Creatively Disrupting The Mathematics With The Humanities

Capstone 2014

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Abstract:

This paper will discuss the importance of incorporating the humanities within a mathematics classroom. An experience was designed for a calculus II class in order to analyze such effects. Students observed a presentation influenced by the history of limits, answered concrete and thought-provoking questions, and shared additional thoughts by writing a journal entry.   The primary purpose of this paper is to analyze what the calculus II students gain after being exposed to this experience. It will also provide one with possible methods that can be incorporated into the classroom.

Introduction

The Mathematics department at Georgia College and State University requires their students to complete a capstone research project. For me, this was a great opportunity to incorporate my philosophy minor as well as the additional studies that I partake in outside of school. I'm currently in school to get a mathematics degree with a teaching concentration and I will also obtain a philosophy minor when I graduate. Besides my love for math, I'm involved and passionate about many things. I'm currently in the process of completing two years of research to present at a diversity conference in the spring of 2015 on Buddhism and Meditation. This presentation will reflect the way studying Buddhism and practicing meditation has affected my journey, my perspective, and my appreciation. I'm a singer/songwriter, guitarist, and performer and spend most of my time outside, as I am a half-marathon runner and mountain biker. After many hours thinking, discussing, and contemplation, I knew that it was necessary for me to create an experience within the mathematics classroom that involved topics such as the hobbies that I'm interested in. That is why I created an experience called Disrupting Mathematics With The Humanities for a calculus two class. In the early stages of this project, I realized that if I had seen how mathematics could be applied to all areas within the humanities while I was in high school, then I am certain that I would appreciate mathematics more than I do now and I could have seen how it affects many aspects of my life earlier in my mathematics career.

Observation

Having had my own experiences in high school mathematics classes, I have been able to reflect upon them and apply what I've learned along my journey to obtaining my mathematics degree. It became clear that up until calculus, I was getting perfect scores on all math exams and receiving awards for placing at the top of my math classes. However, looking back on it now, it's obvious that I was able to regurgitate exactly what my teachers were showing me in class. I did not have the contextual understanding for why I was completing the tasks I was asked to do. It was not until calculus where this severely changed. I was asked to think theoretically and abstractly and I had not had the opportunity to do this in any of my other mathematics classrooms. Fortunately, there have been plenty of opportunities for me to observe how a mathematics class is currently run within multiple high school classrooms. This has led me to witness the strong disconnection that still exists upon our students within the classroom. Taking student's comments, teachers reactions, and the procedure that sets the stage for most classes directed me to believe that something needs to change; the experience that most students are having in mathematics classrooms are monotonous, lack creativity and excitement, and more importantly, the mathematical procedures fail to provide depth, importance, and context to every student. Having this realization, it became clear to me that students should experience the enthusiasm that exists beyond equations and explicit directions; students need to be introduced to the very details of mathematics and the uniqueness of the many applications of mathematics that exists in all aspects of life.

My Vision

It is my goal to create an experience that student's have not had the opportunity to have, to show that it is possible and it is necessary to incorporate the humanities within a mathematics classrooms, and to ultimately prove that by incorporating these ideals in an environment that is known to be repetitive and uninteresting by most of the population, it will influence and encourage creative thinking amongst all students in a mathematics classroom. My vision is to ultimately encourage creative thinking amongst all students in a mathematics classroom by incorporating applications that involve all humanities. I want to create an experience that student's have not had in the past and allow all students to find their own connection with mathematics. I will create an environment where all students are encouraged to ask more than "why", but also "who", "what", "when", "where", and "how". These questions are encouraged in most classes but are not typically found in the mathematics classroom. Lastly, I will show that it is possible and it is necessary to incorporate the humanities within a mathematics classroom. By doing this, more students will see the purpose and the uses of mathematics. Students will be able to continue mathematical education by being able to apply it in more aspects of their own life.

The Experiment

The procedure first began with me receiving approval to conduct surveys, present, and collect data from a Calculus II class at Georgia College and State University. Calculus is typically where high school students are introduced to abstract concepts that are heavily influenced by proofs and the development of theoretical thinking. After observing a few calculus classes and providing one on one tutoring to calculus students, I immediately witnessed students withdrawal completely from the information in front of them; this uncertainty stems from a class that is no longer procedural based for difficulties arise when students are expected to think conceptually when they have not had that opportunity in the past. This, of all classes, is arguably one of the most important classes to influence a student's connection to mathematics. I say this because calculus demands different qualities from a student than say an algebra or geometry class. The concepts are incredibly abstract for it is where a student is introduced to and are expected to simply accept infinity, limits, derivatives, integrals, and other nonconcrete ideas. It is because of this, students are more likely to stop pursuing more mathematics education after one calculus class. It is often forgotten that the time in which calculus was introduced, it was not easily accepted. In fact it shook mathematical and philosophical ground. This is particularly why my experiment was placed in a calculus II course. The students in this class were continuing their mathematical education as they are in the works to become future engineers, mathematicians, and scientists. These students are unique because unlike most of the population, they

are now experiencing a class that involves more rigor, abstract thinking, and the continuation of calculus I.
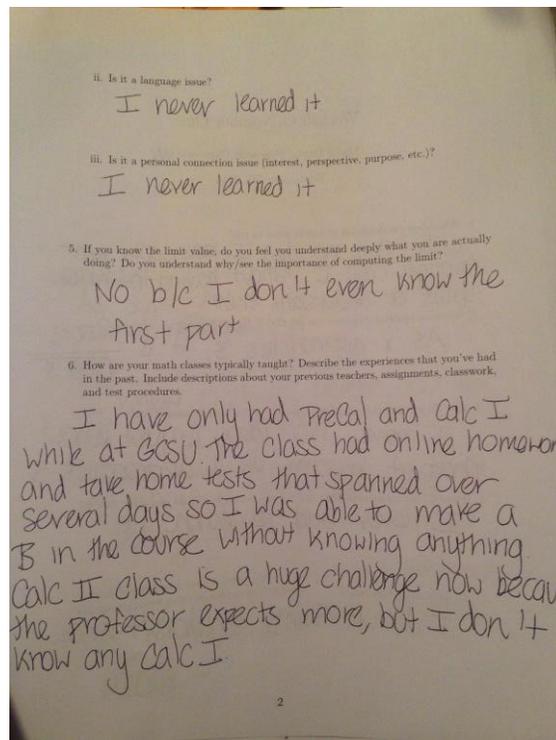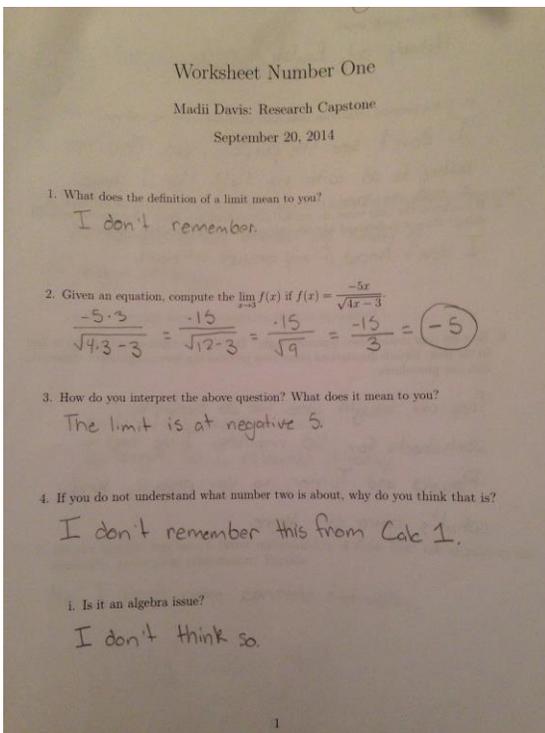
Limits

A topic in calculus that tends to be incredibly difficult to grasp is limits, which happens to be one of the first topics calculus students are introduced to. Although I was presenting to a calculus two course to create an experience most of them have not had before, I felt that it would be beneficial to review limits, with the expectation that they had a good idea of what a limit was and how to solve a limit problem. This is a typical topic for students to grasp when introduced to calculus one which caused me to think that this would be an interesting, thought-provoking, and controversial discussion to have with the class. Thus, I thought that it would be beneficial to conduct two surveys; one would be given prior to the lesson and the second would be passed out to the students to conclude the presentation. This would allow me to see the information that was obtained from their prior experiences as well as the information that was gained after experiencing the environment I was hoping to create.

Survey Number 1

The first survey consisted of a number of free response questions that allowed each student to write as much or as little as they wanted. The purpose of this first survey was to see where they stood prior to the experience I was creating for the class. The survey asked the student to define a limit as well as solve a limit problem, of which reflected an example very similar to a introductory level limit

problem found in their text book. To have a better grasp on each student's

understanding of a limit, I also asked him or her to interpret the problem and share

their understanding of what the question, as well as the answer, actually meant.

Both surveys included questions such as "what does the definition of a limit mean to

you" as well as two example limit problems. I left it up to the student to explain why

they didn't understand what it meant by asking questions such as "is it an algebra

issue", "is it a language issue", or is it a "personal connection issue (interest,

perspective, purpose etc.)". The first survey also aims to understand the kind of

experiences that each student has had in their mathematics classroom. Below is a

student's response to the first survey.

Worksheet Number One

Madii Davis: Research Capstone

September 20, 2014

1. What does the definition of a limit mean to you?

    I don't remember.

2. Given an equation, compute the $\lim_{x \to 3} f(x)$ if $f(x) = \frac{-5x}{\sqrt{4x-3}}$

    $\frac{-5 \cdot 3}{\sqrt{4 \cdot 3 - 3}} = \frac{-15}{\sqrt{12-3}} = \frac{-15}{\sqrt{9}} = \frac{-15}{3} = \boxed{-5}$

3. How do you interpret the above question? What does it mean to you?

    The limit is at negative 5.

4. If you do not understand what number two is about, why do you think that is?

    I don't remember this from Calc 1.

    i. Is it an algebra issue?

    I don't think so.

    ii. Is it a language issue?

    I never learned it

    iii. Is it a personal connection issue (interest, perspective, purpose, etc.)?

    I never learned it

5. If you know the limit value, do you feel you understand deeply what you are actually doing? Do you understand why/see the importance of computing the limit?

    No b/c I don't even know the first part

6. How are your math classes typically taught? Describe the experiences that you've had in the past. Include descriptions about your previous teachers, assignments, classwork, and test procedures.

    I have only had PreCal and Calc I while at GCSU. The class had online homework and take home tests that spanned over several days so I was able to make a B in the course without knowing anything. Calc II class is a huge challenge now becaus the professor expects more, but I don't know any Calc I.

This particular student was able to correctly solve the limit problem but was unable to actually give a reason for the answer or define what a limit was. This is an example where a student is able to memorize a process of solving the limit problem, but not being able to apply any reasoning behind the process. This is a reflection of the experiences that they have had in the past. The student then continued to describe what it has been like taking a Pre-Calculus and Calculus course at Georgia College and State University and stated that "The class had online homework and take home tests that spanned over several days so I was able to make a B in the course without knowing anything. Calculus two class is a huge change now because the professor expects more, but I don't know any calculus one." This is a harsh reality of the processes that exists in high school classes but actually continue in college level mathematics classes. Regardless, this student was able to effectively show be his/her stance on limits prior to the experience. This student, among many shared useful information as they described what it was like in their previous classes. They also displayed their knowledge on limits by solving and explaining as much as they could; in respects to the introductory level problem that they were asked to complete.

Journal Entry

Before I began with my presentation, I asked the students to keep a journal entry. This served as an informal way to assess what the students were thinking and feeling during the presentation. Typically in a classroom, you'll have maybe five students out of thirty who feel incredibly comfortable with talking about
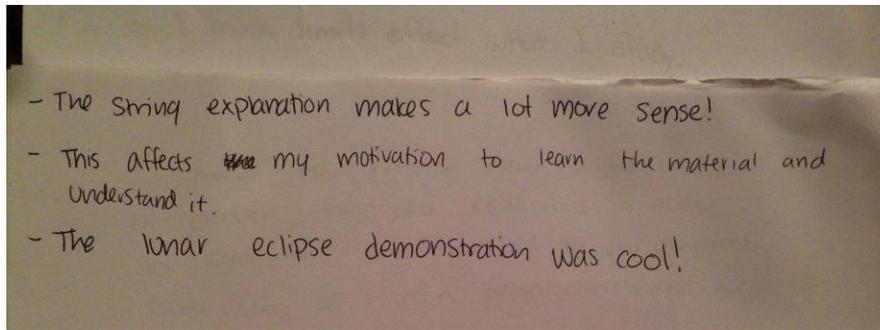
mathematics in front of the class. However, if this is the case, you have a number of students who have not explicitly shared their thoughts with the teacher. To me, this is a problem. As a future teacher, I want every student to be present, involved, and thinking at all times. By incorporating journal entries during this presentation, a teacher is able to get feedback from every student, allowing each of them to share what they are thinking at all times. This entry would be used as a way for each student to freely communicate with me, as an educator. I asked that each student write what he/she was feeling, if they had any problems understanding the materials, or if a student felt uncomfortable asking a particular question, to write it down. I wanted each student to feel comfortable with communicating with me and in order to implement that kind of environment, I felt that it was necessary to ask for student's feedback in multiple ways. This entry was to be completed on a blank sheet of paper to enable the student's creativity and thoughts, I received beneficial feedback from each student; all that was needed was a blank sheet of paper for them to express their thoughts to me.

Figure 1:
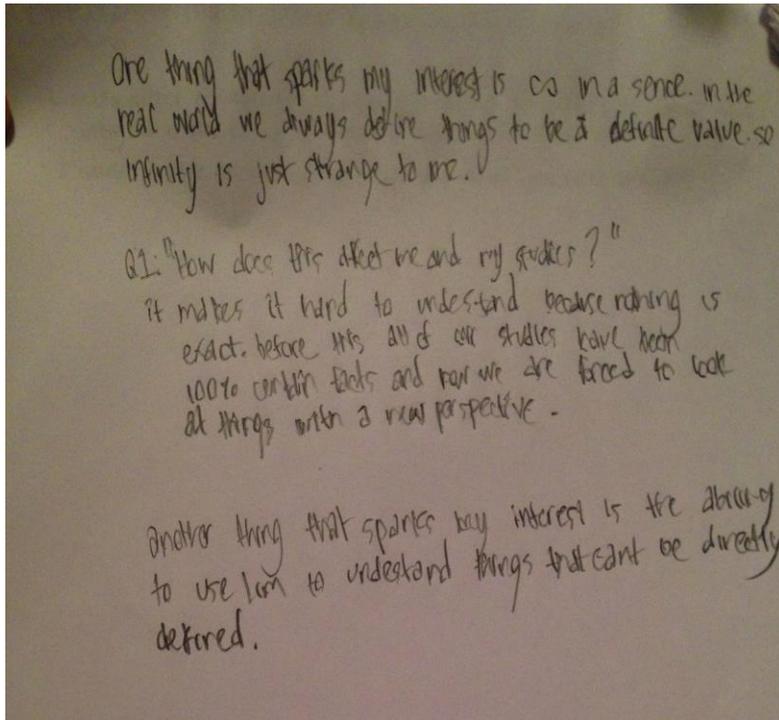
In figure 1, this student expounds upon infinity and reflects how they have not seen it applicable to the real world, however it interests the student. The student also expresses that infinity is a strange concept and that the idea of something not necessarily being defined furthers his interest. In figure one, you see that a question is being asked, but the student continues to attempt at answering his own question and states that they are "forced to look at things with a new perspective."

Figure 2

- The String explanation makes a lot more Sense!
- This affects ~~the~~ my motivation to learn the material and understand it.
- The lunar eclipse demonstration was cool!

In figure 2, this student shares that the use of manipulatives helped them see their own connection and understanding of limits. This was beneficial information because this student shared with me that the experience that I created benefited their understanding of this topic. This wasn't information that I hadn't explicitly received while I was in the classroom with them. Overall, receiving this student's feedback allowed me to understand into more depth how well the class was taking to the experience that I created for them.

Another student used their journal entry to think about the creation of mathematics. They state ""Math was created…" That makes me question how accurate it is. What if our understanding of math is wrong? This would majorly change the way math, engineering, physics etc. are thought of." This is heavily influenced by philosophical thought.

The student's feedback showed me that journal entries were an excellent addition to my experience. I was able to actually see student's thought processes. Whether I agreed with whatever they shared or not, it showed me that they were thinking and interpreting their thoughts to me.

Manipulative Number 1

To begin the official experience, I began with a manipulative to display the idea of a limit. I took a string and cut it in half. I continued this process to display that I can half the string infinitely many times, however as I continue to half it, it gets closer and closer to not exist anymore. The purpose of this was to show the students an example of when something gets closer an closer to zero but never actually reaches zero.

The Presentation

Before going into the classroom, it was imperative that I planned carefully in order to execute this correctly. The most difficult part was creating a presentation that was different, powerful, informative and interesting so that I fulfill my goal of giving students a new experience in a mathematics classroom. Thus, as a future educator, I put myself in the place of a current calculus II student and asked myself "what is a limit", "why is it important", "what is significant about learning limits". From these questions, I began to derive a presentation that I felt was quite different than anything I was ever introduced to in Calculus II. I found it necessary to begin as if it were a regular mathematics power point presentation so that when I actually taught, I could observe their reactions and assess if this is something that they were familiar with or not. That being said, I used the formal definition of a limit and what it means to be a tangent line. I purposely structured these slides similar to slides that may be included in a typical classroom of which the students should be familiar with; it included a standard definition and a simple image that displays the term. This was influenced by what they see in their textbooks or their classroom notes.

However, after this, I wanted to initiate a thorough discussion by asking questions that were meant to expand the definition in order to get them thinking. So, I disrupted the classroom and said "Let's Talk". After asking questions like "what's infinity", "how have you found the equation of a line previously", "what happens when you only have one point and one line", "why is this important", I gave the students time to reflect their thoughts and give them a moment to share them with me via their journal entry if they wanted. I then asked them to describe the experiences that they've had in a mathematics course. We, as a class, collected a number of adjectives that described their feelings. On the board we wrote "precise", "boring", "hard", "important", and "logical". I took their words and said that they displayed mathematics as a monotonous, uncreative subject, and the students agreed.

I then introduced the fathers of calculus: Newton and Leibniz. I expressed to the students that the history of calculus was not at all monotonous, instead it was completely controversial and initiated new perspectives amongst many aspects of society. Calculus was controversial for many reasons, but we discussed the debate on who actually provided the foundation for calculus, as well as why both Newton and Leibniz's approaches were so different. The class and I also spent time thinking of infinity and comparing it to finite and why it was and still is a very difficult concept to grasp and how it created unstable philosophical and mathematical ground.

I proceeded to talk about Isaac Newton and Gottfried Leibniz independently. I consciously thought about how I wanted to display this information, so I designed

the slides by incorporating influences from what students may have seen in a high school history course. Each slide was dedicated to one mathematician, included a picture, and a list of facts about the respective mathematician. In Newton's slide, I included his purpose for calculus, where he applied calculus, and the impactful texts he published. I also included an example that I found in their calculus textbook and the purpose of this example was to display a modern problem that Newton may have applied his studies in order to solve the problem. Similarly, I created a slide for Leibniz. The students were able to see two perspectives of calculus, their own interpretation and application of calculus, and example problems from their own textbook that was influenced by their discoveries. To conclude this PowerPoint, I dedicated a slide that only said "Why Limits". Again, this was a chance for each student to think, reflect and expound upon limits and the personal application that it could have with each student.
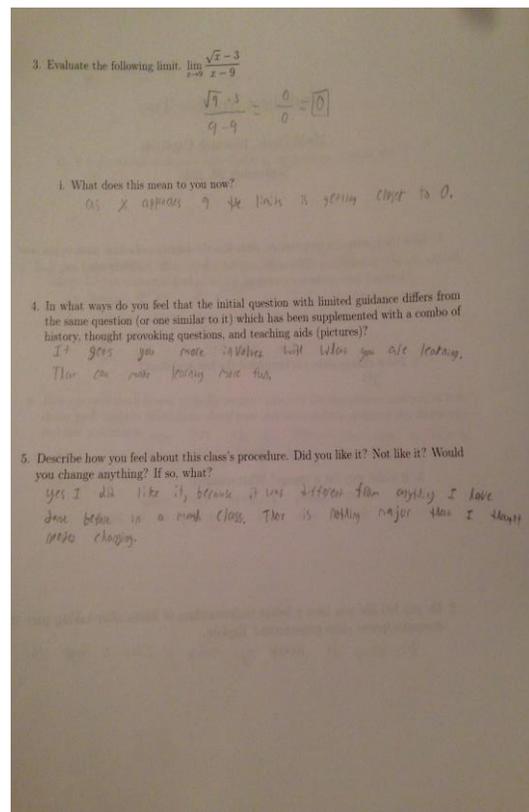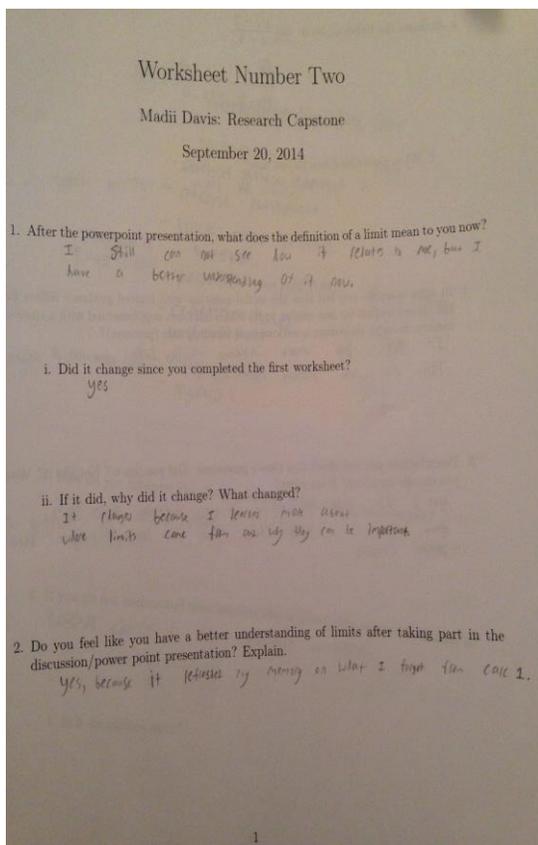
Manipulative Number 2

To conclude the experiment, I created another manipulative.
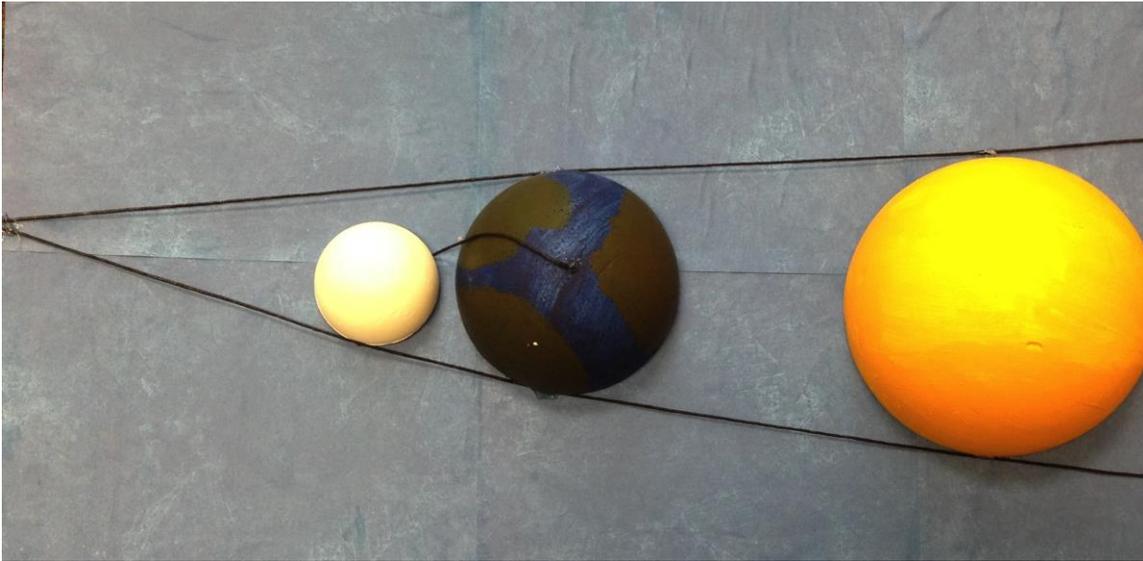
Above is a display I made to represent a lunar eclipse. Similar to calculus, there was a point where the understanding of our world and the solar system was little to none. However, with the development of calculus and the ability to apply it to many aspects of our life, we have been able to grasp an understanding of how it works. For this example, we take the sun and the earth as fixed points and we create two tangent lines of which go through one point on the sun and one point on the earth. The moon rotates around the earth. The space created from the two tangents behind the earth is called the umbra. When the moon falls into the umbra, it creates an illusion as if the moon has disappeared, however we know that this is not the case. The purpose of this manipulative was to give the students a real world application of limits and how the development of calculus and the application of limits have truly changed the way we see the world.

Survey Number 2

Similar to the first survey, the student is asked to define a limit once again. This was to show me if they had gained a new understanding of what a limit is. The second survey included a more difficult limit problem. The purpose of choosing a more difficult limit problem was to see if they could apply anything that they've seen before to a harder problem. I asked the student to interpret what it means and to possibly explain to me, if they were able, why the first limit problem from survey number one differed from the second problem. Lastly, I asked the students to describe how they felt about the experience compared to previous classroom experiences.

Worksheet Number Two

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September 20, 2014

1. After the powerpoint presentation, what does the definition of a limit mean to you now?
   I still can not see how it relates to me, but I have a better understanding of it now.

   i. Did it change since you completed the first worksheet?
   yes

   ii. If it did, why did it change? What changed?
   It change because I learn more about where limits come from and why they are important.

2. Do you feel like you have a better understanding of limits after taking part in the discussion/power point presentation? Explain.
   yes, because it refreshed my memory on what I forgot from Calc 1.

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3. Evaluate the following limit. $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$

   $$\frac{\sqrt{9}-3}{9-9} = \frac{0}{0} = \boxed{0}$$

   i. What does this mean to you now?
   as x appeals 9 the limit is getting closer to 0.

4. In what ways do you feel that the initial question with limited guidance differs from the same question (or one similar to it) which has been supplemented with a combo of history, thought provoking questions, and teaching aids (pictures)?
   It gets you more involved with what you are learning. They can make learning more fun.

5. Describe how you feel about this class's procedure. Did you like it? Not like it? Would you change anything? If so, what?
   Yes I did like it, because it was different from anything I have done before in a math class. There is nothing major that I thought needed changing.

1

Here, a student expresses that the experience created was something they preferred over the typical classroom experience that they have had in the past. The student shared that their understanding of what a limit was has changed as well. Although, this student solves the problem incorrectly, the purpose of the survey was to show that their understanding of limits changed.

My Expectations

Before analyzing the feedback I had received from the students, I did have a number of expectations. I expected the calculus two students to have an understanding of what a limit was. I assumed that each student would be able to solve each limit problem from survey one and two. I also understood that the students probably have not had this experience before which means that their previous experiences could affect what they gain from this particular experience that I was creating. If they hadn't had this experience before, then I expected the students to have had the typical classroom experience where they are constantly taking notes and completing a number of examples to prepare for quizzes and tests. Also, because I wasn't teaching limits, I was creating an experience, I assumed that some students may have expected me to do a number of example problems to support what we were discussing as a class.

Results and Analysis

I analyzed each survey separately based on the student's feedback. The calculus two class had twenty students. The students were pursuing a physics,

chemistry, engineering, computer science, or mathematics degree. It is important to state that not every student came from the same calculus one teacher.

Survey number one showed that 65% of the class could correctly solve the limit problem. However, only 26% of the class could correctly solve the problem and explain what it means. Virtually every student expressed that they have only had the "typical classroom" experience.

Survey number two showed that no calculus two student could correctly solve the limit problem that would typically be introduced in a calculus one class. However, on survey number two, we see that 70% of the student's understanding of limits changed favorably.

The journal entries were an incredible and useful addition to this experience. I received beneficial feedback from all students. Each student expressed their own thoughts and feelings about limits and how it affected their personal interests and students expressed their understanding of the manipulatives. I saw a few students ask their own questions and proceed to expound upon their own individual thoughts. From this, I was able to examine their thought process.

Overall, 83% of the class explicitly stated that they preferred this experience over the typical classroom experience and I did not clearly ask them to answer that sort of question. This implies that the percentage could be higher. However, 30% of the students shared that they wanted more examples. This was expected, as it is a reflection of what they have seen in the past.

Conclusion

By incorporating the humanities in mathematics, one is able to create an experience completely opposite of a "typical classroom". Involving other subjects in the mathematics classroom will allow all students to not only find a connection to mathematics, but it also encourages all students to find their own connection to mathematics. For this particular example, I incorporated history, science, and literature. However, one could even incorporate music, art, philosophy, physics, chemistry, and more. Given the opportunity to teach for an extended time period, and not fifty minutes, I am certain that each student will benefit tremendously from having an experience similar to this in a mathematics classroom. My purpose was to give students the opportunity to think differently about mathematics. If I had more time within the classroom, I would of course incorporate examples for I find that necessary in mathematics. However, I do not find it necessary for a class to be structured heavily around them. Overall, this was a great experience as I was able to see my vision through. I conducted an environment that encouraged creative thinking amongst my students, I created an experience that students have not had, and I've shown that it is possible and necessary for all teachers to incorporate the humanities within a mathematics classroom.