude towards the subject. This realization pushed students to be more open to change and be able to take steps to overcome their own difficulties. This benefit was noted by a majority of the students in their final evaluation (Borasi, 1989).

- **An increased knowledge of mathematical content**

Journals can be used to ask students to write about a specific topic they have seen in class. In their study, Borasi and Rose found that “Restating concepts and rules in one’s own words can in fact facilitate their internalization. . . Writing about a mathematical topic can also help identify learning difficulties and problems, and recognize connections previously unrealized” (Borasi, 1989). While this benefit was not as widely recognized in the student evaluations, the researchers conclude that providing prompts that require students to write this way is beneficial (Borasi, 1989).

- **An improvement in learning and problem-solving skills**

Students also realized through their journals how they do math. It allowed them the opportunity to see their strategies for problem-solving, reflect on the good and the bad, and then be able to improve their shortcomings. This created a higher awareness of their thinking processes. “By asking the students to report in their journals how they solved a problem or approached the study of a topic, they can be encouraged to become introspective of how they do and learn mathematics, and consequently be brought to identify more general heuristics to solve mathematics problems as well as to realize the possibility of alternative approaches to the same learning task” (Borasi, 1989).

- **Steps toward achieving a more appropriate view of mathematics can be taken**

The experience the researchers had with the journals suggests that providing prompts that allow students to question the nature of mathematics can broaden their view of mathematics all together. These beliefs that students have about math are made clear and in some cases can be reevaluated as the journals progress. Two of the students in the study said in their final evaluation that the experience had changed their view of mathematics. (Borasi, 1989)

Potential Benefits of Journals for Teachers

- **More appropriate evaluation and remediation of individual students can result**

This can result from the teacher reading the individual journals. As the teacher reads, he/she gets a look into a student’s mind that they may have never seen before. This allows the teacher to better understand the issues individual students possess and be able to respond to the students in a more effective way.

- **Immediate changes and improvements in the course itself can be made**

If the majority of the class is showing the same misconception in their journal, the teacher can re-evaluate the way material is being presented and make sure to go over the concept again. Evaluation can also come from a teacher asking his/her students to write about what they did or did not like about a lesson. This entry will still help students strengthen their verbalization skills, and it will provide the teacher with feedback that can help improve the teaching of the course itself. Borasi and Rose conclude that positive responses can strengthen a teacher's beliefs, while negative responses remind the teacher of the varying types of students and the need for individualizing instruction (Borasi, 1989).

- **Long-term improvements in teaching approach and methodologies may be induced.**

Teachers can use the journals as a way of getting student feedback on the course. The feedback received can be used as an evaluation of the course and can help the teacher make changes to the teaching style accordingly. This was a benefit mentioned by about half of the students in their final evaluations (Borasi, 1989).

Potential Benefits of Journals Between the Teacher and Student

- **More Individualized teaching can be achieved**

Students will each have responses from their teacher, which are specifically for them. This creates an opportunity to individualize teaching. The journals create a dialogue between each student and the teacher, allowing the teacher to provide support for all the students' concerns. "In our experience, the journals certainly allowed the unfolding of an unprecedented dialogue between teacher and individual students, witnessed by the exchange of questions, responses, comments, and remarks in the journals throughout the semester." (Borasi, 1989).

- **A more caring classroom atmosphere may be created**

The dialogue built through journal writing creates a positive rapport between the teacher and student. Having a teacher take the time to respond to each student's entry shows the level of care that teacher has for his/her students. Connell is quoted in Borasi and Rose's paper stating that through his research he found, "students' motivation is positively correlated with the degree to which they perceive the teacher to care for them," (Borasi, 1989) meaning by providing these journals and responses, the students are motivated to work harder. This open-dialogue builds trust and respect, which will positively affect student participation. Teaching a group of motivated students who are willing to participate, is much better than trying to teach students who simply do not care. (Borasi, 1989)

Through their research in the college math course all of these benefits were shown in some way or another and they concluded that "Journal writing in fact introduces new important dimensions in the mathematics classroom: by writing in the journals, students make use of writing as a learning tool in the context of mathematics; by reading students' journals, teachers access a wealth of information usually unavailable to them; and by commenting on students' entries, responding to specific questions and posing new ones, teachers engage in a unique and continuous dialogue with each individual student throughout the course. In turn, each of these elements has the potential to provide a variety of benefits for mathematics instruction." (Borasi, 1989)

Using Journals to Increase Mathematical Proficiency

Adding It Up states, "Mathematical Proficiency, as we see it, has five components, or strands:

- *Conceptual Understanding*- comprehension of mathematical concepts, operations, and relations
- *Procedural Fluency*-skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *Strategic Competence*- ability to formulate, represent, and solve mathematical problems
- *Adaptive Reasoning*- capacity for logical thought, reflection, explanation, and justification
- *Productive Disposition*- habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.”
(National Research Council, 2001)

A TIMSS research group conducted a follow-up to the 1999 Video Study that compared the teaching methods of 7 high-achieving countries. Through this study they found the United States placed an apparent emphasis on applying mathematical procedures versus examining mathematical relationships (TIMSS Video Research Group, 2003). To improve the mathematical proficiency of students, teachers need to strive to incorporate understanding mathematical relationships into the classroom. Writing can be used to increase this understanding.

Writing in Mathematics: An Alternative Form of Communication

There was a previous study that evaluated the use of journals based on the strands of mathematical proficiency. This study was called *Writing in Mathematics: An Alternative Form of Communication for Academically Low-Achieving Students* conducted by Juliet Baxter, John Woodward, and Deborah Olson. They looked at the effects of writing in a 7th

grade classroom. The classroom included 28 students, one-third of which qualified for special education. The researchers focused in on the responses of four of the students which qualified for special education. The main questions they focused on were:

- 1)What does writing reveal about the students' Conceptual Understanding?
- 2)What does writing reveal about the students' Strategic Competence?
- 3)What does writing reveal about the students' Adaptive Reasoning?

The teacher applied writing about once a week and aimed the prompts towards “relating to the mathematical topics being studied in class, improving students’ awareness of their own thought processes, and facilitating students’ “personal ownership” of knowledge” (Baxter, 2005). She started off providing prompts that would allow the students to share their opinions and feelings, then moved on to having them write about their mathematical thinking, and finally she progressed to justification. While the students participated in the journals, the researchers focused on four students, all of which qualified for special education. The four students were all very passive in class, but 3 out of 4 of them seemed very involved in their journals. After examining these 4 students over the course of the journals it was concluded that in 3 of the cases teachers were able to gauge whether or not a student had conceptual understanding of the topic. The impact the journals had on these students’ conceptual understanding is unclear, but it did provide the teacher a look into their level of understanding. There was also a correlation found between success in the journals and a student’s productive disposition (belief that one can learn and do math). When the teacher makes an effort to build personal relationships with the students, they begin to believe more in themselves, which can lead to an increase in performance. (Baxter,2005)

METHODS

The purpose of my research is to be able to observe the effects journals have in the math classroom. I aim to discover the benefits of journals for the student and teacher, if the journals help students increase their conceptual understanding, and how applying the journals change the way students feel about writing in mathematics.

During the Fall of 2016 I began my research on the effects of writing in a mathematics classroom. I chose to research these effects by assigning journals in a college pre-calculus class, as well as, conduct pre- and post-class surveys to see how the students' opinions of mathematical writing changed. On the first day of class a survey was given to gauge the students past experience with writing in math, their opinions of writing, and their ability to explain their thought process. The same survey was given to the students at the end of the 9 weeks. My goal for the surveys was to evaluate the change in the student's opinions towards using journals in class.

Between the pre- and post- surveys, the students were assigned journal prompts throughout the semester. A total of six prompts were assigned to be completed in class. The journals were then evaluated on a scale of 0-5, 0 being the lowest, 5 being the highest. The table below shows a general description of each score that could be given.

0	1	2	3	4	5
Student did not attempt any explanation	Response was unclear, incorrect, and missing a lot of details	Response was semi-clear, missing details, and incorrect (could be correct, but if the explanation is	Explanation was clear, but was missing details and correct. (Could be incorrect but if the explanation	Explanation was clear, correct and missing very few details	All parts of the prompt were answered. The explanation was clear, correct, and included

		very weak still give a 2)	is still good rate a 3)		plenty of detail.
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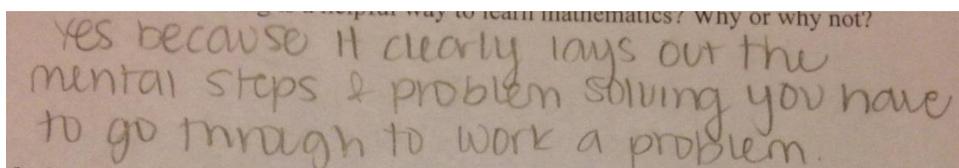
Each student received a score and response for each of his/her entries. The prompts were given mostly on Fridays and pertained to the concepts covered in class that week. The students were allowed between 10-15 mins to complete each entry. My goal with the journal entries was to find evidence of student and teacher benefits, as well as, find what the responses could tell me about each particular student's understanding of a concept.

Data Analysis

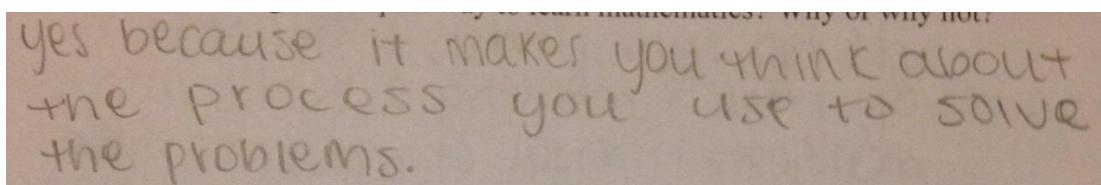
After examining my data, I split my findings into the benefits for the student, the benefits for the teacher, the benefits for the relationship between them, and the information collected from the surveys.

Benefits for the students

The benefits for the students came straight from the responses to the surveys. The first benefit I found was that writing in the journals allowed students to see the process they use to work problems. These two students wrote,



yes because it clearly lays out the mental steps & problem solving you have to go through to work a problem.



yes because it makes you think about the process you use to solve the problems.

This is an important benefit because if students are able to see how they work problems they are able to reflect on the validity of their own work. They could also use this to decide if

there is a better or simpler way to complete the particular task. When reading through the surveys I noticed this was one of the most common benefits the students recognized. I also had many students acknowledge the next benefit I found, which was the journals encouraged the students to have a deeper understanding of the concept. This can be seen in these two comments taken from student surveys.

Yes, it forces me to actually know what and why we do things instead of just memorizing a formula

I think it was helpful because it made me describe the principles of a certain concept. It allowed me to fully understand a concept + be able to connect to different concepts even more

When I read these comments I thought they really said a lot about the effect of journals, because the goal of every educator is to have their students understand the concepts behind the material being taught, so for the students to recognize this type of progress from journals is pretty incredible.

The last student benefit I found was that the journals allowed the students to self-assess.

Two students wrote,

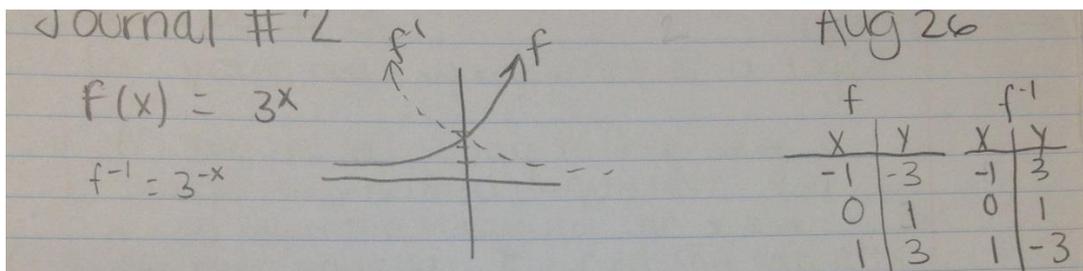
Yes, it helps realize what questions you actually have

Yes - it helps me find out if I truly know what the topic entails. Also it helped me to know what stuff I needed to go back on to look over.

Being able to self-assess is a valuable skill that the students need. Allowing them to be able to see their own error provides them an opportunity to take responsibility over their own learning experience.

Benefits for the Teacher

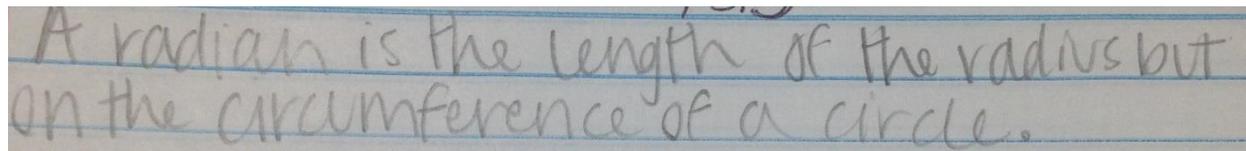
The benefits for the teacher I found while reading and responding to the students' journals entries. The first benefit found was that teachers could see common misconceptions the students were having. For example, when reading through the journals I found a lot of students misunderstanding inverse function notation.



In the above entry we had a student misunderstand the notation for an inverse function $f^{-1}(x)$.

The negative 1 exponent was mistaken for multiplying the variable by -1, which is incorrect.

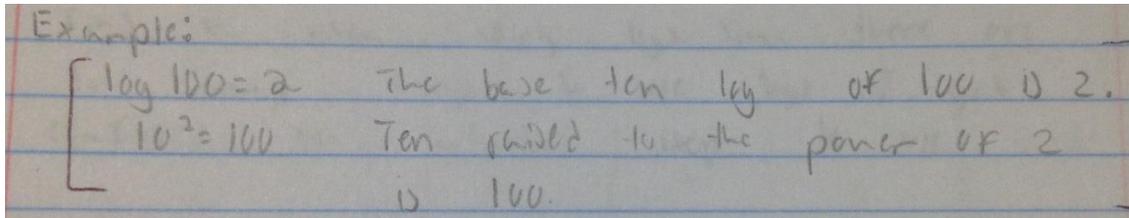
Another misconception I found was about radians. A radian is a unit measure, and 1 radian is the measure of an angle whose corresponding arc length is equal to the length of the radius. The misconception that was happening with a lot of students was that they were seeing the radian as an arc measure instead of an angle measure. This is demonstrated by the student below.



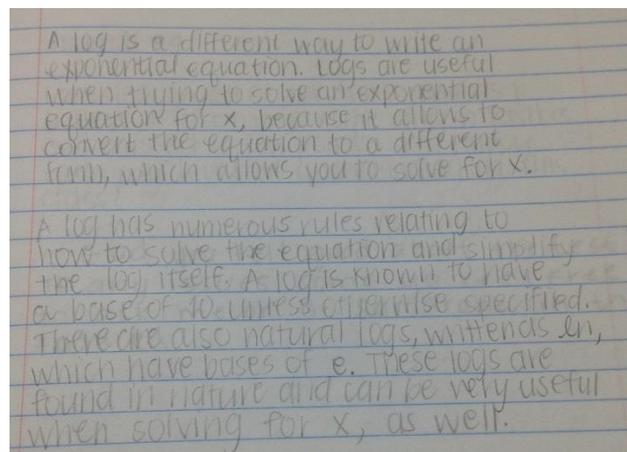
Being able to identify these misconceptions was very helpful because these were misconceptions that were not noticed in class. While reading through the journals, when I noticed a mistake being made by the majority of students I would e-mail Dr. Santarone about it. In the instance

with the inverse notation, she even said that was something she wouldn't have seen in class. The journals provided a way to catch these mistakes early, and correct them before the test.

The second benefit I found was that through the journals a teacher could gauge a student's level of understanding. For example, in the pictures below the students were asked to describe in their own words what a log is.

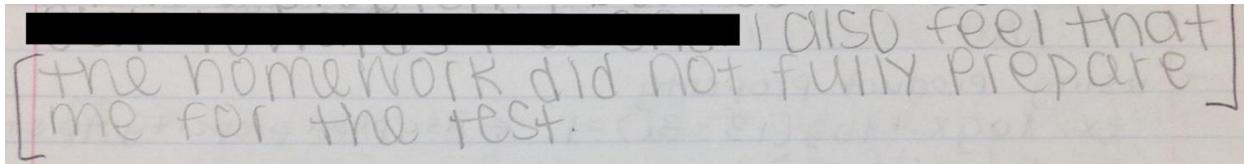


In this picture, the student provided the definition they were given in class and an example of how to apply logs in a simple way. This response is very procedural and does not show any deeper understanding from the student.



However, this second response talks about what logs are, what they can be used for, and how they are helpful. This student did not just state the definition, but she made appropriate connections and showed a more conceptual understanding of the topic.

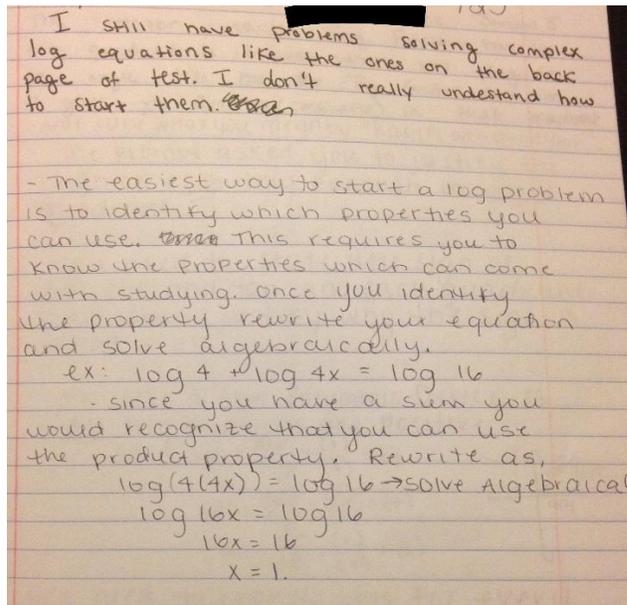
The last teacher benefit I found was that student feedback could be taken from the journals and used as an informal course evaluation. For example, one student wrote



Teachers can use student feedback to make improvements to the course or to validate their own teaching methods.

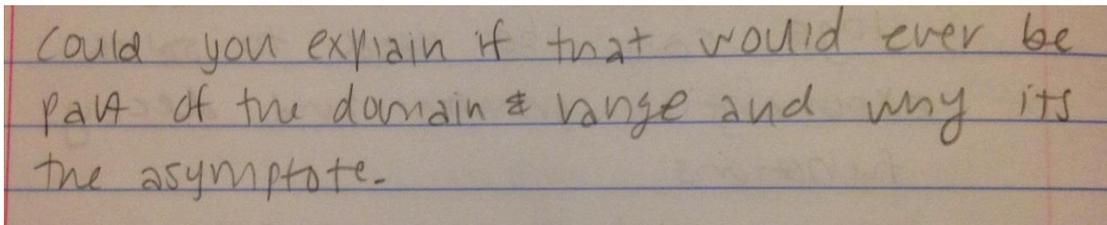
Benefits for the Relationship Between the Teacher and Student

I pulled the benefits for the relationship between the teacher and the student from the journal entries and the responses given. The first benefit was that participating in the journals provided an opportunity for individualized instruction. An example of this is given below.



The student wrote about an issue he/she was having with a concept and was provided an explanation, as well as, an example of how to complete this particular task. This also helped with students who did not necessary want to speak out in class. Some student were embarrassed to ask questions in class, so the journals provided a way for these students to communicate. This leads

me into my last benefit I found from journals, which was that the journals created a dialogue between the teacher and student. For example, in the following student's journals she wrote her entries very conversationally, and despite the student and I not having face to face interaction, we were still able to have a dialogue in the journals.



could you explain if that would ever be part of the domain & range and why its the asymptote.

This is another way journals can help students that do not like to speak out in class. The shy students who do not enjoy talking in front of their classmates are still able to express themselves in their entries without having to feel uncomfortable, and this can strengthen the teacher-student relationship.

Survey Information

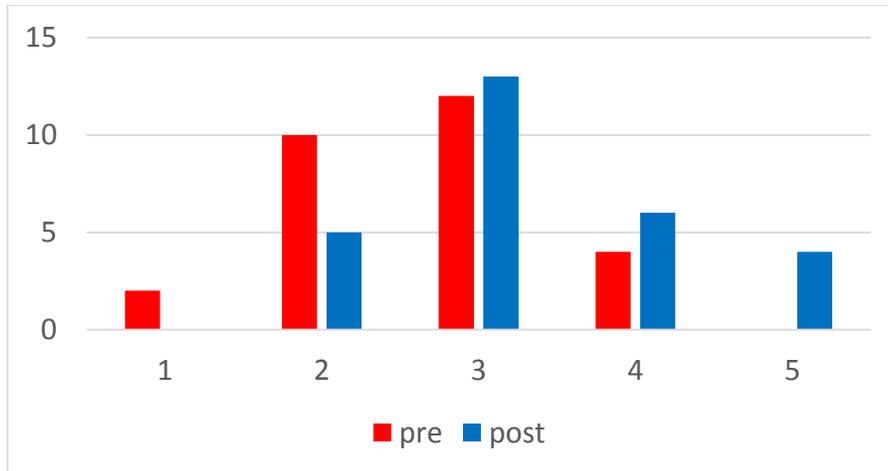
The final pieces of data I gathered came from the student responses in their surveys. The students were asked 3 questions and I received the following results:

Do you think writing is a helpful way to learn mathematics? Why or why not?

On our pre-survey 25 out the 28 students said yes, and on the post-survey 27 out of 28 of the students said yes. When first looking at the pre-survey results, I was very surprised at how many students felt writing was helpful. Even though we started with a majority of the students stating they thought it was helpful, we still observe a small increase in this student opinion.

On a scale of 1 to 5 (1 being the lowest, 5 being the highest) how important to you is writing in mathematics?

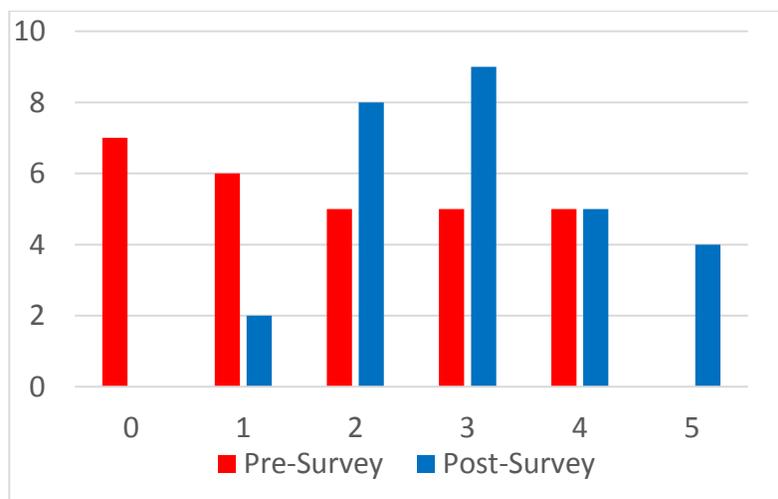
I divided their responses into the chart below



We have the student ratings from the pre-survey represented in red, and the ratings from the post-survey in blue. In the pre-survey I noticed a majority of students ranging between a 2-3. Eight weeks later, when the students were given the post-survey, we can see the scores lying mostly in the 3-4 range. So again, there was some improvement in student opinion over the course of the study.

Complete the pattern problem (Included on the Survey (Problem 4))

To compare the students' scores on this particular problem I created the chart below.



Again we have the pre-survey scores represented in red, and the post-survey scores in blue. By looking at our pre-survey scores we see the majority of them lying in the 0-1 range. For our post-

survey a majority of our students scored in the 2-3 range, with some even receiving a perfect score of 5. This once again shows some improvement in our students' writing abilities.

- 18/28 scored higher on the post-survey than the pre-survey
- 6/28 scored the same on the pre- and post- survey
- 4/28 scored lower on the post-survey than the pre-survey.

The findings of my research support the application of journals in a math classroom. My goal of this research was to discover the benefits of journals for the student and teacher, see if the journals helped students increase their conceptual understanding, and observe how applying the journals changed the way students felt about writing in mathematics. My research clearly stated the benefits, as well as, the changes in the student opinion over the course of the study, but there was no evidence of an actual increase in conceptual understanding. I found that by reading the journals I could gauge whether or not a student had conceptual understanding over one specific topic, but since the prompts were based on different concepts, I was unable to gauge an actual increase in conceptual understanding. There was an improvement in their scores, but there is no way to know if this improvement was in their understanding or simply in their ability to express themselves.

Future Research

Ideally I would be able to continue this research to collect more data on the effectiveness of math journals. If I were to apply journals into another class I would definitely change a few aspects of my study. The first thing would be to prepare some sort of training for what writing in math actually looks like. In the beginning of this semester, most of the students had never experienced writing in math, so they struggled with knowing exactly what I was looking for in their entries. Providing them with a practice activity would provide them with a better

understanding of what I was looking for, which may make the journaling process easier. I also would provide multiple journal prompts over one topic. I found in this semester that I could not gauge an increase in conceptual understanding because all of the entries were over different content. By providing journals over the course of one topic, I would hope to be able to see the progress of a student's conceptual understanding. Lastly, I would assign more prompts. I only had time for the students to complete 6 journal entries, but ideally I would want to assign more prompts. This could help strengthen the data I found, or bring out aspects of journals that I did not see this semester.

Conclusion

In conclusion, my experience through this study has been very influential on my own teaching beliefs. I have found that journals can be a great way to incorporate writing into the math classroom, and they can help strengthen students' ability to write. The journals allow students to see math in another form, which can help them make steps towards a more appropriate view of mathematics. The students themselves even recognized the journals as being a helpful tool when working in their math course. As teachers, we should want to provide many different ways for students to learn and I believe journals are a great addition to any math class.

Survey Pre-Calculus Fall 2016

Name: _____

Thinking back on what we have done this semester, answer the following

1. Do you think writing is a helpful way to learn mathematics? Why or why not?

2. On a scale of 1 to 5 (1 being the lowest, 5 being the highest) how important to you is writing in the mathematics classroom?

3. Below is the first 4 terms of a pattern. How many dots would be in the tenth term of the sequence? Explain in detail how you used the picture to determine your answer.



Journal Prompts

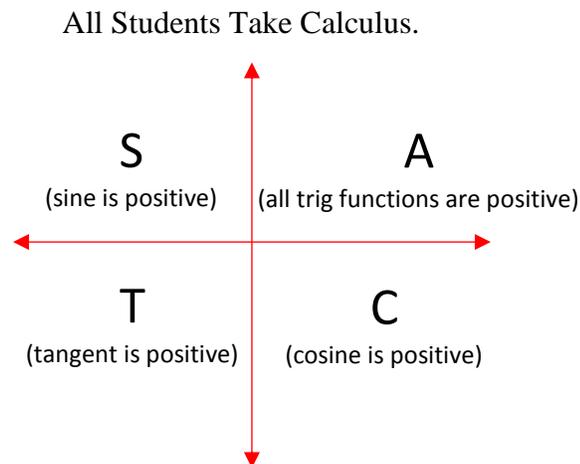
Week one: -Explain in your own words what the term function means. Give an example of a real life function and explain in detail why your example qualifies as a function.

Week Two: Sketch the graph of $f(x)=3^x$. On the same coordinate plane sketch the inverse of this graph. Explain your thought process and the method you used to create your new graph.

Week Three: In your own words, what is a log?

Week Four: "After getting your test back, is there any concepts we've gone over this semester that are still unclear? Include any questions or concerns you have about these concepts or problems. Be specific in your response."

Week Five: Some students use this pneumatic device to help remember which Trig Functions are positive or negative in each quadrant.



Using ideas, we discussed in class, explain **WHY** each trig function (sine, cosine, tangent) has its designated sign within each quadrant.

Week 6: Thinking back to the activity we did on Wednesday, What is a radian? Draw an angle that has a measure of approximately 1 radian and explain how you know the angle has this measure. (Do NOT convert to degrees)

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