Symmetry Analysis of Inca Textiles and Ceramics

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History of Textiles and Cloth of the Inca

The textiles of the Inca are considered to be of major importance to their civilization. The intricate work and time spent on these garments are associated with their culture, tradition, politics, spirituality, and several other components linked to Inca society. With the absence of a writing system, the Incas were very dependent on these materials to pass down their own historical and cultural information. These textiles were constructed with individual and personal experiences. In other words, the designs in these textiles were based upon the weaver's experiences, understanding of the world, history, and their own memories. Several of these garments carried political and social identity [5]. Textiles were graphic indicators of several things including ethnic groups, marital status, rank in society, affiliation, and economic status. It was mandatory for the Incas to wear only the style and symbols of their social standing. As a result, only a select few and those of higher rank were adorned with tocapu, which were small geometric square patterns.[4]

The "unku", which was the traditional Andean male tunic, was a very common garment used in everyday dress. Some of these could have also been found on the elite, especially used in festivals and ceremonial events. Woven into these unkus were tocapus, which are square designs with abstract geometric forms. The majority of these patterns were arranged in rows. Many

scholars think the tocapus have aspects of a written language [2]. There is very little understanding however of the specific meaning of these abstract symbols.

The unku designs have been identified by four major styles of Pre-Colonial Inca. These include the Inca key, black-and-white checkerboard, diamond key, and the tocapu waistband. The Inca key and checkerboard are the two most common designs. These have been associated with the military and the elite. The tocapu unkus been affiliated with the Inca of high status, which would have made them heavily restricted.

The manufacturing and distribution of textiles in the Inca Empire was of a high regulation and standard. Along with projecting social affiliation, textiles were also used as political currency and were used in textile exchange between groups. For example, the possession of an enemy's textile represented victory. A warrior's success was linked to his garments [4]. Possession of the enemies clothing suggested harm and danger. Textiles were not only linked to politics and society, but also played a huge role in the religious ceremonies of the Inca culture [2]. There were also used in the adornment of the dead.

After the Spanish conquest, the regulation of textiles became less strict. The Spanish completely removed cloth restrictions. As a result, anyone could wear particular clothing that identified their social ranking. Therefore, former non-elite were allowed to dress and adorn themselves as they pleased, which completely obliterated the distinction of social rank with textiles [4]. Consequently, the style and significance of the unku began to change dramatically. Although the textiles have been modified over hundred of years, the Inca's have still maintained a strong textile culture. This has been able to preserve their traditions and is still portrayed as a vital function in their civilization.

The ceramics and pottery of the Incas were also of great importance to the civilization. Several Inca style ceramics include different variations, which are considered to be an indicator of the particular region is was produced in [6]. Here another argument arises with the Incas lack of a writing system. Many scholars suggest that these ceramic visuals played a key role in the communication of ideas about state authority and identity [7]. Previous analyses show that the elements on these ceramics reflect how ideas about person identity and society in the Inca Empire were conceptualized [6]. Similar to the textiles, the symbols, motifs, and iconography of the ceramics reflect several aspects of class identity and status in the Inca Empire. The ceramics we are interested in observing are those containing similar geometric designs to those of the tocapu.

Characterization of Symmetry: Planar and Three-Dimensional Ceramics

Planar Isometries:Textiles

An isometry of the plane is a transformation f of the plane that preserves a distance defined by the following: If d is a metric on the plane, then $d(f(x), f(y)) = d(x,y)$ for any points x, y. Three types of planar isometries are translations, rotations, and reflections. It is true for each of these that the image preserved by these distances must be equal to each other. Symmetries are associated with invariance of image. We define a set S to be symmetric with respect to f if $f(S)=S$. In other words, a set is symmetric if it maps to itself.

A translation can be defined as the movement of each point of a plane of equal distance, while traveling in the same direction. Each translation is defined by two constants, which we denote a and b. The translation, denoted by $t_{a,b}$, which is defined by $t_{a,b}$ (x, y) = (x + a, y +a) for each point (x,y)[1]. We can describe a translation as a motion of a plane that moves every point a specific distance in the same particular direction along a straight line. Note that image of a line must be parallel to it. We observe translational symmetry to be characterized as unbounded, with an unbounded reoccurring pattern. However, the textiles and ceramics we observe only contain bounded translations.

A reflection can be defined as the reflection of the plane at any point, about the x or y axis. It can be characterized by a figure that is reflected across a particular line, which creates a "mirror" image. Characterization of a reflection would require these patterns to create an image that reverses the orientation of the original pattern. The reflected image must be congruent to the original image. We can observe several patterns to be characterized by reflectional symmetry.

Let c,s be real numbers such that ($c^2+s^2=1$). A rotation of the plane about the origin by $r_{c,s}$ sends the coordinate (x,y) to the point (cx-sy, sx+cy). A rotation about (0,0) is defined by $r_{c,s}$ (x,y)= (cx-sy, sx+cy). Therefore, the main characteristic of a rotation is that the transformation of each pattern must be rotated about a fixed point, called the center, by a precise amount and direction[1]. One way to think about these different rotational symmetries is to characterize how they are arranged around a fixed point and their different periodicities. We can define an "n" rotation as the correspondence to a 360/n degree angle of rotation, or a rotation through an angle θ. The different patterns are characterized by the degrees of rotation around its fixed center. The different rotational symmetries are characterized by the angle measurement.

(Thus, symmetry with respect to a rotation through angle $\theta = 180$ would be considered a 2-fold symmetry). Symmetry under R_{180} is 2-fold symmetry. Other examples include the following:

$\Theta = 180$: 2-fold → $c = \cos(180) = -1$, $s = \sin(180) = 0$

$R_{180}(x,y) = (-x, -y)$

$R_{90}(x,y) = (-y, x)$

$\theta = 120$: 3-fold

$\theta = 90$: 4-fold

$\theta = 60$: 6-fold

We will also note that symmetry under R_{60} is also under R_{120}, R_{180}, R_{240}, etc. This holds true for any rotation containing any multiple of 60.

We can form a rotation matrix that rotates the points about the origin of the plane, through an angle θ.

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

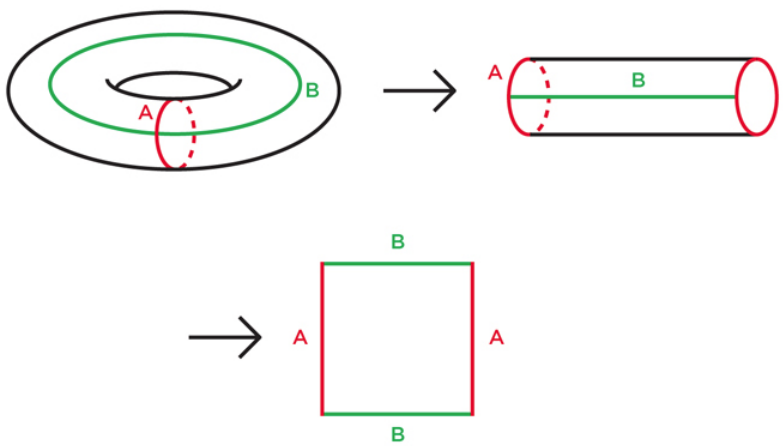
This is used for a rotational direction in the plane. For three-dimensional space rotations we use the matrix:

$$R_y = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

These are found in both the planar symmetries of the textiles, as well as the symmetry of surfaces seen in the ceramics.

Symmetry of Surfaces: Ceramics

The symmetry of three-dimensional surfaces corresponds with the coordinates of any points (x,y,z). They are similiar to planar isometries; however, the symmetry of these surfaces can be interpreted in different ways. The translations seen in these ceramic surfaces might be represented as a repeating pattern around a specific shape, which can also be characterized by rotational symmetry. It also preserves a distance between two points x,y, however the patterns are repeated on the surface, often in a circular shape. Rotations now are about a line of the plane (x,y,z), called the axis of rotation. This is often seen in both the shape of the surface, as well as the design patterns.



Assume there is a translational pattern on the place. If we were to take the top and bottom edges and connect thing, we could form a cylinder. Thus, the cylinder would also have the same translational pattern repeating around the surface of its shape.

Reflections of these surfaces are still considered to be a figure that is reflected across a particular plane, creating a mirror image. This is most often only seen from the shape of the ceramic. Symmetries in place are stilled defined as f(S)=S, a mapping of a set onto itself. However, we have two things to consider. One being the shape and structure of the pottery and the other being designs created on the ceramics.

ANAYLSIS

We can characterize the Incas to have used several reoccurring patterns and symmetries seen throughout their textiles and ceramics. Their symmetries can be separated into groups, and can be characterized differently according to their dimensional shape. We observe several of our own individual photos from Museo de Larco, Museo de Art, Precolombino, and Museo Inka. Other photos we observe are illustrations by Christiane Clados, depictions of tocapu, and Guamon Poma's sketches.

We also observe several patterns to be rotational, both planar and spatial. The rotational symmetry is seen more commonly in the ceramics. However, notice that they are characterized by the patterns and not always by the shapes and color coordination. Some of the bowls and vases are completely rotational with their shapes, and others might only contain reflectional rotation about the y axis.

Planar: Symmetry in Textiles

Several textiles used in Guaman Poma's "The First New Chronicle and Good Government" portray clear depictions of Inca tocapus. This work was written as a critique of Spanish colonial rule that expressed a view of local nobles of the Spanish conquest. Poma draws parts of his critique from different Inca social and environmental structures. He includes several illustrations and graphics that suggest how Inca rulers were dressed. These certain illustrations contain numerous repetitive patterns and symbols that are very similar to those in the many

concrete textiles and ceramics. We observe the differences between the symmetry in the individual tocapus, as well as the repeating patterns involving multiple tocapus.

The majority of the illustrations used in this book contain finite translational symmetry. Note the periodicity of threes as well as the common reoccurring patterns. The movement of the patterns is of equal distance that travels in the same direction.

Translations

[Guamon Poma: Drawing 25. The first Inca, Manco Capac Inca]

This particular translation, as seen in many other illustrations is characterized by its diagonal direction shift. These images repeat in orders of three and contain a non-alternating

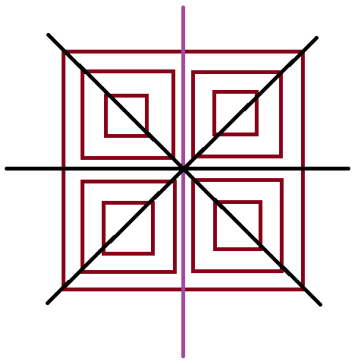
pattern. There is also an idea of constant “slope” that we can identify to be traveling down 2 and right 1. We also note that the individual patterns contain 2-fold rotational symmetry.

In several other patterns we observe this translational pattern of multiple tocapus, and the rotational symmetries of each individual squares. In several of the Inca textiles, we also observe alternating patterns and an increase in the number of repetitions for each arrangement. Several reoccurring patterns that appear to be translations are the maze-like pattern and stair patterns, as seen below. Translations from these textiles can be observed within individual squares, as well as within the entire textile. We can observe patterns that contain equal images that have shifted in some way in a specific distance, and in a specific direction. We can characterize these translations by their direction, and the number of times the images repeat. Several noted translations are those along the diagonal, and shifts that create alternating patterns.

[National Museum of Archeology, Anthropogy, and History]

We observe the tunic on the right hand side to contain a translation on the diagonal. We can also note a slope of down 1 right 1. This can be observed separately for each of the black and white squares. Note on the left tunic contains this same idea of "slope". However, with this pattern we take into consideration the color and pattern of each square. If we observe two individual adjacent squares of different coloring, we also see a reflection about the vertical axis.

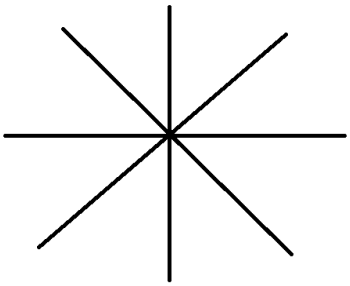
Reflections

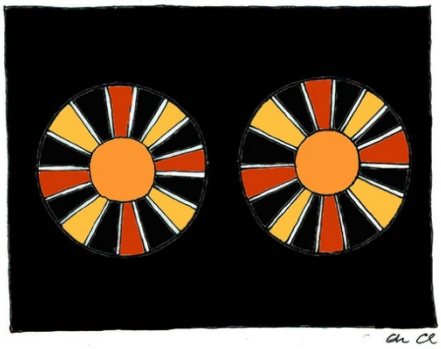


The illustration of the Fourth Inca, Mayta Capac Inca provides an example of several reflectional symmetries. There are reflections across the vertical, horizontal, and diagonal axes of this particular pattern. This would be considered to have 4 lines of reflection, as well as 4-fold rotational symmetry within squares and within the entire textile. This also reappears in the "star" pattern seen throughout these illustrations. The examples below demonstrate an image and its reverse across the vertical, horizontal, and diagonal axis.

We can observe this image to be of several different types of rotational symmetries. If one were observe the symmetry with respect to color, it would be considered 4-fold rotational

symmetry. We could also group the individual sections, which would be seen as 16-fold symmetry. We can look at groups of both 2 and 3, with separators. For this particular image, it depends on whether one ignores the color with respect to its particular rotations.

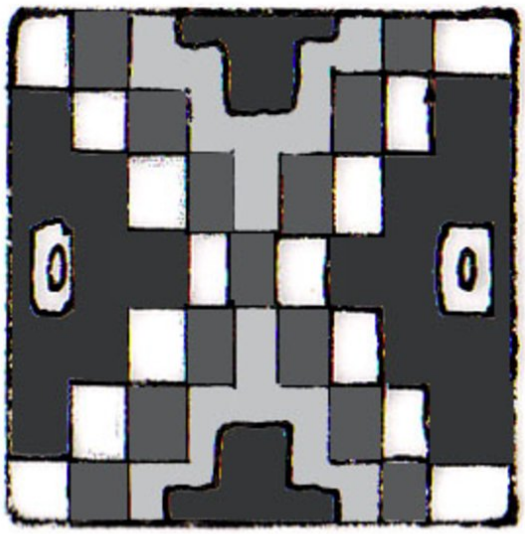




[Museo de Larco]

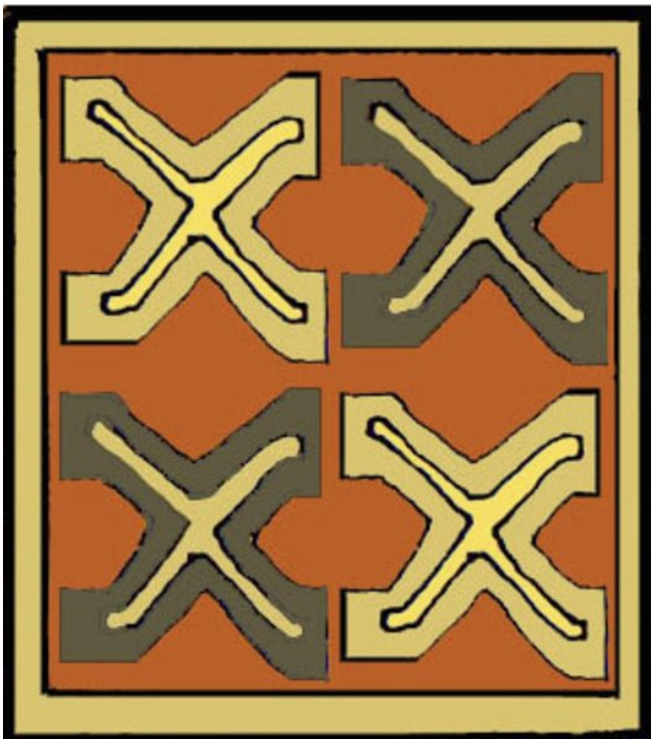
The star pattern above is characterized by rotational symmetry. With respect to color, we consider this to be 4-fold rotational symmetry with R_{90}. We could also consider this to be 16-fold rotational symmetry. Dividing the petals into groups of 3 would also create other possibilities for rotations. These star shaped club heads were used in war, which were often depicted in Inca ceramics.

Variations of this similar polygonal pattern of the "X" creates a reflection about the vertical axis, The different variations of the "x" pattern creates a mirror image along both the horizontal and vertical axis. Depending on the angle between the lines, note that this particular pattern may not be also characterized by 2-fold rotational symmetry. For this image, we will note that the corners of the tocapu are not precise depictions.



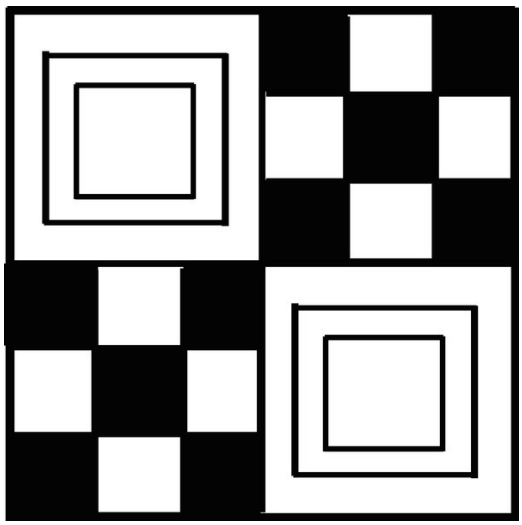
[Christiane Clados: Barthel 7 Tocapu Variation, Colonial Period Tunic]

Observe the reflection along both the horizontal and vertical axes. Note this reflection is not along the diagonal axis because of the ends of the “X”.



[Christiane Clados: Barthel 18 Tocapu]

We note the symmetries in each if the 4 individual sections of the square. Observe the reflections about the horizontal and vertical axes. In the entire square, we also observe the same reflections about the horizontal and vertical axes. It is not very common to find the same symmetries within both the whole and each section. However, note that these symmetries are not associated with the color. The color is only associated with the individual patterned reflections. We could also consider this to have a diagonal translation of the repetitive “x” pattern. If we were to believe this tocapu to be repeated on a textile, we could interpret this symmetry as a diagonal translation if a repeating pattern is considered (for example on the torus).



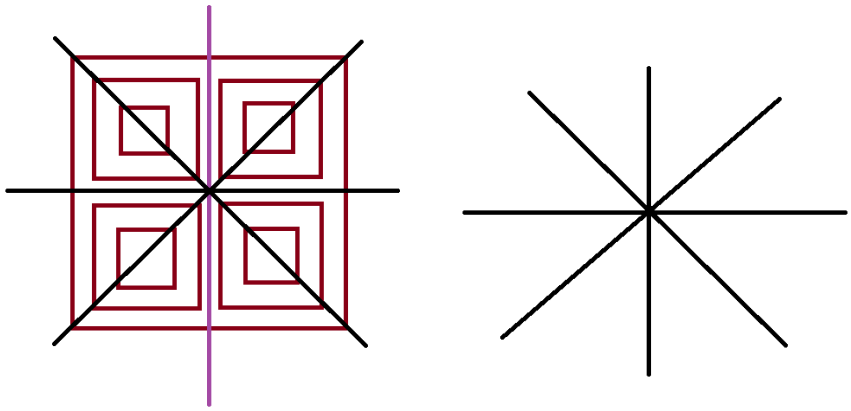
[Christiane Clados: Tocapu- Nested boxes and checkerboard forming a quartered Tocapu]

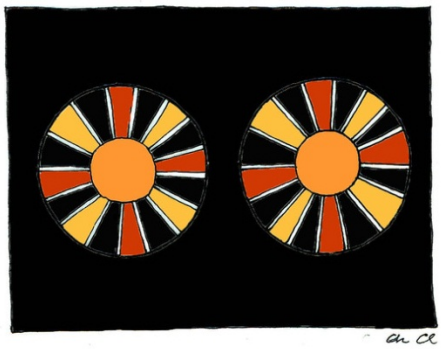
We also observe tocapu to contain reflections across the diagonal in the illustration above.

Rotations

The illustration of the Fourth Inca, Mayta Capac Inca also provides an example of 4 fold-rotational symmetry. However, this polygon could also be categorized by 8-fold rotational symmetry. This particular pattern has a fixed center where each of the angles of rotation intersects. Therefore, this polygon can be characterized as a rotation through 90 degree or 45 degrees. The following rotation matrix rotates the points about the origin of the plane through the angle of 90 degrees.

$$R= \begin{bmatrix} \cos(90) & -\sin(90) \\ \sin(90) & \cos(90) \end{bmatrix}$$





[Christiane Clados: Flowers or Stars- Keru. Inca-Colonial Style]

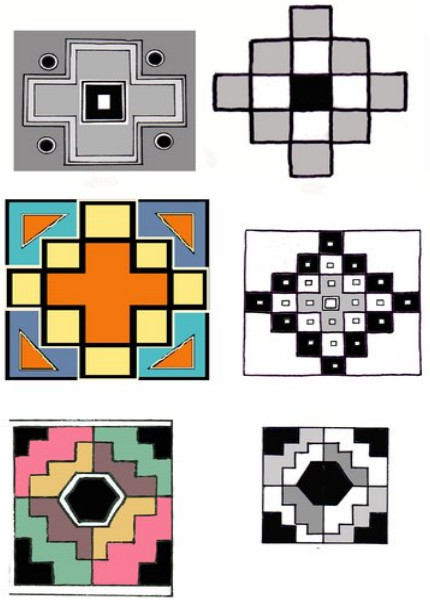
The flower/star pattern is also n example of 16-fold rotational symmetry, but not with respect to color. The following rotation matrix rotates the points around the origin of the plane through the angle of 45 degrees.

$$R=\begin{bmatrix} \cos(45) & -\sin(45) \\ \sin(45) & \cos(45) \end{bmatrix}$$

Several other variations seen throughout Inca textiles are characterized by the same n-fold rotations. Some of the following are examples of these.

[Christiane Clados: Diamond or Cross- Decoration of a four-cornered hat]

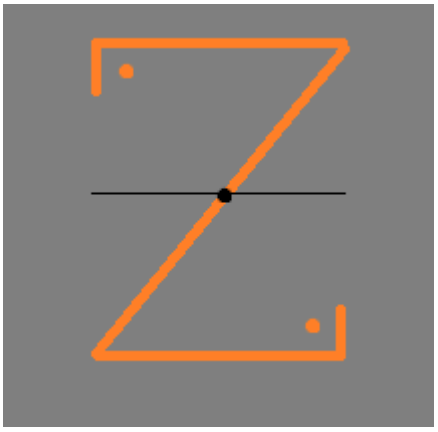
The cross, the four cornered hat, and the square/ rectangular motifs are reoccurring designs that are characterized by a 4-fold rotational symmetry [3]. Thus, the pattern reoccurs every time the image is rotated 90 degrees about the fixed center. However, we note the motifs containing a hexagon in the middle. We observe these particular patterns to contain only 2-fold symmetry. The hexagon throws off the 4-fold symmetry because the greatest common divisor of 4 and 6 is 2. Thus, it creates an illusion of 4-fold symmetry. From observations, the textiles studied contain simple rotations and the ceramics are composed of more complex rotations. The textile patterns seem to create mostly 2-fold and 4-fold rotational symmetries.

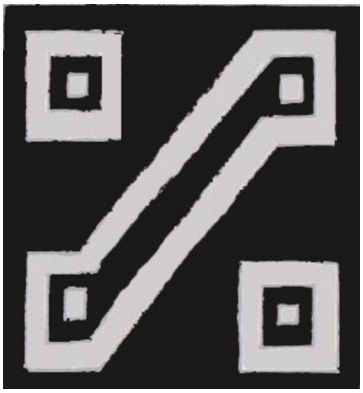


[Christiane Clados- Square/ Rectangular Motif]

Note that the first two illustrations are symmetric with respect to color as well. Observe that the other tocapu squares are only with respect to pattern. Note that the bottom two designs appear to be the same type of illusion patterns again.

This symbol is characterized by 2-fold rotational symmetry. Therefore, this polygon can be characterized as a rotation through 180 degrees. Similar patterns often appear in the textiles.




[3]

[Christiane Clados: Barthel 24 Tocapu]

[3] [Man's tunic(uncu) with Inca Key Tocapus varied in color. Staatliches Museum fuer Voelkerkunde Muenchen]

We observe the individual square and the entire tunic to be different symmetries. The first square is of 2-fold, while the upper half of the tunic can be considered a finite translational

pattern, which repeats every 2 squares. The translation alternates color throughout the textile. We also observe the entire textile to contain a reflection across the vertical axis. However, note that this does not include symmetry with respect to color. Also note the periodicities seen in this textile. We observe there are 6 horizontal lines at the bottom, 8 rows of squares, and 10 columns of squares. Note that $6^2 + 8^2 = 10^1$, is a Pythagorean triple.

Ceramics and Symmetry in 3-Dimensional Space

Several ceramics observed from the various museums in my own photos contain both similar and very different characterized symmetries. We first note the symmetries in space to have several differences compared those in the plane. (In contrast to the textiles). Several of these ceramics have differing symmetry according to the spherical shapes to the ones characterized by the designs on the actual shapes. We observe several of these Inca artifacts to have very similar symmetries in a different way of characterization.

Translations and Rotations

Note that the "translational" patterns on the ceramics often are seen as patterns on the outside or inside of the ceramic—these are rotations. These patterns are very common on bowls, which usually shares the same symmetry with the shape as well as the pattern. The translations occur in a directional pattern that is formed around the bowl in a circular shape. If one were to take the pattern and set it into a plane it would look very similar to a line of translational shifts of

a reoccurring image.

[Museo de Larco]

This image contains a translational pattern that repeats around the bowl in a circular motion. Note that the lines of these rectangles have curvature. We can compare these patterns to those depictions of Guaman Poma's sketches. He also has individual squares that contain these rectilinear shapes with curved lines. We also note the periodicity of this ceramic. We observe it contains 8 lines of rectangles, and then a rotational fold of 6. Note this reoccurrence of two numbers seen in the earlier Pythagorean Triple.

Reflections

We observe the ceramics to have mostly reflections about the vertical place. Note that some of these may be with respect to shape only, both shape and pattern, or only with respect to patterns. We will consider the axis of reflection to be along vertical line straight down the middle of each ceramic.

[Museo de Larco]

These vessels show reflection symmetry with respect to both its shape and the geometric patterns.

[Museo de Larco]

This vessel contains a reflection about the vertical axis. Note that the ceramic has a reflection only with respect to its shape and not with respect to pattern.

[Museo de Larco]

Observe the reflection of symmetry also about the y-axis. Note this reflection is also only characterized with respect to its shape. However, observe the rotational/ translational pattern that is seen on the outside of the vessel. We observe the pattern to be considered a translational

reoccurring print. Without respect to the shape, we take the center of rotation to be the vertical line in the middle of the pot. Hence, we would characterize this pattern as 8-fold rotational symmetry. Therefore, there is a rotation occurring every 45 degrees. However, note the 10-fold symmetry seen from the top circular pattern. We see this reoccurrence of illusion, where the ceramics seems to be 8-fold. However, the greatest common divisor of 8 and 10 is 2. Thus, this particular vessel has only 2-fold rotational symmetry. We observe that the symmetries within this piece are characterized differently with respect to its shape and pattern. Also note the reoccurrence of 6, 8, and 10.

Conclusion

There are several different ways of characterizing these textiles and ceramics based on shape, color, and pattern. Several of the same patterns seen in the planar symmetries also are characterized with spatial surfaces. They are many similarities and differences; however it is fascinating to see the connection between both artifacts. We begin to notice that several of these patterns contain very similar symmetries which could suggest to us some unknown element about the Incas.

References

[1] Stillwell, John. The Four Pillars of Geometry. (2000). 3.6 Isometries p. 57-61.

This chapter in the text offers brief definitions and well-written explanations for the types of symmetries discussed in the Characterization section. It provides formulas, pictures, and examples corresponding to each explanation for translations, rotations, and reflections.

[2] Frame, Mary. What Guaman Poma Shows Us, But Doesn't Tell Us, About Tukapu. Nawpa Pacha. Journal of Andean Archaeology Volume 30, Number 1.

This article provides a good source of pictures with illustrations including tocapu garments and how they were used. It provides information on the relationship, organization, and connection through the patterns seen through these illustrations. It suggests how these textiles represented rank, time periods, symmetries, partition of groups, and reoccuring shapes. Ultimately, it discusses the organization and interpretation of the textiles.

[3] tocapu.org. Analysis of Prehispanic Cultural Signs Database. Images.

This provides illustrations and good visual representation of specific tocapu patterns. It provides categories of these patterns, and provides the specific time period and culture. From this source I was able to identify distinct patterns used from personal photographs of these textiles.

[4] Seguin, Alexandra. “Politics and Identity through Clothing: The Significance of Inca Textiles & Garments in Empire and Colonial Andes as Visualized Through the Unku.”

This provides great background information about tocapu, and the “unku”. Gives information about several characteristics it portrayed, how it was symbolized, and the importance of the role of textiles in the Inca Empire. Gives examples about how textiles were used in society, how they were distributed, and other components it represented.

[5] Centre de Textiles Tradicionales del Cusco http://www.textilescusco.org/eng

This is the website for the Center of Traditional Textiles in Cusco, Peru. It provides very useful information about the weaving and importance of textiles. We were able to visit this museum during our study abroad trip, and it offered very useful information about the importance of textiles in Andean society.

[6] Costin, Cathy Lynne. “Material Symbols of Status, Class. and Power in The Inka Empire.”

This provides great background information about the style and variability of Inca ceramics. This article specifies what the ceramics could have represented and what they could reflect about Inca society. This gives more details about what the symbols and motifs on these ceramics could have reflected about class identity and status.

[7] Bray, Tamara L. “Inka Pottery as Culinary Equipment: Food, Feasting, and Gender in Imperial State Design.”

This abstract shows how Inca ceramics were examined in terms of their functional importance. It also provides information about the correlation to politics, gender, and the consolidation of Inca state power. This provides useful information for the background section on ceramics.