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## **Discovering Discovery Learning**

### **I. Introduction**

Education is in the midst of great change. The role of the teacher, the common core, and the way the content is being taught to students are just a few examples of areas that are undergoing this change. Specifically, the concept of discovery learning is making its way to the classroom. I was first introduced to the idea of discovery learning in my Math Education courses. When I first heard this, it was just a new way to allow students to understand topics. It was not until my placement for one of my classes, Project Focus, that I saw discovery learning in action. I was interested in how the typical role of a teacher shifted, and how the cognitive demand increased for the students. The students became more of the center. I remember when I was shown this type of learning; I was frustrated with the teacher breaking what I considered normal which made it difficult for me to come to a conclusion by my own opinion. I have also heard many secondary school teachers' thoughts on discovery learning which for the most part have been more on the negative side. It seems necessary to explore further the effect that discovery learning can have. My motive for doing this study comes from personal experiences which I have previously described.

## **II.RESEARCH**

According to google, to discover something implies that we are bringing to light an idea, uncovering something we have been searching for, or to simply come across a new idea. Combining this and education leads to discovery learning. Discovery learning is a method used in the classroom that encompasses many positive techniques. Experience, communication, and interaction are just a few aspects of discovery learning. This type of learning allows students to take responsibility for their own learning and understanding. It breaks the traditional norms where the teacher is the major role, and allows the students to engage in a way they have not before (Meece, 2002). In 2006, an article was published that believed this type of learning had a negative influence in the classroom. Kirschner, Sweller, and Clark state, “The goal of this article is to suggest that based on our current knowledge of human cognitive architecture, minimally guided instruction is likely to be ineffective”(Clark, 2006, p.3). After gaining more understanding on discovery learning and realizing how controversial this subject is, I became more interested in exploring this concept.

“Discovery learning can have a lasting impact because learners not only experience the content, they also improve.”(Rillero, 2013,pg.2). After multiple experiences in a classroom and being a student myself, I know what it mean to strictly learn material for a test, and then immediately forget about it. Experiencing content, instead of being handed content, leads to a certain level of improvement which Weimer believes is vital. Along with experience and improvement, students become more involved and engaged. Weimer states,

Constructivism prescribes a whole new level of student involvement with content. It makes content much more the means to knowledge than the end of it. It and the empirical

way of psychology change the function of content so it is less about covering it and more about using it to develop unique and individual ways of understanding (Weimer, 2002, p.12).

So often, it is about the content. Weimer (2002) believes that it is not about just the content, but rather about gaining understanding.

## **Learner – Centered Teaching**

Discovery learning falls into the category of Learner- Centered Teaching, also known as student-centered teaching. In a traditional classroom, the teacher has most of the power and the students do more listening. The teaching style is more lecture based. When it comes to Learner-Centered Teaching, there are two perspectives/roles that need to be examined: the student perspective/role and the teacher/role perspective.

### **The teacher's perspective\role**

It is usually assumed that when the students become the center, the role of the teacher decreases to nothing. However, this is not true. For successful discovery learning, the teacher has to be creative along with understand how to develop lesson plans in which the students can learn from interacting and experiencing. Learner- Centered teaching should not be used as a means of survival or an escape from what is considered traditional teaching, but rather a commitment to develop an experience to remember. From a constructivist's perspective, the teacher has to be creative so that he/she can invent lesson plans that will further the students' conceptual understanding (Reinhardt, 2000, p.54). The teacher must let the students answer their own questions and challenge each student. Reinhart shares the following statement,

When I was in front of the class demonstrating and explaining, I was learning a great deal, but many of my students were not! Eventually, I concluded that if my students were

to ever really learn mathematics, they would have to do the explaining, and I, the listening. My definition of a good teacher has since changed from ‘one who explains things so well that students understand’ to ‘one who gets students to explain things so well that they can be understood’(Reinhardt, 2000, p.54).

It is important to note that the realization this teacher made is vital to student learning. This becomes its own assessment: can a student explain the material? , can a student answer another student’s question? Is the teacher seeing the students interact in a way that expresses the students understand of what is going on? So, from the teacher perspective in a Learner-Centered classroom the teacher is not the dominant figure. In fact there is a balance of power between the students and the teacher. Some people have difficulty accepting this balance of power. Weimer states,

Many who object the ideas of radical pedagogy do so on the ground that if faculty relinquish control, they abrogate legitimate instructional responsibility. Students, they say, end up running the class and teaching themselves, leaving the teacher no viable role in the educational process. It is true that this educational ultimately dispenses with the teacher. The goal is to equip the students with learning skills so sophisticated that they can teach themselves (Weimer, 2002, pg. 29).

It is true that this could cause chaos, but this is where the teacher would need to have enforced classroom policies; basically classroom management becomes another huge responsibility for the teacher. The teacher must create an environment that allows such activities and experiences to take place. The teacher plays a major role in the class room, even when the approach is student-centered. Kilpatrick, Swafford, and Findell agree on the following role of a teacher,

Effective teachers have high expectations for their students, motivate

them to value learning activities, can interact with students with different abilities and backgrounds, and can establish communities of learners. A teacher's expectations about students and the mathematics they are able to learn can powerfully influence the tasks the teacher poses for the students, the questions they are asked, the time they have to respond, and the encouragement they are given—in other words, their opportunities and motivation for learning. (Kilpatrick, 2006, pg. 9)

The role of a teacher is far more than lesson planning. Another role is they must determine the level of thinking required which is known as cognitive demand. This will be further explained in the student perspective portion. However, it is important to note that this is another role the teacher must take on to fully commit to partake in discovery learning. Every part of a task must be analyzed to incorporate cognitive demand.

### **Student's perspective/role**

The generation of children today has grown up with technology. Nothing against technology, but it makes things easier. Along with that, I have seen many children that are handed everything they need/want. This has become a problem in the classroom. The teacher hands out a list of steps, the students follow them, and then they circle their “answer”. Challenge is a necessary part of the classroom, and sometimes there tends to be a lack of it. Listen while the teacher lectures, do your homework, and memorize for the test. In a Learner-Centered classroom, the student is given responsibility. They get the chance to explore and control what they learn. In a couple of case studies, students were pleased with the way a Learner-Centered model affected the classroom. Meece states, “Students reported more positive forms of motivation and greater academic engagement when they perceived their teachers were using Learner-Centered practices

that involve caring, establishing higher order thinking, honoring students' voices, and adapting instruction to individual needs."( Meese, 2002, p.110). When viewing the student's role in Learner-Centered teaching, we begin to introduce different levels of cognitive demand.

According to Stein (2000), cognitive demand is the level of thinking it takes students to successfully engage in a task. He continues on by explaining what these difference types of cognitive demand are. Stein states, "Not all mathematical tasks provide the same opportunity for student learning" (Stein, 2000, p.17) Stein outlines four different types of cognitive demand:

- Memorization – Students will perform multiple different problems anywhere from 10-30 times. These problems are extremely similar. This goes along with the drill method which is where a teacher "drills" facts into their students' mind.
- Procedures without Connections – This is on the same level as memorization. (lower level of cognitive demand). The students are asked a question where they can repeat steps that were previously given to them. They do not make connections, but rather perform certain steps without much understanding.
- Procedures with Connections – This is where we see higher level of cognitive demand. Students are aware of what is going on, and they are able to make connections to the bigger picture. Their conceptual understanding deepens here. We also see that they are able to understand various representations.
- Doing Mathematics. This requires fewer problems with more exploration. Students develop the procedures, and are not immediately given the steps. We can develop a decent understanding of what this looks like in a classroom by what VandeWalle has to say, "Doing Mathematics means generating strategies for solving problems, applying

those approaches, seeing if they lead to solutions, and checking to see if your answers make sense.”(p.13). There are a few important expectations to remember

- Persistence, effort, concentration
- Collaboration between students
- Listening to other students
- Learning from mistakes
- Making connections

These are all parts of Doing Mathematics that the students will be experiencing (VandeWalle, 2013). Each task has its own purpose. It could be to engage in reasoning or enforce a procedure. On the contrary, the science behind this matter shows a different side. Clarke (2006) states,

We have known at least since Peterson and Peterson (1959) that almost all information stored in working memory and not rehearsed is lost within 30 sec and have known at least since Miller (1956) that the capacity of working memory is limited to only a very small number of elements (p.76).

It seems that as humans we need some type of memorization to reinforce the knowledge, but Stein sees memorization as lower cognitive demand. Throughout my study and personal research, Stein’s cognitive demand levels will be referenced.

### **Learning-Centered Study- Middle School Edition**

There was a previous study (Meese, 2002) that focused on Middle School students and teachers. It involved 2,200 students and 109 teachers. The purpose of the study was to see the benefit Learner-Centered teaching can have in the classroom. The students and teachers had to complete surveys that allowed them to assess these practices. The study included rating scales where the academic success, level of cognitive demand, classroom performance, and

demographics could be examined. Meese determined three different goals that were based on prior research. The way Meese (2002) assessed the results were by the following categories: “Mastery goals, a desire to improve ability; Performance goal, a desire to demonstrate high ability and performances; and work avoidance, a desire to complete a task with minimum effort” (p.112). After analyzing the student and teacher surveys, they were able to come to a conclusion. Meese states,

The analyses revealed several interesting findings for middle school education. Both teachers' and students' ratings of learner-centered practices were correlated with measures of student motivation and achievement, but patterns of relations were stronger for student ratings. Only teachers' reported support for higher order thinking showed a positive relation to student outcome (p.113).

The students' mastery goals showed the highest positive relation. Meaning they were able to gain and improve their overall ability. Meese was able to conclude that the students preferred Learner-Centered, and after all the students' voice matters the most.

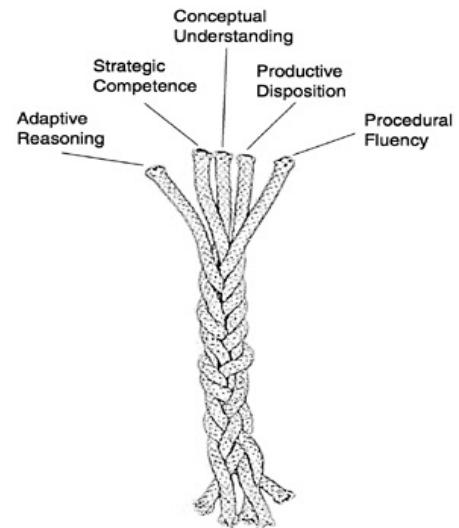
### **III. Methods**

Throughout my own personal research, I aim to discover how cognitive demand plays a role into the classroom, specifically mathematics, explore the benefits of teacher-centered and student- centered approaches, observe discovery learning in a college setting, and consider student responses to this type of learning. Overall, I want to discover the advantages and disadvantages of discovery learning, and come to a conclusion of whether or not teachers should be using this method in the classroom. These results are important to me, since I am a future educator.

In fall 2015, I plan to begin my research on discovery learning. I am choosing to explore Discovery Learning with college students from two pre-calculus classes. In my particular case, I hope to explore these ideas by allowing students to discover a connection between the triangles within the unit circle and the graphs of the trigonometric functions. In the end, I will be able to come to a conclusion of which type of lesson plan gave the students more Mathematical Proficiency. This is a term used by \_\_\_\_\_. Mathematical Proficiency is a term that incorporates five different intertwined aspects with a mutual dependence. The book, Adding It Up, defines each aspect individually, but also expresses how they should exist together. Kilpatrick states,

Mathematical proficiency, as we see it, has five strands:

- conceptual understanding—comprehension of mathematical concepts, operations, and relations
- procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence—ability to formulate, represent, and solve mathematical problems
- adaptive reasoning—capacity for logical thought, reflection, explanation, and justification
- productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (Kilpatrick, 2001, pg.5)



I will be able to examine which branches of Mathematical Proficiency they students are demonstrating and experiencing in each lesson plan. It is important to note that the NCTM learning principle states, “Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.” As a future educator, I am curious to see which lesson plan most satisfies this principle.

### **Teacher-Centered Lesson: Plan**

For my research, I taught two college pre- calculus classes. One class was taught in a traditional manner where definitions and formulas are given. This is considered more of a teacher-centered lecture based class. At the beginning of class, I had the students fill in a unit circle. They were given a unit circle and 3 minutes. This allowed me to see how much of the unit circle had memorized. We discussed the unit circle which is a topic that has been previously discussed in class. Our discussion was a review that reminded the students of the unit circle and how each x and y coordinate relates to the trigonometric function. This allowed the students to work individually; they made tables of the coordinates, and then graphed the coordinate points strictly from the table (See Appendix A for handout).

$x = \theta$ radians	$y = \sin \theta$	(x, y)	$x = \theta$ radians	$y = \sin \theta$	(x, y)
0	0	(0, 0)	$\frac{\pi}{6}$	$\frac{1}{2}$	( $\frac{\sqrt{3}}{2}$ , $\frac{1}{2}$ )
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	( $\frac{\sqrt{2}}{2}$ , $\frac{\sqrt{2}}{2}$ )	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	( $\frac{1}{2}$ , $\frac{\sqrt{3}}{2}$ )
$\frac{\pi}{2}$	1	( $\frac{\sqrt{3}}{2}$ , 1)	$\frac{4\pi}{3}$	- $\frac{\sqrt{3}}{2}$	( $-\frac{1}{2}$ , $-\frac{\sqrt{3}}{2}$ )
$\frac{2\pi}{3}$	$\frac{\sqrt{2}}{2}$	( $-\frac{\sqrt{2}}{2}$ , $\frac{\sqrt{2}}{2}$ )	$\frac{5\pi}{3}$	- $\frac{\sqrt{3}}{2}$	( $-\frac{1}{2}$ , - $\frac{\sqrt{3}}{2}$ )
$\frac{3\pi}{4}$	$\frac{1}{2}$	( $-\frac{\sqrt{3}}{2}$ , $\frac{1}{2}$ )	$\frac{7\pi}{6}$	$-\frac{1}{2}$	( $-\frac{\sqrt{3}}{2}$ , - $\frac{1}{2}$ )
$\pi$	0	(-1, 0)	$\frac{4\pi}{5}$	$-\frac{\sqrt{5}}{2}$	( $-\frac{\sqrt{5}}{2}$ , 0)
$\frac{5\pi}{6}$	$-\frac{1}{2}$	( $-\frac{\sqrt{3}}{2}$ , - $\frac{1}{2}$ )	$\frac{11\pi}{6}$	$-\frac{1}{2}$	( $-\frac{1}{2}$ , - $\frac{\sqrt{3}}{2}$ )
$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	( $-\frac{\sqrt{2}}{2}$ , - $\frac{\sqrt{2}}{2}$ )	$2\pi$	0	(2π, 0)

Using the unit circle, fill in the table with the exact values of sine.

Plot these points on the graph below.

The cognitive demand in the first lesson was procedures without connections, because the students were not making connections to the triangle within the unit circle. For example,  $\sin(\beta) = \text{opposite/hypotenuse}$ . We claimed procedures without connections, because the students can

aimlessly go through the steps. There must be a certain level of thinking and understanding. At the end of the class, the students were given an assessment.

### **Teacher-Centered Lesson: Observations**

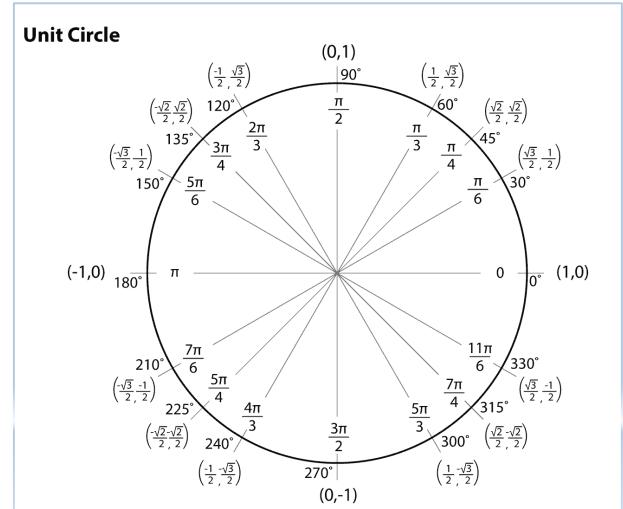
After teaching the teacher-centered class, I made a few observations. First off, there was less students than we expected. We expected 34 and only had 17. At the end, we had to reconsider the way be analyzed out data. The students were quiet in this class or either on their phone. I tried to ask probing questions, but I received little response to these questions. At the end of class, the students were satisfied about their time in class. We will look at their responses later on in the paper.

### **Student-Centered Lesson: Planning**

The second class and third classes were those who participated in the Discovery Learning Activity. They were also given the blank unit circle and three minutes. We then discussed the unit circle. Our review looked different in this class, because they had to be prepared for the Activity (See Appendix B. for Instructions). This discussion will incorporate a review, but also a quick demonstration that will help them see the triangles with three sides that are measured in distance. For example, we discussed how we get to the first

coordinate point  $(\frac{\sqrt{3}}{2}, \frac{1}{2})$ . That required us to go over  $\frac{\sqrt{3}}{2}$ , horizontally from the x-axis, and up  $\frac{1}{2}$ . We also

recalled that the hypotenuse is one. Then the students will participate in an activity where the



goal is for the students to discover the relationship between the graphs and the unit circle, by the measurements of the triangles. This was considered a student-centered approach, and the students were to experience discovery learning. The activity allowed the students to develop an understanding of why the sine and cosine graphs look like they do. After that they were given spaghetti noodles. The students then measured the base of every triangle with the spaghetti noodle. They will place the measured noodles on a coordinate plane in the order of the least angle measure to the greatest. As they do this, they will begin to actually construct the graph of cosine. Then they repeated this by measuring the height of each triangle. This resulted in the development of the sine graph. We classified this level of thinking as Doing Mathematics. We claimed Doing Mathematics, because their thought had to be justified, explained, and represented. They were able to take a circle, create the triangles, and then use those triangles to discover the graphs of sine and cosine.

### **Student-Centered Lesson: Observations**

When students first walked into class, they were shocked to see construction paper, glue, and noodles. They then began to make assumptions that this was going to be “easy”. It was not until they started working in their groups that they realized this required thought and focus. I observed one student, in particular, do this activity from memory. Rearranging the noodles without participating in the activity. In thus specific case, the student took the cognitive demand from Doing Mathematics to Memorization. When we walked around observing their “noodle graphs” we made the following observations:

- Mislabeled axis- The location of zero was not at the intersections of x and y.
- Skipped over zero- The graphs did not go through the x-axis, but rather skipped it.

- Extends to Infinity- After  $2\pi$ , the graphs went upward to infinity.
- Stops after  $2\pi$ - At  $2\pi$ , the graph did not continue.

All of these observations led to great mathematical conversations we were able to have with the students.

## IV. Data Analysis

At the end of both classes, the students received an assessment. The assessments were the same for both classes. I was interested in only two of the five aspects of Mathematical Proficiency, so I created my assessment accordingly. The assessment had two parts: procedural fluency and conceptual understanding.

### **Assessment: Procedural Fluency**

The first four questions were strictly procedural. (See Appendix C for assessment). These first four problems were graded strictly for right or wrong. For every correct answer, the students received 1 point; however, if they answered in incorrect, they received zero points. The purpose of these four problems was to see if they could use their unit circle properly to apply the procedures.

### **Assessment: Conceptual Understanding**

The next set of questions will focus more on their thought process on how the unit circle and trigonometric function graphs are related, and this part represented the conceptual understanding portion. Basically, I wanted to see if the students understand the “why” behind the concepts. Eight questions fell into this category. These questions were assessed solely on

conceptual understanding. They received somewhere between zero points and three points. A student received zero points if they gave a blank answer or restated the question, one point if they simply made an observation, two points if they were able to make two connections and three points if they were able to make the connections and offer a justified explanation(See Appendix D for rubric). They were able to receive half a point if their answer does not satisfy a full point. It is important to notice that conceptual understanding is present when a student received two or three points. This portion of the assessment allowed me to see if the students who were taught a formula based lesson are aware of what is happening conceptually and if the students who participated in a discovery learning activity are able to put into words what they discovered.

### **Assessment Analysis: Conceptual Understanding**

After both classes took their assessments, I examined the assessments thoroughly. In four problems, I saw a significant difference in the level of conceptual understanding. The following problems demonstrate the different levels of conceptual understanding between my student-centered class and my teacher-centered class:

- Question 5: The value that corresponds to  $\sin \frac{\pi}{4}$  is  $\frac{\sqrt{2}}{2}$ . What does this mean

Teacher-Centered: 1pt.

The value that corresponds to  $\sin \frac{\pi}{4}$  is  $\frac{\sqrt{2}}{2}$ . What does this mean?

The y value on the unit circle at  $\frac{\pi}{4}$  is equal to  $\frac{\sqrt{2}}{2}$

Student-Centered: 2 pts

The value that corresponds to  $\sin \frac{\pi}{4}$  is  $\frac{\sqrt{2}}{2}$ . What does this mean?

when you draw a triangle from (0,0) to ( $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$ )  
and down to the x-axis, the sin or y-value  
of this point is  $\frac{\sqrt{2}}{2}$ . Also the opp value of the triangle.  
hyp

Notice that the student made an observation pertaining to the unit circle, while the student below was able to make a connection to the triangle within the unit circle along with applying the definition of sine properly.

- Question 7: Sine increases from 0 to  $\frac{\pi}{2}$ . Why does this happen?

Teacher-Centered: 0 pts.

Teacher-Centered: .5 pt.

On the interval 0 to  $\frac{\pi}{2}$  radians, the sine graph is increasing. Why?  
Yes, don't know it just is

On the interval 0 to  $\frac{\pi}{2}$  radians, the sine graph is increasing. Why?  
The y values are increasing

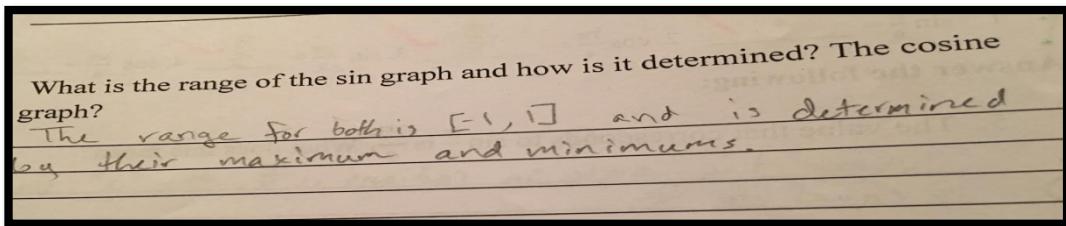
Student-Centered: 2pts.

On the interval 0 to  $\frac{\pi}{2}$  radians, the sine graph is increasing. Why?  
because the y value | height of triangle  
is getting bigger / taller

Both of the Teacher-Centered responses above demonstrate a lack of conceptual understanding. On one hand, we have a blank response and on the other hand we have an unspecific observation. The student-centered response shows the students actually picturing the triangles within the unit circle.

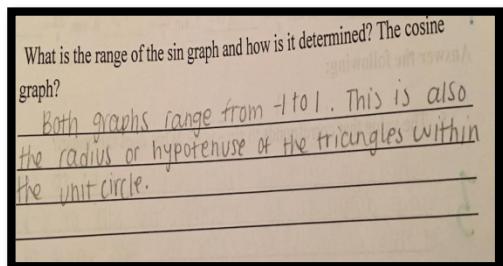
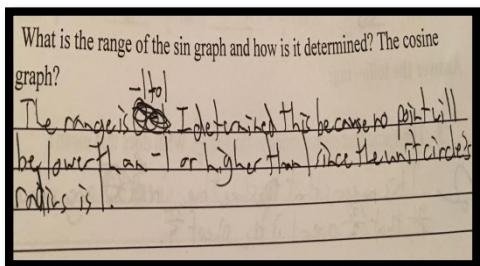
- Question 9: What is the range of both the graphs ? Why?

Teacher-Centered: 1 pt.



Student-Centered: 2pts.

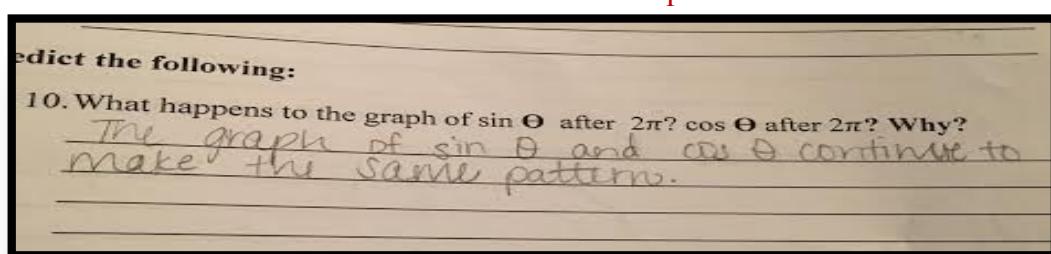
Student-Centered: 3 pts.



The answers above show us three different levels of conceptual understanding. The student who received one point made an observation by looking at the graph. The next student, scored two points, because he made a connection to the radius of the unit circle. The answers are gradually building to lead to a three point answer. This student shows a full conceptual understanding, because she is able to notice the radius and also the triangles' hypotenuse.

- Question 11: What happens after  $2\pi$ ?

Teacher-Centered: 1 pt.



Student-Centered: 3 pts.

10. What happens to the graph of  $\sin \Theta$  after  $2\pi$ ?  $\cos \Theta$  after  $2\pi$ ? Why?

*sin -> it repeats* *→ You've went though the whole unit circle and whenever you go though it again you get the same result*  
*cos -> it repeats*

The majority of the answers for my Teacher-Centered class looked like the one above.

Most of the students knew that it continued in the same pattern, but they did not know “why”.

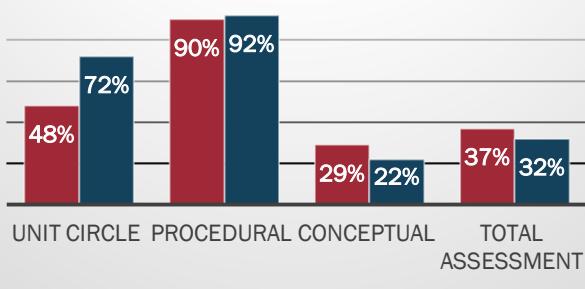
Since knowing why demonstrates a level of conceptual understanding, the students in my teacher-centered class were solely able to observe the graphs. The students who participated in the Student-Centered Class show a much greater level of conceptual understanding. This student in particular connects the idea that you can keep going around the circle to determine what happens after  $2\pi$ .

In all of the answers, it was evident which students seemed to have a deeper conceptual understanding. I then chose to look at the classes as a whole.

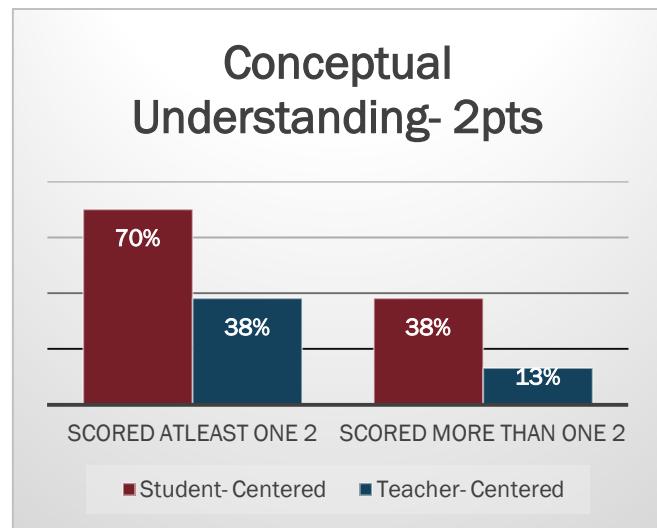
### Overall Analysis

When I first analyzed my data, I noticed that my teacher-centered class actually has 72% of the unit circle memorized. Even though the students in this class had the unit circle memorized, it seems to not have influenced their other score

### Student Centered vs. Teacher Centered



categories. While only 48% of the students in the student-centered classes had the unit circle memorized, they were able to answer the other questions in the assessment without recalling from memory. Next, I looked at Procedural Fluency. In both classes, the students were able to perform procedurally which was good to see.



Following analyzing memorization and procedural fluency, I examined their Conceptual Understanding. I was surprised by the results only yielding only 6% higher in the student-centered classes. So, I took a closer look. I noticed that conceptual understanding is not present until a score of two points. When considering the average of one and two, it is not going to be a significant amount. I then looked at the percent of students who scored at least one two, and the percent of students who scored more than one two. These results showed an accurate depiction of conceptual understanding. This chart, to the right, allows us to see the significant difference between the conceptual understandings.

### Student Experience

After observing this data we concluded that a teacher copying and pasting information into a student's brain is proving to be less beneficial for the students. Even though the assessment demonstrated a difference that supports a Discovery Learning, Student-Centered approach, I wanted to consider student reaction. At the end of their assessment, they were asked how their experience in the class was. From my teacher-centered class, I received responses that

either complemented me as a teacher, expressed desires to know more, or contentment because class was easy to follow. Some examples of those responses are as follows:

How was your experience in today's class?

I'm really confused but that might be my graphing abilities. I also don't know why we did the fill in unit circle thing. You were nice though 😊

**Evaluate your Experience.**

How was your experience in today's class?

Overall, I like the activity. The graphs and charts were helpful. I wish she had explained the reasoning, the "why" behind the concepts more.

These responses were not all negative, but they definitely displayed confusion or desire to know the reason why we can do what we do. Next, I looked at responses from my student-centered classes. These responses expressed excitement that they had retained information, understood a connection, or enjoyed being interactive. They are as follows:

How was your experience in today's class?

It was interesting, I didn't ~~completely~~ really understand it there's relationship for it

to the  
unit  
circle

Was this learning before? If yes, when?

How was your experience in today's class?

It was different experience and I probably won't forget the graphs now.

How was your experience in today's class?

I liked it today because now I understand why cos is x and why sin is y

These are a few examples that demonstrate how beneficial this type of learning was to the students. It was encouraging to see that the students gained so much for an experience. It is clear that the students left class with a lasting impression. It is also encouraging to see the student make such an important connection to the reasoning of why sin corresponds with y and cosine corresponds with x.

## V. Conclusion

In conclusion, this research experienced proved to be beneficial to my future experience as a teacher. The results of my research, in fact, demonstrate that when the students are the center of the classroom and have a chance to participate in an experience, they will have a deeper conceptual understanding of the concepts. When students have this deeper understanding it helps with their future. Students leave each class with a lasting impression which allows them to continue to recall what they have learned. It also allows the students to have a firm foundation, and allows them to make connections to other mathematical concepts. It was evident by the data

I collected that the students responded better to the activity in which they were the center. If students prefer this type of learning, it is important that we strive to enact this type of learning into our classrooms especially since the assessments show a deeper conceptual understanding. Benjamin Franklin says this quite well. He stated, “Tell Me, I forget, Teach Me and I may remember, Involve me and I learn”. Applying these results to a classroom influences students in a positive manner. After all of my research, I am able to conclude that when students involve themselves in a lesson; they not only take the responsibility for their learning, but they leave each day with a lasting impression.

# VI. Appendix

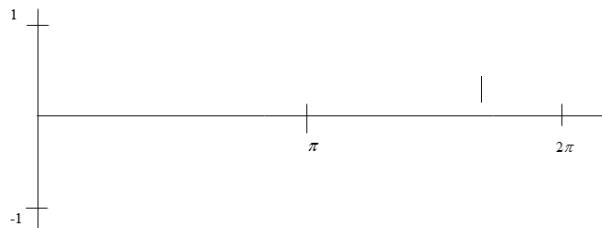
## Appendix A. Lecture Worksheet

### Graph of the Sine Function

Using the unit circle, fill in the table with the exact values of sine.

$x = \theta$ radians	$y = \sin \theta$	$(x, y)$	$x = \theta$ radians	$y = \sin \theta$	$(x, y)$
0			$\frac{\pi}{6}$		
$\frac{\pi}{6}$			$\frac{5\pi}{6}$		
$\frac{\pi}{4}$			$\frac{4\pi}{3}$		
$\frac{\pi}{3}$			$\frac{3\pi}{2}$		
$\frac{\pi}{2}$			$\frac{5\pi}{3}$		
$\frac{2\pi}{3}$			$\frac{7\pi}{4}$		
$\frac{3\pi}{4}$			$\frac{11\pi}{6}$		
$\frac{5\pi}{6}$			$2\pi$		
$\pi$					

Plot these points on the graph below.

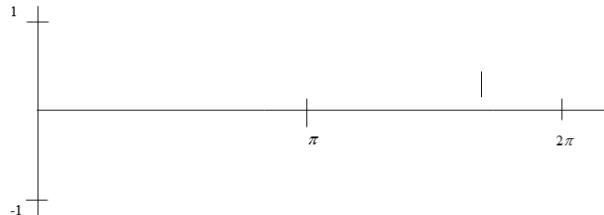


### Graph of the Cosine Function

Using the unit circle, fill in the table with the exact values of cosine.

$x = \theta$ radians	$y = \cos \theta$	(x, y)	$x = \theta$ radians	$y = \cos \theta$	(x, y)
0			$\frac{7\pi}{6}$		
$\frac{\pi}{6}$				$\frac{5\pi}{4}$	
$\frac{\pi}{4}$				$\frac{4\pi}{3}$	
$\frac{\pi}{3}$				$\frac{3\pi}{2}$	
$\frac{\pi}{2}$				$\frac{5\pi}{3}$	
$\frac{2\pi}{3}$				$\frac{7\pi}{4}$	
$\frac{3\pi}{4}$				$\frac{11\pi}{6}$	
$\frac{5\pi}{6}$			2 $\pi$		
$\pi$					

Plot these points on the graph below.



## **Appendix B. Discovery Learning Activity**

Today, you will be discovering the graphs of  $\sin\theta$  and  $\cos\theta$ . To complete this activity you need the following:

- A group of 3 people
- Construction paper & glue
- Spaghetti noodles
- Paper copy of the unit circle

The Instructions to this Activity are as follows:

1. Make sure your group has all materials needed to complete the Activity.
2. On your piece of construction paper, create a graph with an x and y axis.
3. Label the x- axis with the values of theta on the unit circle. (begin with 0, end with  $2\pi$ .)
4. Beginning with the graph of sine, we are interested in the side length
5. opposite of  $\theta$  of the triangles within the unit circles (or y coordinate on the unit circle).

This means the distance a given point on the unit circle is from the horizontal axis.

**Ex. At  $\frac{\pi}{6}$ , the y value is  $\frac{1}{2}$ . So your group would measure the vertical distance with the spaghetti noodle from the x axis on the unit circle to the point  $(\frac{\sqrt{3}}{2}, \frac{1}{2})$ . Break the noodle to represent the distance and then place the noodle vertical at  $\frac{\pi}{6}$ , on the graph you created.**

6. Repeat this process for the rest of the angles to create the sine graph.

7. Once you have discovered the sine graph, repeat this process on a new sheet of construction paper with the cosine function. (hint. For cosine, we are interested in the adjacent side length of the triangles within the unit circles (or x coordinate on the unit circle)

Class Discussion:

Key features	$\sin\theta$	$\cos\theta$
Intercepts		
Maximum		
Minimum		

Other notes:

## Appendix C. Assessment

**Compute the following:**

1.  $\sin \frac{\pi}{6} = \underline{\hspace{2cm}}$     2.  $\cos \frac{7\pi}{4} = \underline{\hspace{2cm}}$     3.  $\sin \frac{2\pi}{3} = \underline{\hspace{2cm}}$     4.  $\cos \frac{5\pi}{4} = \underline{\hspace{2cm}}$

**Answer the following:**

5. The value that corresponds to  $\sin \frac{\pi}{4}$  is  $\frac{\sqrt{2}}{2}$ . What does this mean?

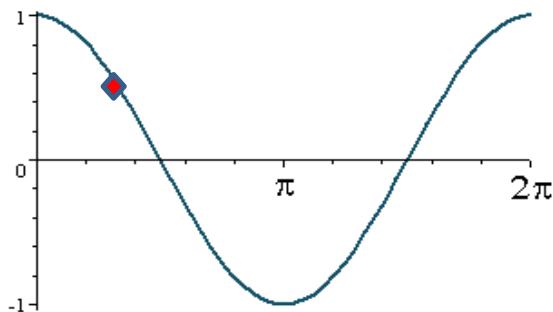
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6. The coordinate point  $(\frac{\pi}{3}, \frac{1}{2})$  has been marked in red on the graph of cosine. Explain the meaning of this coordinate point.



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7. On the interval  $0$  to  $\frac{\pi}{2}$  radians, the sine graph is increasing. Why?

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8. On the interval  $0$  to  $\pi$  radians, the cosine graph is decreasing. Why?

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9. What is the range of the sin graph and how is it determined? The cosine graph?

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**Predict the following:**

10. What happens to the graph of  $\sin \Theta$  after  $2\pi$ ?  $\cos \Theta$  after  $2\pi$ ? **Why?**

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11. What happens to the graph before  $\sin \Theta$  before  $0$ ?  $\cos \Theta$  before  $0$ ? **Why?**

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12. What do you think the graph of  $2\sin \Theta$  would look like? **Why?** (Draw the graph).

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**Evaluate your Experience.**

How was your experience in today's class?

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Have you seen the graphs of sin and cosine before? If yes, when?

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## Appendix D. Rubric

Question #	0 pts	1 pt	2 pts	3 pts
<b>Question 5</b>	Blank or restatement of question.	Mention the y coordinate & reference the unit circle.	State the connection of the definition of sine according to the triangle within the unit circle	State the connection of the definition of sine according to the triangle within the unit circle and thoroughly explain in this specific case.
<b>Question 6</b>	Blank or restatement of question.	Stated proper x value in reference to the unit circle	State the connection of the definition of cosine according to the triangle within the unit circle	State the connection of the definition of cosine according to the triangle within the unit circle and thoroughly explain in this specific case.
<b>Question 7</b>	Blank or restatement of question.	Mention the y coordinate & reference the unit circle.	Mention the increase in distance from x –axis or mention the length of the opposite leg is increasing.	State the connection of the opposite side lengths of the triangle within the unit circle and thoroughly explain how this makes the graph appear increasing
<b>Question 8</b>	Blank or restatement of question.	Mention the x coordinate & reference the unit circle.	Mention the decrease in length of the adjacent side.	State the connection of the adjacent side lengths of the triangle within the unit circle and thoroughly explain how this makes the graph appear increasing
<b>Question 9</b>	Blank or restatement of question.	State the correct maximum and minimum and/or reference that the graph visually	State the correct maximum and minimum with reasoning of the unit circle.	Correct maximum and minimum followed by a reasoning of the unit circle. Further

				justification necessary including a reference the triangles
<b>Question 10</b>	Blank or restatement of question.	Realizes it continues in the same manner, no explanation.	Realizes it is continues in the same manner, because the unit circle is continuous	Realizes it is continues in the same manner, because the unit circle is continuous. Proper justification, and possibly predictions of exactly what happens after $2\pi$ .
<b>Question 11</b>	Blank or restatement of question.	Realizes it continues in the same manner, no explanation.	Realizes it is continues in the same manner, because the unit circle is continuous. (understands the negative concept)	Realizes it is continues in the same manner, because the unit circle is continuous. Proper justification, and possibly predictions of exactly what happens before $2\pi$ .
<b>Question 12</b>	Blank or restatement of question.	States stretches with a proper graph.	Realizes there is a stretch because it is taking the length of the opposite side and doubling it.	State the connection of the definition of sine according to the triangle within the unit circle and thoroughly explain in this specific case how this would change the graph.

## VII. References

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