

The Comparison of Japanese Mathematics Education and United States Mathematics Education

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### Abstract

Japanese students outrank United States students in mathematics based on their standardized testing scores. My capstone explores the reasons why. I compared a typical 8<sup>th</sup> grade Japanese lesson to a typical 8<sup>th</sup> grade American lesson. A Japanese lesson typically uses problem solving based learning. It typically includes an environment that encourages productive struggle and has high levels of communication and cognitive demand. This type of lesson tends to produce higher retention. For my study, I taught U.S. students a Japanese lesson and got feedback on what they thought and how much they learned. I evaluated the levels of productive struggle, communication, cognitive demand, and retention of the lesson.

*Keywords:* Japan, Mathematics, Education, Productive Struggle, Cognitive Demand, Communication, Retention

## Introduction

The Trends in International Mathematics and Science Global Study (TIMSS) released by the International Association for the Evaluation of Education Achievement's international study center found that in general, Japanese students rank significantly higher in mathematics than students in the United States (US Department of Education, 2015). There are a few theories as to why this is. As a future mathematics educator in the United States I am passionate about learning successful teaching techniques. Through the TIMSS, Japan has proven to have success in math education so I explored more into why they outperform students in the United States. I did not expect to come up with a definite reason for this difference in achievement but I planned to investigate and analyze what makes education in Japan different than education in the United States. Based on research thus far (Mastrull, 2002) Japanese lessons tend to teach more by exploring solutions to a problem rather than memorization or step-by-step instructions. In my study, I deeper researched the benefits of problem solving based learning. Specifically, I wanted to answer the following research questions:

- Is productive struggle a key element in successful learning and thus better conceptual understanding?
- Is communication a key element in successful learning and thus better conceptual understanding?
- Which levels of cognitive demand do Japanese style lessons imposes compared to those of a typical United States lesson?
- Are students able to better retain the concepts learned through a Japanese style lesson?

I believe this study is important in moving the United States mathematics field forwards.

Curriculum has advanced in the past few years with implementing the Common Core but there is always room for improvement. With my study, teachers will better understand teaching techniques to invoke higher levels of cognitive demand in their classrooms. This, then, will encourage students to pursue the mathematics field because they can learn, understand, and retain math concepts better.

### **Literature Review and Theoretical Framework**

#### **A Japanese Lesson (TIMSS)**

According to the 2015 TIMSS, the average mathematics scores of 8<sup>th</sup> grade Japanese students was 586 compared to the average mathematics score of 8<sup>th</sup> grade U.S. students of 518. The average overall content domain scores of Japanese students was 588 and the average overall content domain score of U.S. students was 516. Mastrull (2002), a U.S. secondary school teacher, describes her research comparing Japanese Education to United States education. She describes that Japanese students spend on average 22 days more in school compared to U.S. students. Their curriculum and textbooks are nationally standardized so that every student learns the same material. “The Japanese assume that learning is the product of effort, perseverance, and self-discipline rather than of ability.” (Mastrull, 2002). The curriculum is quite different in Japan. Topics from elementary are not repeated. Algebra and geometry are the main focuses of junior high and they move onto probability and statistics with their solid background in geometry. According to the TIMSS, Japanese lessons have higher expectations which challenge the students while covering less than 10 topics per year compared to the 35 topics in the U.S. eighth grade textbooks. This means teachers spend more time per topic encouraging students to develop a deeper understanding of each concept. Japanese educators believe that a deeper understanding

of fewer topics is more beneficial than a shallow understanding of many topics. The following statistics were developed by researchers who studied videotaped eighth grade lessons in Japan and the U.S.:

- In 96% of U.S. math lessons, students practice a procedure, as compared to 46% in Japan. In only 1% of U.S. math lessons do student formulate the procedures themselves, as compared to 44% in Japan.
- 0% of the videotaped U.S. eighth grade math lessons had instances of deductive thinking, as opposed to 61% of the Japanese lessons

Overall the atmosphere, curriculum, and stimulation is much different in a typical Japanese mathematics lesson. Another large aspect of Japanese learning is exploring concepts through problem solving based learning. A problem is any task or activity for which the students have no prescribed or memorized rules or methods. A problem must begin where the students are and the problematic aspect must be due to the mathematics that the students are to learn. Lastly, a problem must require justification and explanations for the answers and methods. (Van de Walle, Karp, & Bay-Williams, 2010) The students are given a problem that they have no known solution for. They are expected to use their previous knowledge and build upon that. The students are given the time to struggle and present their solutions to the class at the end of the lesson.

### **Levels of Cognitive Demand**

The researchers [of The Third International Mathematics and Science Study] conclude that U.S. math and science curricula lack focus and coherence and, compared to other high-achieving nations, have lower expectations. The results demonstrate that American school children are not being challenged enough. U.S. eighth graders are being taught at a

seventh grade level compared to many of their international counterparts (Mastrull, 2002, p.3).

A large part of education is the level of cognitive demand of a lesson. Cognitive demand refers to the level and kind of thinking required of students in order to successfully engage with and solve the task. (Stein, Smith, Henningsen, & Silver, 2000) There are four levels of cognitive demand. The first and lowest level is memorization. One example of a task that only demand memorization are ones that cannot be solved using procedures because a procedure does not exist or it is too long to be completed in the time frame. A task involving exact reproduction of previously seen material is another example of a lesson that only demand memorization. The second level of cognitive demand is procedures without connections. Tasks that required the second level of cognitive demand are algorithmic. They use a procedure that is specifically called for. There is no connection to the concepts or meaning that underlie the procedure being used. The higher levels of cognitive demand are procedure with connections ad doing math. Lessons that demand procedures with connections are ones that focus a student's attention on using procedures for the purpose of developing deeper levels of understanding of concepts. They are usually represented in multiple ways. Tasks that require students to do math involve requiring complex and non-algorithmic thinking. They encourage students to explore the nature of math concepts and relationships. Some factors associated with teachers maintaining high-levels of cognitive demand are

- Teachers using questions and comments to press students reasoning and explanation
- Teachers supporting students in monitoring their progress and performance
- Teachers allowing sufficient time for tasks

Some factor associated with the decline of high-levels of cognitive demand are

- Teachers emphasizing complete or correct answers rather than the meaning behind them
- Teacher reducing the complexity of the task by providing procedure or routines
- Teachers not allowing sufficient time

(Stein, Smith, Henningsen, & Silver, 2000). This highest level of cognitive thinking is the goal for all mathematics lessons. Students should be challenged to make connections on their own.

### **Conceptual Understanding**

Conceptual understanding is the comprehension of mathematical concepts, operations, and relations (Kilpatrick, Swafford, & Findell, 2001). Students must grasp mathematical ideas in an integrated and functional way. Students with conceptual understanding have knowledge exceeding methods and procedures. They recognize why mathematics is important and see the applications for concepts that they learn. Conceptual understanding leads to retention. Because students make a connection between concepts, they understand and remember the material easier. “A significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes.” (Kilpatrick, Swafford, & Findell, 2001) The level at which a student has conceptual understanding can be shown through the extent of the connections they have made. When a student can see deeper similarities between mathematical situations they have less to learn and thus less material needs to be covered. As previously mentioned, Japanese students cover significantly less topics, yet continue to outperform U.S. students. This could be due to Japanese students having higher levels of conceptual understanding. Two important aspects of conceptual understanding are productive struggle and communication.

**Productive Struggle.** Productive struggle refers to the struggle in learning mathematic concepts, procedures, and ideas to have a meaningful understanding of math. Math is about creativity and problem solving. Students should be encouraged to explore their abilities to creatively solve problems through productive struggle. Warshauer (2015) describes some strategies to encourage productive struggle include: asking purposeful questions to help students reflect on the source of their struggle and focus their thinking, encourage students to engage in the process of critically thinking about the problem and concepts as opposed to focusing only on obtaining a correct solution, give students time to manage their struggles, and creating a classroom norm that shows students that struggle is an important part of learning. A teacher that uses productive struggle should anticipate what the students might struggle with and be prepared to support them through the struggle. They should help the students realize that errors and confusion is a natural part of learning. Teachers can respond in various ways. If they tell the students how to complete the problem, cognitive demand is lowered. Teachers should strive to guide the students or ask for a detailed explanation to keep levels of cognitive demand high. Productive struggle is useful to students by helping them vocalize the questions they have related to the source of their struggles. They can then help other students without telling them the answer. Wisconsin middle school educator Butturini (2011) describes eight teaching habits that block productive struggle:

1. Calling on students who know the right answer.
2. Praising students for their smarts.
3. Creating bulletin boards to display high achievement.
4. Focusing on teaching procedures and formulas.
5. Making student responses right or wrong.

6. Giving easier work to struggling students.
7. Following a strict schedule for covering new material.
8. Making students feel okay about not being a “math person”.

These practices discourage students to explore math outside their comfort zones. While studying Japanese lessons in the TIMSS, productive struggle is often shown through the students.

The 1999 TIMSS study also analyzed textbooks in use in these thirty-eight countries. Typical eighth grade mathematics textbooks in America cover more than 35 topics, compared to fewer than ten topics in both Japan. We need to concentrate on teaching each topic in a way that gives students time to struggle with the problem being addressed before being told the answer. This is the only way that they can acquire the knowledge and skills that will be useful for them later. (Mastrull, 2002, p.4)

**Communication.** The Nation Council for Teachers of Mathematics (2000) suggest that communication is a vital part of conceptual understanding. It allows students and teachers to share ideas and clarify understanding. When a student is asked to communicate their ideas, they are forced to articulate a clear and convincing argument for their way of thinking. Asking students to share their solutions with other students not only benefits the listening students but also the student sharing their answer. Conversations about mathematical concepts helps students to make connections about those concepts. It also allows students to better their mathematical language and learn the significance in being exact with their language. Another benefit of communication in mathematics is that it lets students organize and consolidate their thinking. A large part of encouraging communication in the classroom is the teachers’ ability to guide the discussion. If the students are uncomfortable speaking, they will have a more difficult time

expressing their understanding. Communication can be both verbal and written. The Japanese lesson I based my project on involved lots of discussion between students and between the teacher and students. I also asked the students to explain their answers by writing down their thoughts.

### **Culture**

Dr. Ji Seun Sohn is a criminology professor at Georgia College. She was born in Korea but lived in Japan for 6 years. She described the culture of Japan and how it relates to education. “Students are more respectful to professors. There is a different culture in terms of the relationship between students and their teachers in Asian countries.” Researchers in the TIMSS found that U.S. lessons were interrupted 28% of the time, compared with only 2% in Japan. A large part of quality education is time. Dr. Sohn also accounted her personal experience with the private education in Japan. “The private education system is very common in Japan. So after we finish school we go somewhere to learn something like math, piano, or other things. I did not like it but when I was younger, I didn't have an option to not go. In that aspect, I like it here better. Some [U.S.] students get a private education too but that is not the majority.... Public education systems should be enough to educate people.”

### **Methods**

I began my study by giving a lesson and surveying the MAED 3100 middle grades cohort students and the MAED 3119 secondary mathematics education major students. I chose these groups because of their mathematical knowledge and their desires to be future teachers. I felt as if these groups would benefit from seeing this type of lesson and be able to give evaluative feedback based on their experiences. There were 15 students in the MAED 3100 course and 6 in

the MAED 3119 course. The lesson that I taught was inspired by a lesson that was taught to Japanese 8<sup>th</sup> grade students in the TIMSS. Before presenting my lesson I ensured the students had the necessary prior knowledge to be successful in my lesson. In a previous class, the students explored the fact that triangles with the same base and height have the same area. Then, I presented the following problem that the students had not seen and were not previously shown something similar:

Rin makes Yuna a homemade cake for her birthday. Although it is an odd shape, Yuna loves it very much. Rin asks Yuna to draw a line with red icing indicating the piece of cake she would like. Rin wants to give Yuna the same size piece as she wants but only make one straight cut. Where should Rin cut the cake?

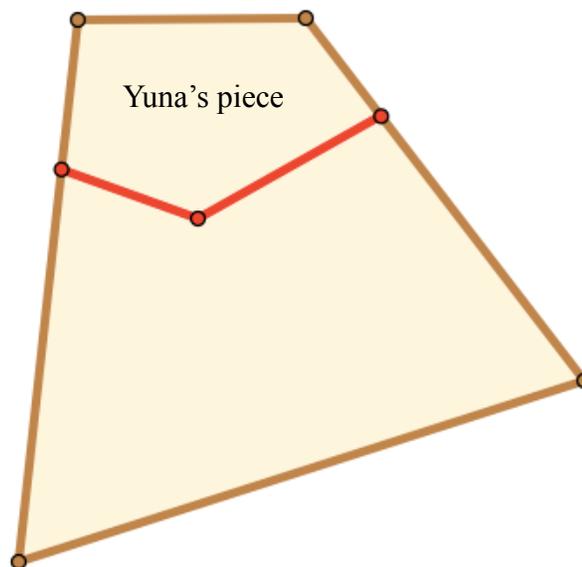


Figure 1

The goal of this problem is to find a method to change the two red line segments into one line or line segment while keeping the area of each section of the cake the same. The students were required to make their own connections to their previous knowledge that a triangle with the same base and height have the same area. The students were required to explore relationships and various ways of solving the problem. I had to clarify what the problem was asking because a few students were confused. I allowed the students to work in groups of four for approximately 15 to 20 minutes. Then I walked around the room discussing their work and giving small hints to guide the students in the right direction. After the 15 to 20 minutes passed, I gave the entire class the hint to work the following triangle indicated with blue dashed lines:

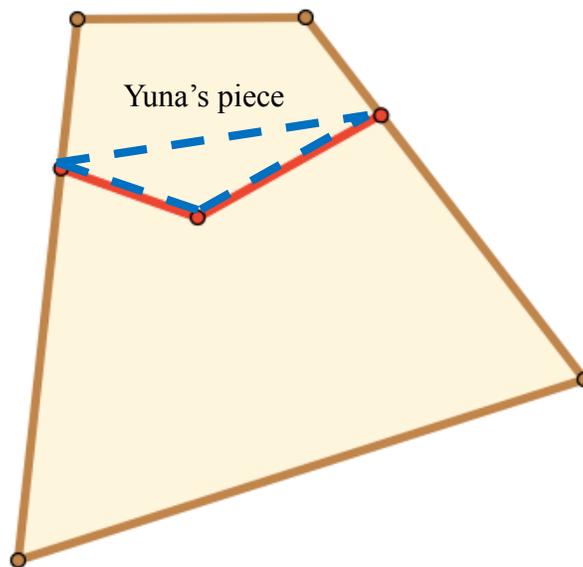


Figure 2

I suggested that they attempt to change this triangle but keep the area the same. After about 40 minutes, some students began to find the solution. I continued to walk around the room and help until each group found a solution. I then asked a student to present their solution to the class. I

asked if anyone had any different solutions to present. A second student presented their different solution. At the end of the lesson, we had a discussion on this style lesson. I asked if the students liked this problem solving type of lesson and what they thought about their struggle in finding the correct solution. Next, I gave the students a list of survey questions. The questions were as follows:

- Do you believe this method of teaching is more effective, less effective, or equivalent to teaching by step by step instructions? Why?
- Do you believe the students would deeper understand the concepts with this type of problem solving lesson? Why?
- Do you believe the struggle in finding your own solution is good? Would you rather be told step-by-step instruction on how to solve these problems? Why or why not?
- Do you think you were more engaged in the lesson than in a more traditional step-by-step instructional lesson? Why do you think that is?
- Would you consider teaching your future students with a problem solving based lesson?
- Do you believe a student sharing their method in solving the problems helps the other students to better understand the concepts? Does this benefit in the understanding of the student that is sharing? Why or why not?
- As a recent high school graduate, do you believe your mathematical learning experience was more procedural or conceptual? What year did you graduate high school?

I allowed 15-20 minutes for them to answer the questions. In addition to the lesson, I performed a few more assessments that measured the level at which students retained the information taught in my lesson and their conceptual understanding of my lesson. One was a homework problems

for the students to complete directly after the lesson and other was a follow-up evaluation three weeks after I taught the original lesson. Both of which the students were asked to complete individually. The homework problem was as follows:

Alisha and Luke are neighbors. Alisha wants to build a fence with three sides around her property. A land surveyor surveys Alisha's land and determines her property line. The property line is drawn below. Alisha notices that building a fence on the line would make four sides. Therefore, Alisha talks to Luke and the two agree that she can make her fence with three sides as long as the area of both their properties stay the same. Draw where Alisha should build her fence and explain why?

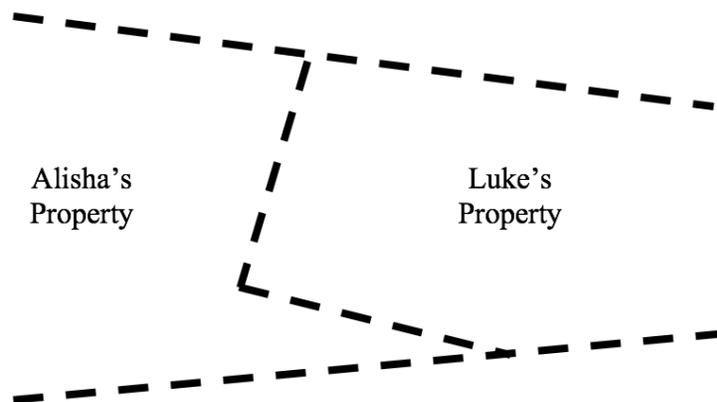


Figure 3

The follow up evaluation had two parts. The first prompted the students to answer and explain the original problem about the cake presented in my first lesson. The second was a problem that allowed me to see the extent at which students could apply their conceptual understanding gained through my lesson to a different problem. The second problem was as follows:

Draw a triangle that has the same area as the quadrilateral.

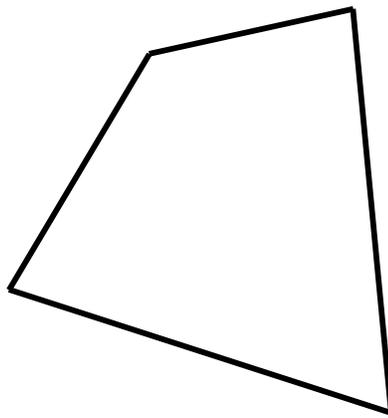


Figure 4

I chose to collect data this way because it allowed me to see the effectiveness of a Japanese style lesson. I could see productive struggle and determine if it was beneficial. I could determine the level of communication and cognitive demand that the lesson required. Lastly, I could measure the retention of concepts from the students.

I analyzed my data in a couple different ways, most of which were qualitative. One way I analyzed my lesson was through evaluating the level of communication and cognitive demand the students presented. In addition, I evaluated my lesson through the survey responses. I was able to see what the students thought about productive struggle. I was able to gain statistics about

their previous mathematical experiences and opinions on my lesson. Another way I analyzed data was through the homework problem and follow up evaluations. I gave a score to each answer on a scale of 0-2. Zero indicating the student got the incorrect answer and did not understand the concept. One indicating the student came to the correct answer but did not explain it correctly and thus did not understand the concept or they got the explanation correct but did not answer the problem correctly. Two indicates the student had the correct answer and showed that they understood the concept.

One major limitation of my study was time. If I had more time to perform my study, I would have taught a lesson both in a “traditional United States style” and “Japanese style” and compared the two. Another limitation was accessibility to middle-grades students. If I was able to, I would have taught this lesson to eighth grade students as well and analyzed their responses to the lesson.

## **Findings**

### **Conceptual Understanding**

**Communication.** I evaluated the level of communication in my lesson. I allowed the students to work in groups to find a solution. They communicated amongst each other their ideas and possible methods to come to a solution. After I noticed the students were unable to come to the correct answer, I began to give hints. These hints sparked more communication between the students and me. Eventually each student came to the correct solution. I asked two students to present their answers. They communicated their answers and how they came to the solution in front of the class. The first student who presented their solution in the MAED 3100 course began by labeling vertices on the cake A, B, and C. She then drew a line connecting points B and C which created a triangle with the three vertices. She drew a parallel line to the segment BC

through A. She mentions, “Okay so now we have two parallel lines, right?” She points to the segment BC and says, “And this is our base. We can move A wherever we want. If we drag A this way or this way (she shows A moving along the parallel line), it still has the same area. So now we have to make a cut. Where should we put it? So we move [A] all the way over here. This is our new A.” She points out the original triangle ABC. “So this is our first triangle right here.” She draws out the new triangle with the new point A. “And this is our new triangle right here.” She asks the class, “Do they have the same area?” The class responds, “Yes”. She asks, “How do you guys know that.” One student replies, “From our lesson yesterday.” The students presenting asks what the lesson was and a student replies, “Triangles with the same base and same height have the same area.” She responds, “Wow, genius. So, our cut is going to be right here.” She motions where the new cut would be. This is an example of great communication because she used correct language, gave a thorough and correct explanation of the math used in the problem, and students answered other students’ questions. Having the students work in groups, probing guidance through hints and having two students present their answers sparked great communication within my lesson. I, as a teacher, was able to conclude that most if not all of my students understood the concept I taught. Another example of communication through my lesson was the students’ written explanations on their homework answers and survey responses. One students conveyed, “Using properties of triangles and parallel lines, we know that the area of triangle ABC equals triangle CBD because we have only moved one of the vertices of the triangle to another location on the line k. In other words, we have kept the same base and height and thus preserved the area.”

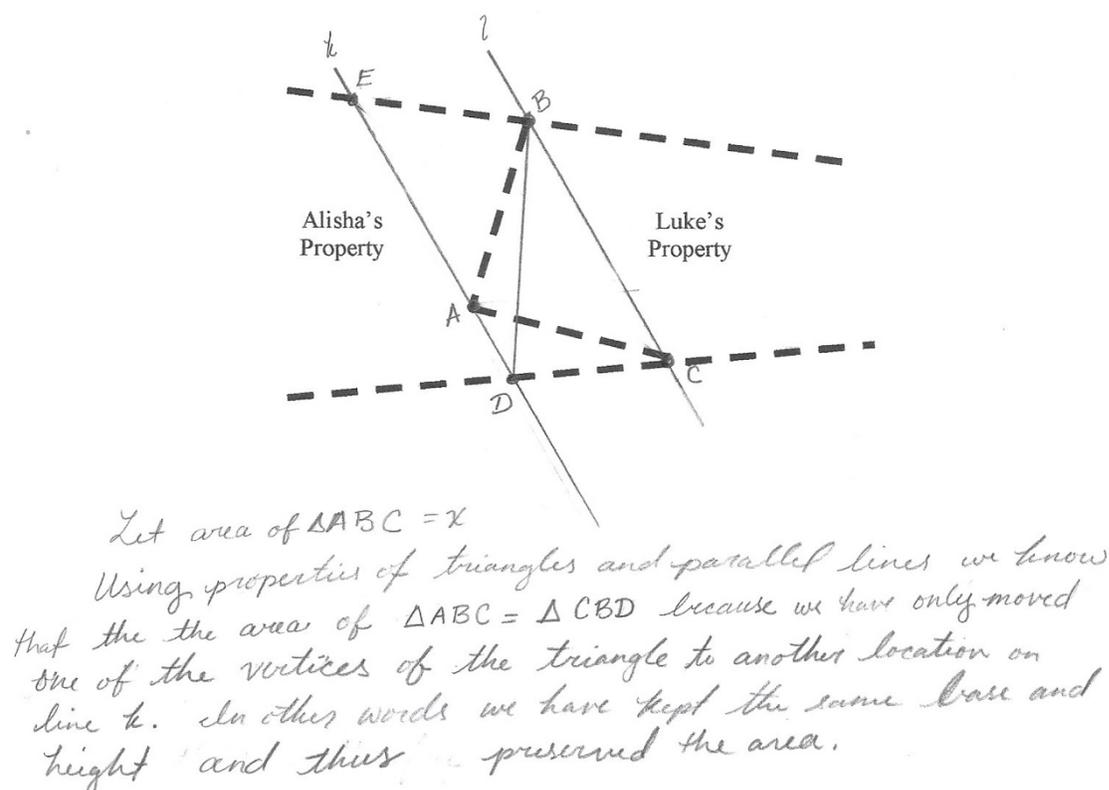


Figure 5

This is a good example of written communication because precise mathematical language was used, the explanation is clear, labeled, and correct, and her thoughts were expressed with two different ways of saying them.

**Productive Struggle.** I believe that each student experienced productive struggle through struggling to carrying out a process. The students were stuck due to the inability to use a process or straight forward formula to solve the problem. The MAED 3100 class took a while longer to come to the correct solution compared to the MAED 3119 class. This may be due to the MAED 3119 class's greater experience with upper level math courses. When I introduced the first question, no student could solve it right away. Some students thought to use angle measurements. Some thought they could solve the problem by measuring the sides of the piece of cake. The following pictures are some examples of methods students attempted:

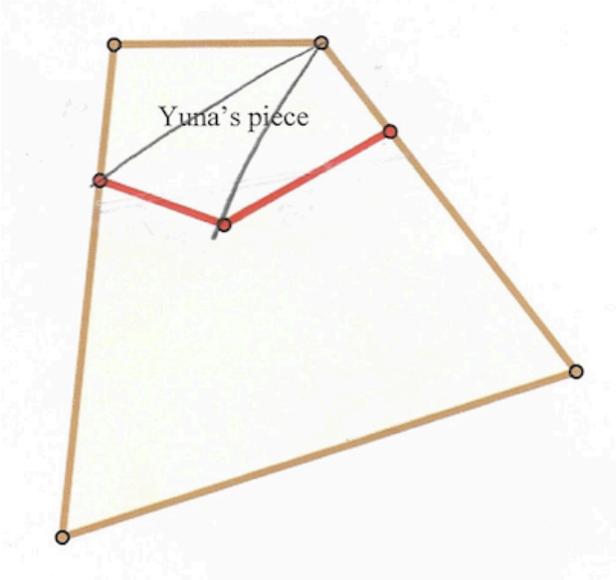


Figure 6

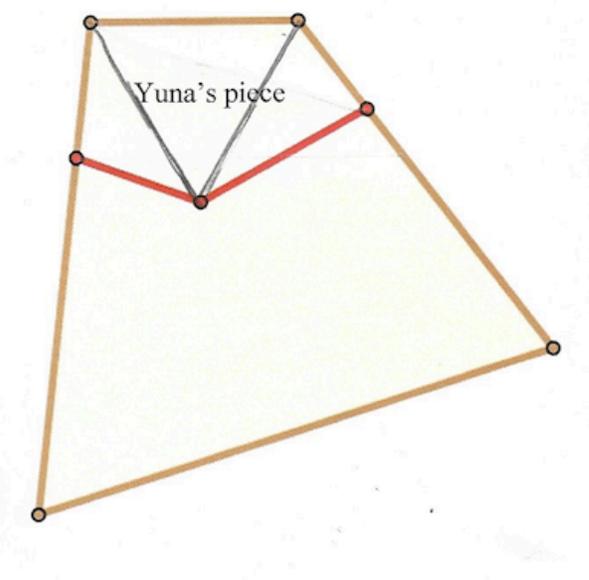


Figure 7

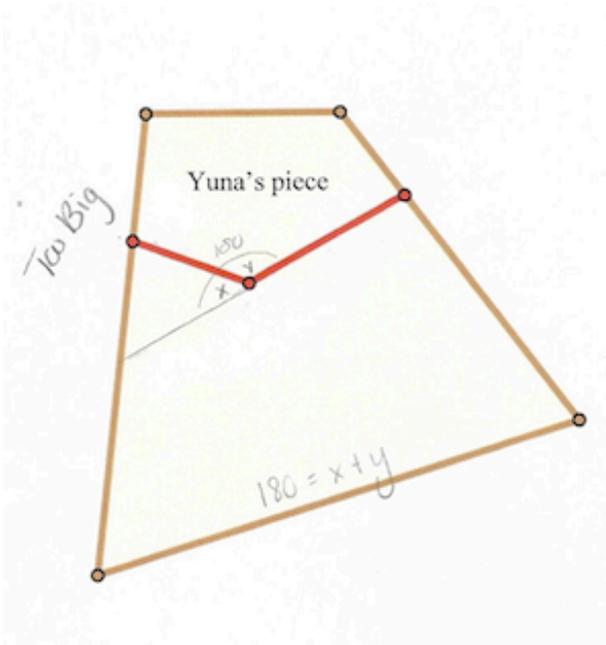


Figure 8

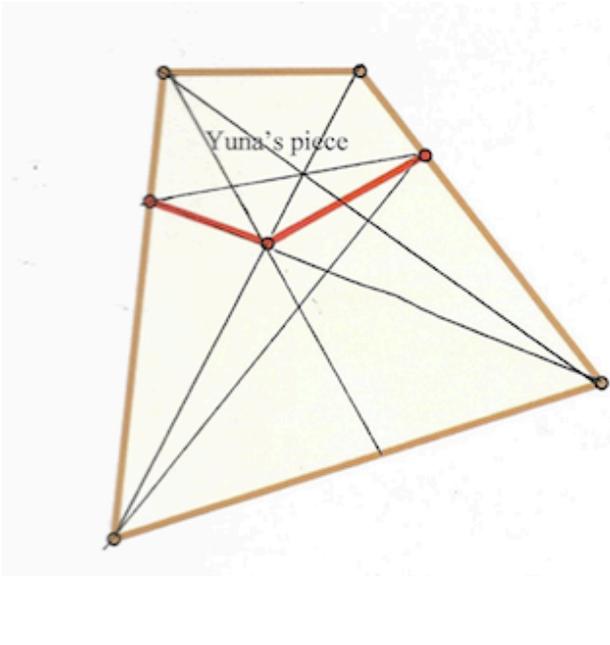


Figure 9

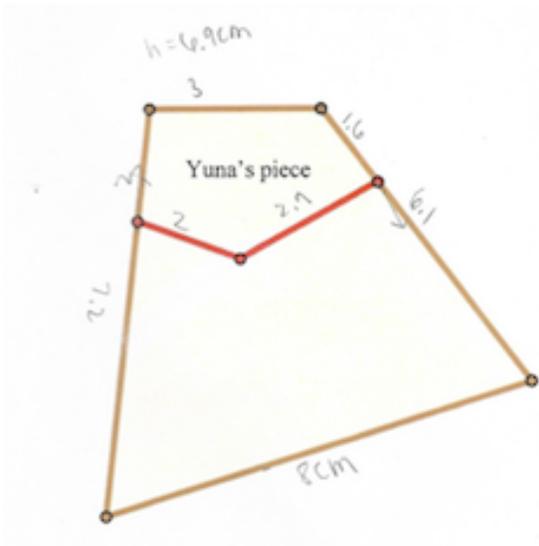


Figure 10

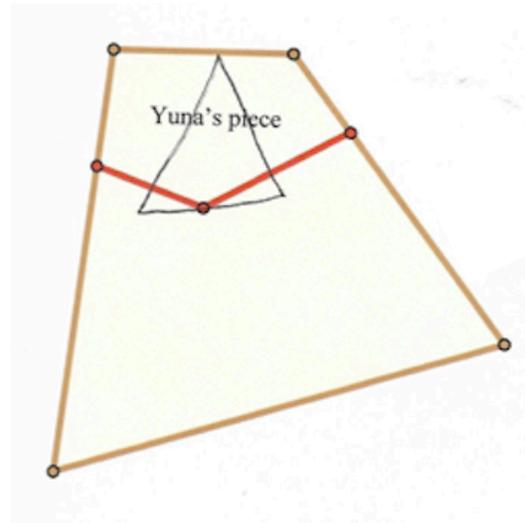


Figure 11

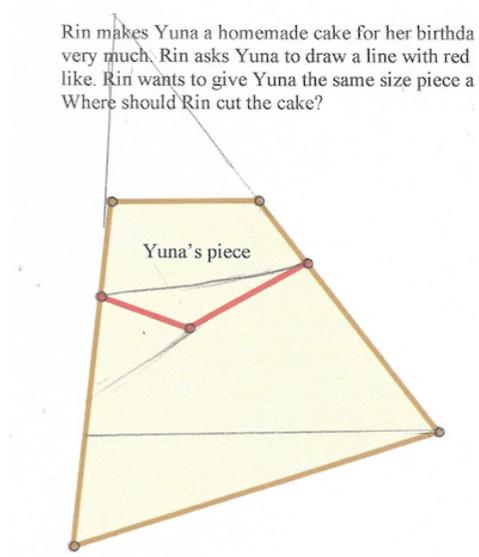


Figure 12

These methods show students drawing triangles within the shape, drawing angle measures, and attempting to measure the sides to find the area. This demonstrates to me that even

though these methods would not bring the students to the right conclusion, they were trying.

They struggled through the problem to eliminate methods that did not work. Then I gave a hint.

This sparked some students to use parallel lines but they were unsure where these lines needed to be.

Here are some attempts students made with parallel lines:

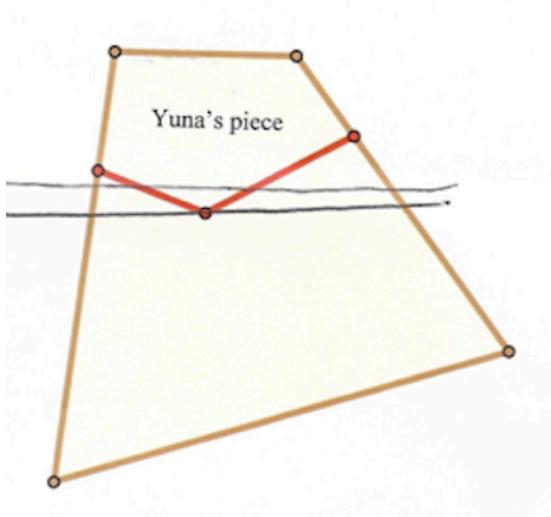


Figure 13

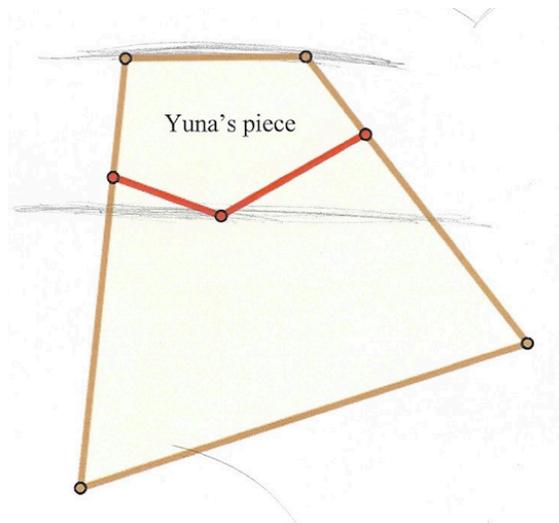


Figure 14

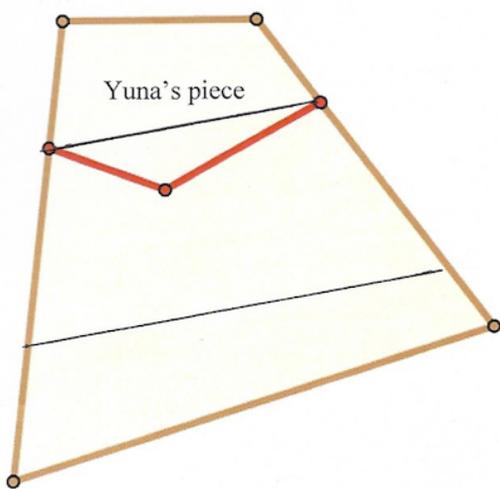


Figure 15

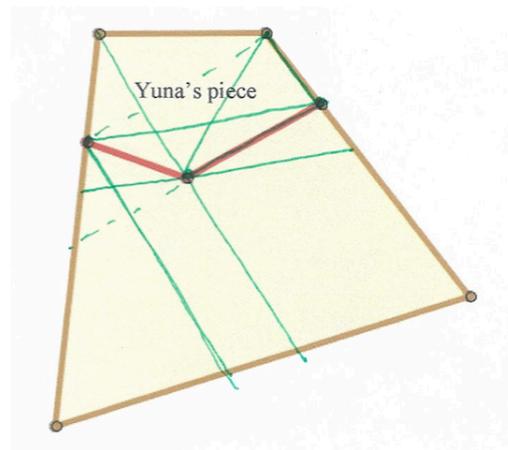


Figure 16

Eventually a few students came up with the correct solution. I continued to walk around occasionally guiding a student to figure out the problem. Each student was able to come up with the correct solution. The following images are ways the students came to the correct conclusion:

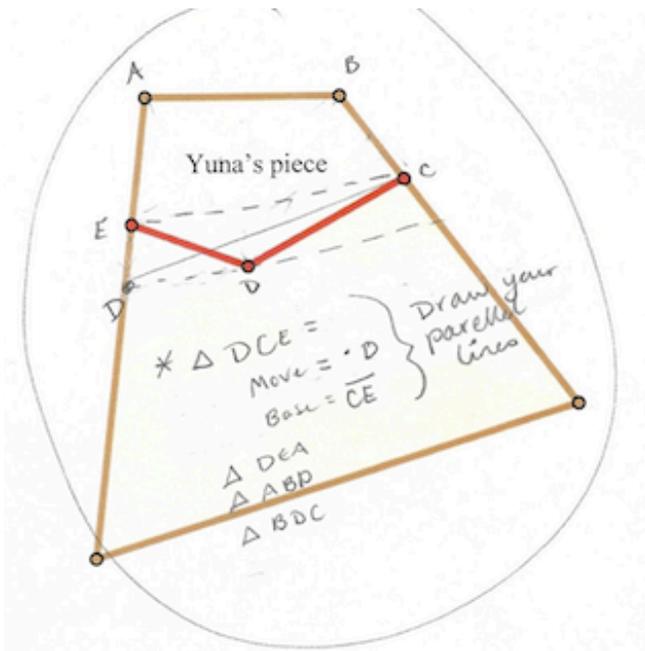


Figure 17

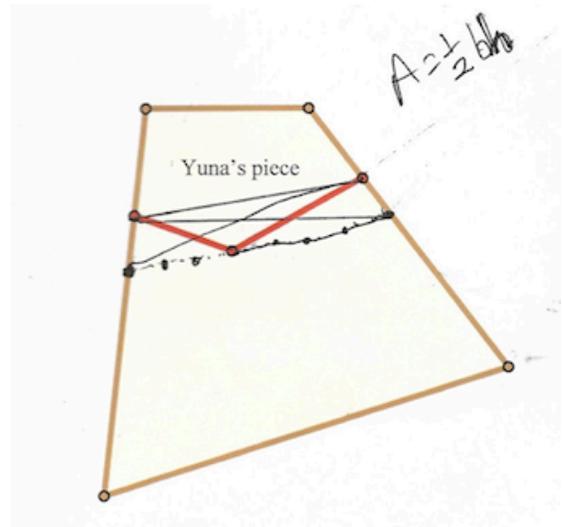


Figure 18



Figure 19

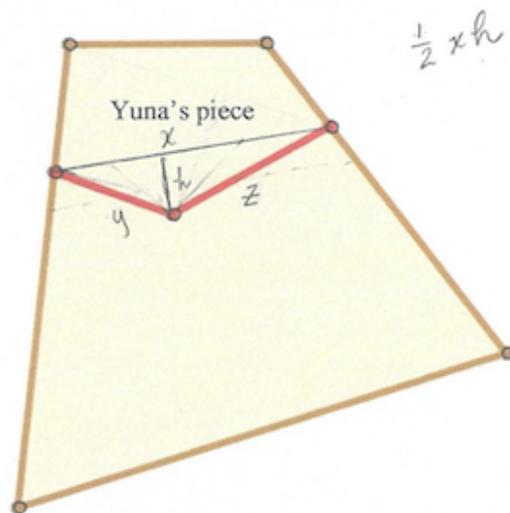


Figure 20

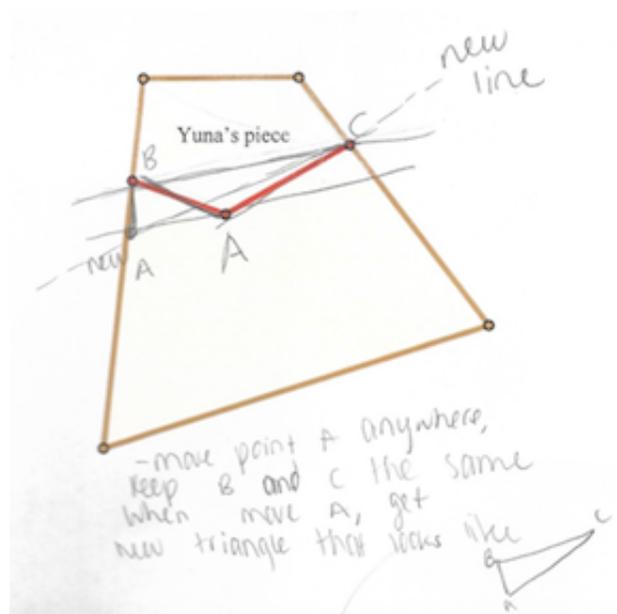


Figure 21

These students used their previous knowledge to show that the new triangle they made has the same area. The student in Figure 18 and 19 used her knowledge of the formula to find the area of a triangle to determine that she needed to keep the  $x$  and  $h$  the same to keep the area the same. She shaded the new piece of cake to visualize her answer. After my lesson, I asked the students what they thought about the struggle they experienced. The survey questions read: Do you believe the struggle in finding your own solution is good? Would you rather be told step-by-step instructions on how to solve these problems? Why or why not? Out of the 14 responses in the MAED 3100 class, nine (or 64.3%) said they liked the productive struggle and would rather be taught with problem solving based lessons. Five students (or 35.7%) said that they preferred to be taught using step-by-step instructions. Out of the 5 responses in the MAED 3119 class, all 5 students said they liked the productive struggle and would rather be taught with problem solving

based lessons. Here are a few of the survey responses to this survey question:

I prefer step by step because that is how I grew up.

Figure 22

I like step by step because I learn better when there is a process especially in math and science. Problems like this make me frustrated & I don't want to continue because I don't know where to begin.

Figure 23

In the end, I would have rathered step by step because I had no clue how to do it, but I enjoy these problems more when I am able to solve them myself.

Figure 24

Its alot easier to have step by step instruction but it doesnt create deep understanding like what is gained through struggling. This type of learning enables students to approach other new problems, where step by step learning leads us to the exact type of problem we have seen before

Figure 25

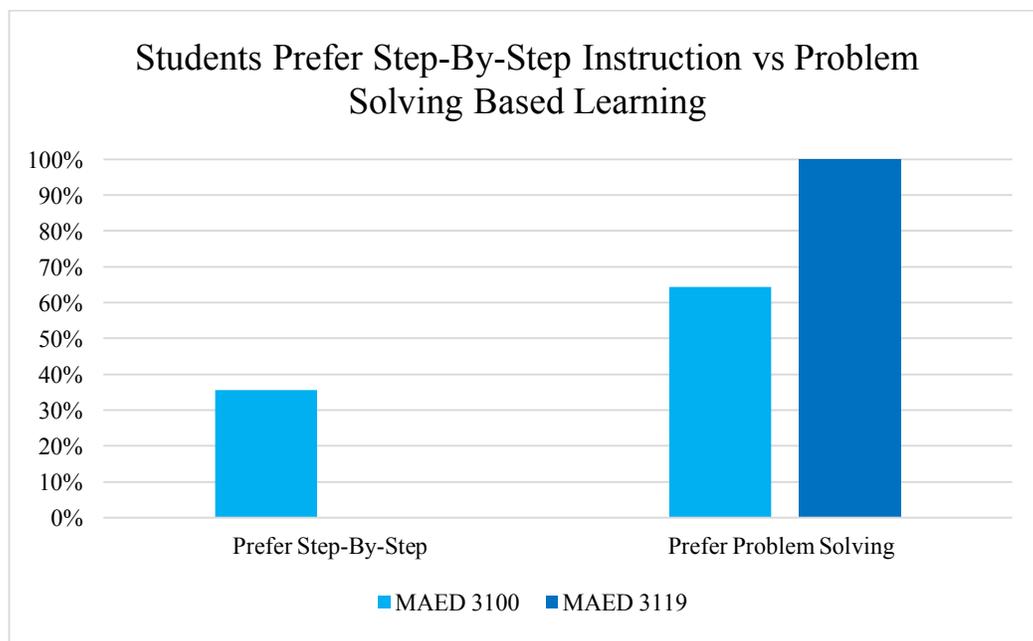


Figure 26

From these survey responses, I conclude that most of the students saw the benefit of this type of learning but some were frustrated that it was too challenging. One student said, “[Problem solving based] learning enables students to approach other new problems, where step-by-step learning limits us to the exact type of problem we have seen before.” Another student mentioned “I don’t want to continue because I don’t know where to begin.” That shows me that students in the United States would rather give up than attempt to think creatively using their previous knowledge. This may be due to how U.S. students were taught growing up. I asked the students if their high school experience was more conceptual or procedural. 84.6% of the MAED 3100 class and 83.3% of the MAED 3119 class had more procedural high school math experiences.

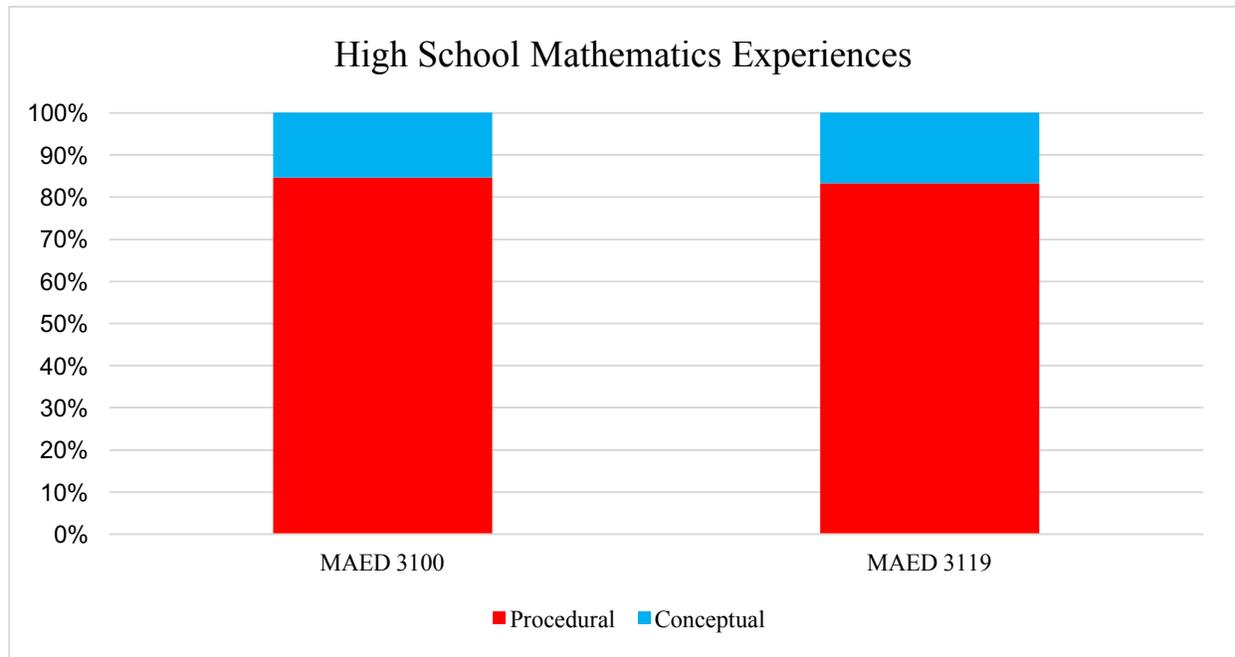


Figure 27

**Gaining Conceptual Understanding.** Overall, I believe this lesson provoked productive struggle and communication leading to a better conceptual understanding. Another survey question that I asked was, “Do you believe this method of teaching is more effective, less effective, or equivalent to teaching by step by step instructions? Why?” Here is one of the responses:

I believe this method of teaching is more effective long term because students will then learn to apply lessons to future problems. Students will be a lot less likely to “run away” from changes (big or small) of similar problems.

Figure 28

I think this method of teaching is <sup>more</sup> beneficial for students for several reasons. This method allows students to be actively involved in the lesson while also forcing them to think beyond "What's the next step?" Students have to take knowledge they already know and figure out how to put it together to effectively solve the problem.

Figure 29

More effective because it allows students to think through things themselves before being told a solution. Developing independent and peer-supported thinking is very important in mathematics.

Figure 30

This shows that the student thought the lesson developed conceptual understanding and would allow students to apply the lesson learned to future situations. Ten out of the 14 responses (or 71.4%) of the MAED 3100 class agreed this lesson was more effective. Two students argued it was less effective and two said it is equivalent. In the MAED 3119 class, 83.3% said the lesson was more effective. One students said it was equivalent.

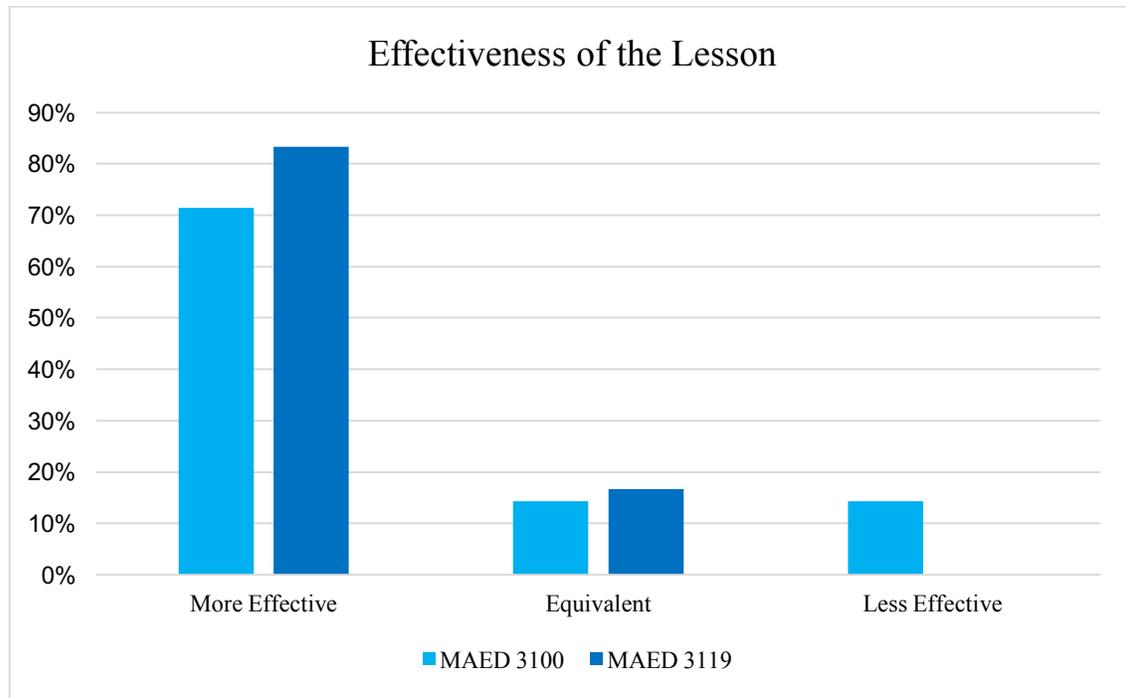


Figure 31

Going off this, I asked the students: “Do you believe the students would deeper understand the concepts with this type of problem solving lesson? Why?” All 20 students answered yes. Here are some of the responses:

Yes, I know I do. I see the real world application. If my friend wanted to have cut the same area of cake the only way I would be able to do it is some formula that's way harder. This way shows you a more simple way to do something and had to do it in real world context.

Figure 32

Yes, because you have to logically understand it rather than simply memorizing.

Figure 33

Again, this demonstrated the greater level of conceptual understanding the students believed this lesson has compared to a lesson taught through step-by-step instruction. It is interesting to see how even when five students say they would prefer to be taught through step-by-step instruction, all the students surveyed think that problem solving based learning yields deeper understanding of concepts. This shows me that even when students know a better way of teaching is possible, they would prefer to simply be told what to do and not struggle through finding solutions. When I asked the students if they would consider teaching their future students this way, two responded maybe and the rest said yes.

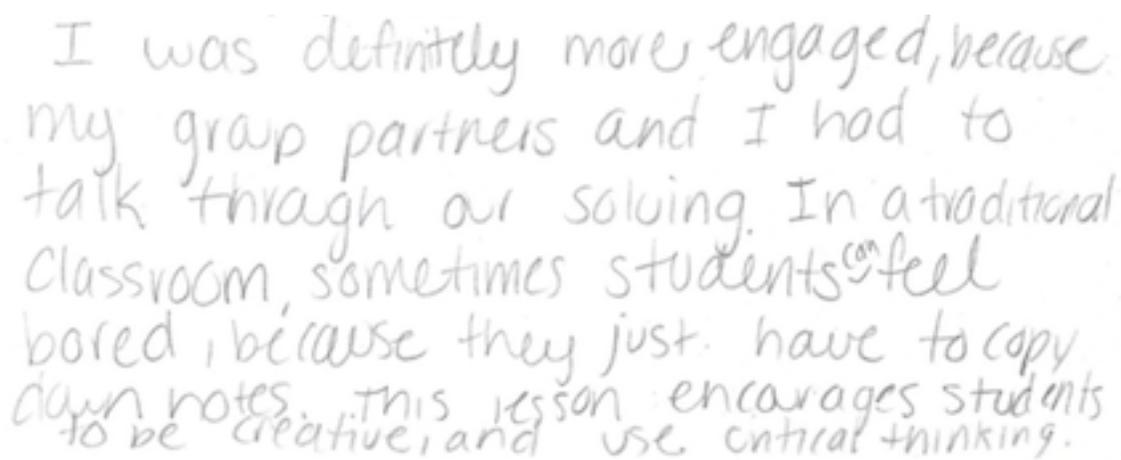
### **Cognitive demand**

The Third International Mathematics and Science Study concluded that U.S. children are not being challenged enough in their classes. This is based on the statistic that students practice a procedure in 96% of U.S. math lessons, as compared to 46% in Japan and in only 1% of U.S. math lessons do student formulate the procedures themselves, as compared to 44% in Japan. In addition, over 80% of students in my study were taught math procedurally, I speculate the level cognitive demand of a typical U.S. class to be one of the lower levels, memorization or procedures without connections. I speculate that a U.S. teacher would teach this lesson by using formulas or step-by-step instruction and telling the students how to solve problems or using directed guidance. I think that the teacher would not feel that they have time to let the students discover this concept on their own. They would possibly present to the students how to solve the problem then allow the students to do similar problems on their own. This is consistent with a lower level of cognitive demand. I planned my lesson to be one where students experience doing math. I believe the actual cognitive demand of my lesson after implementation was also doing

math. I never told the students how to solve the problem, they all figured it out on their own.

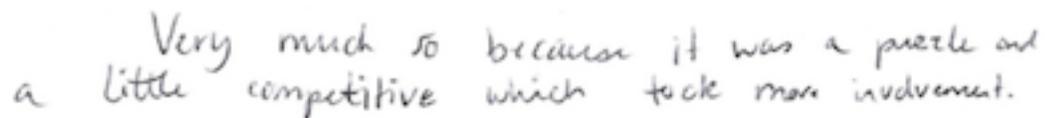
This lesson encouraged students to explore the nature of math concepts and relationships.

Another large part of cognitive demand is engagement in the lesson. All my students were actively trying to figure out the problem. However, I did notice a few got stuck and became discouraged. I asked “Do you think you were more engaged in the lesson than in a more traditional step-by-step instructional lesson? Why do you think that is?” Most responses were positive.



I was definitely more engaged, because my group partners and I had to talk through our solving. In a traditional classroom, sometimes students can feel bored, because they just have to copy down notes. This lesson encourages students to be creative, and use critical thinking.

Figure 34



Very much so because it was a puzzle and a little competitive which took more involvement.

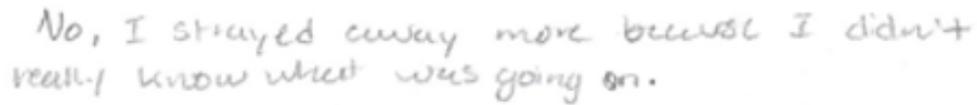
Figure 35

Other responses suggest that the challenging nature of the lesson discouraged the participants from wanting to continue.



not really it was just a lot of guessing

Figure 36



No, I stayed away more because I didn't really know what was going on.

Figure 37

I believe that overall most students were very engaged in this lesson.

### **Retention**

I evaluated the level of retention with three assessments. The first was a homework assignment that used a problem the students had not seen before but it used the same concept. On the rating scale from 0 to 2 in the MAED 3100 class, 8 students received 2's, one student received a 1, and two students did not explain their answers. In the MAED 3119 class, 3 students got 2's, 2 students got 1's, and one student received a zero. Note that the students who did not explain their answer could have understood the concept and could have been able to explain their solution but I cannot verify this. I decided to give these students a 1. Overall 64% got a 2, 29% got a 1, and 7% got a 0.

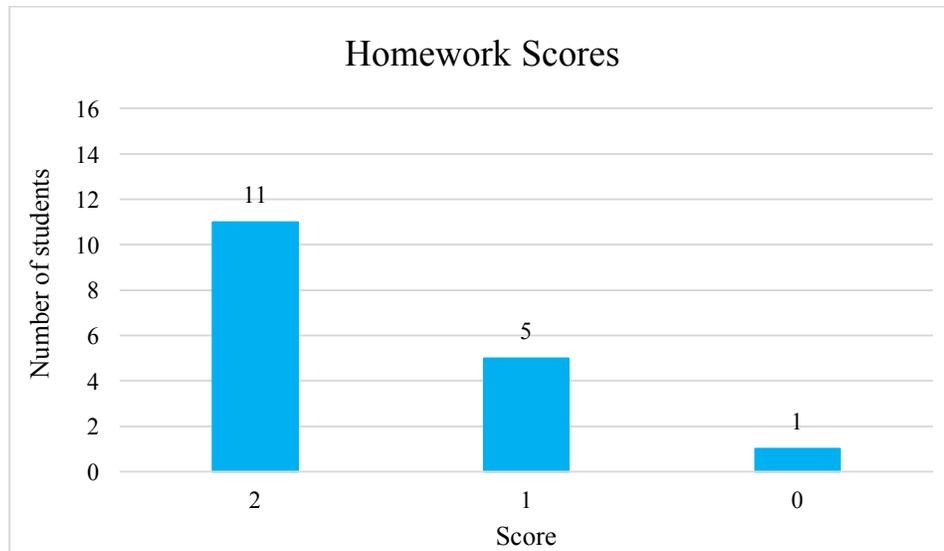


Figure 38

The average score of this homework assignment is 1.6, or 80%. The following are a few examples of homework answers.

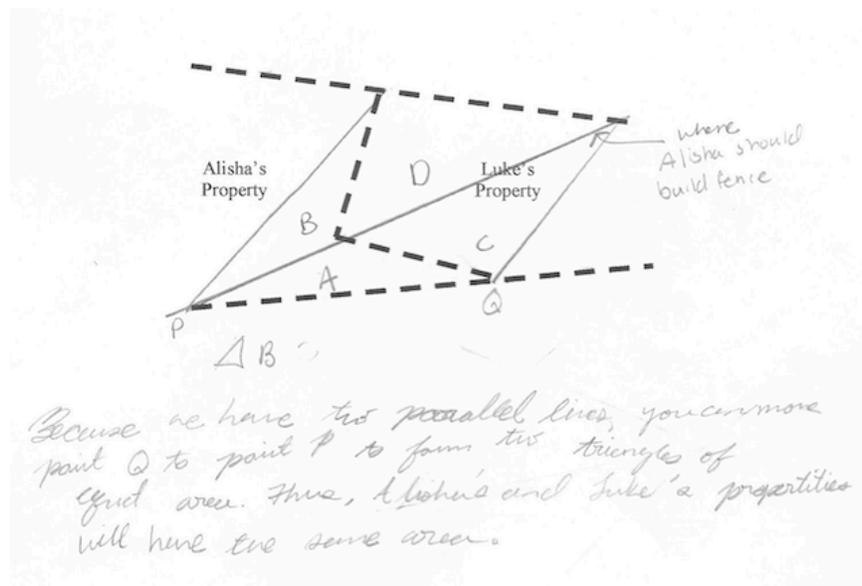


Figure 39

This answer received a 0 due to an incorrect line and incorrect explanation. The student did not grasp the concept and did not use her previous knowledge to solve the problem.

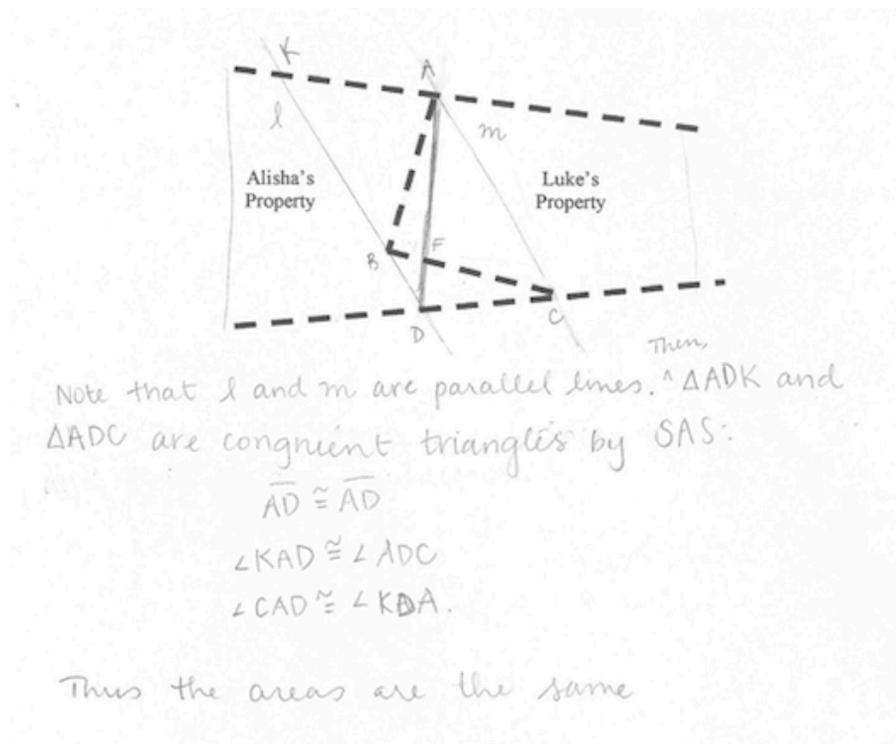


Figure 40

This answer received a 1. This student drew a correct line but did not explain the reasoning correctly. I believe the student understood the concept but was not clear on the reasoning behind this idea.

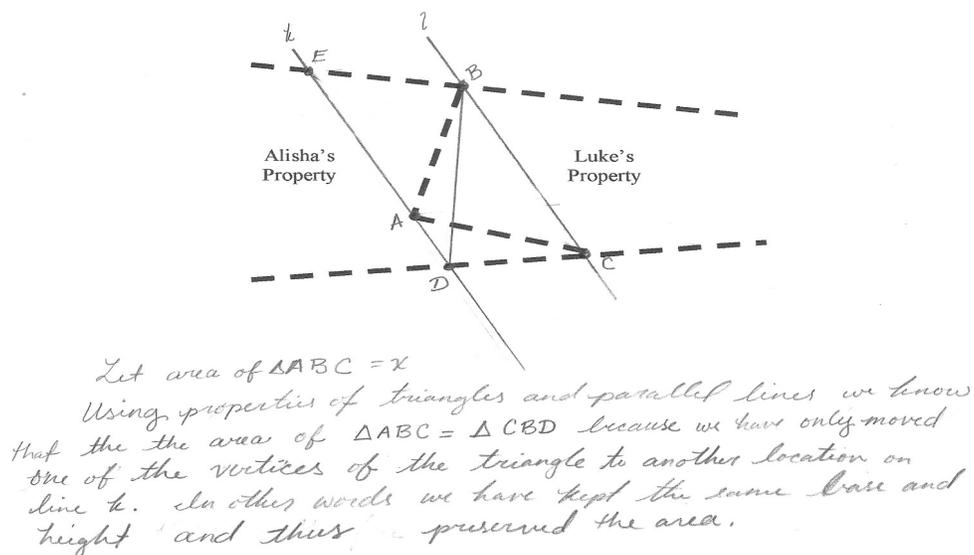


Figure 41

This student received a 2. They had to correct line and explained the reasoning well. They went as far to restate their idea in another way.

I evaluate the students level of retention again 3 weeks after I taught the lesson. I gave them the same problem we went over in class. When working individually, 12 students in the MAED 3100 class were able to complete the problem again and explain their answer correctly. One students received a 1, and 2 students received a 0 for not answering the problem. In the MAED 3119 class, all but one student was able to complete the problem again. The average score of this assignment is 1.7 or 85%.

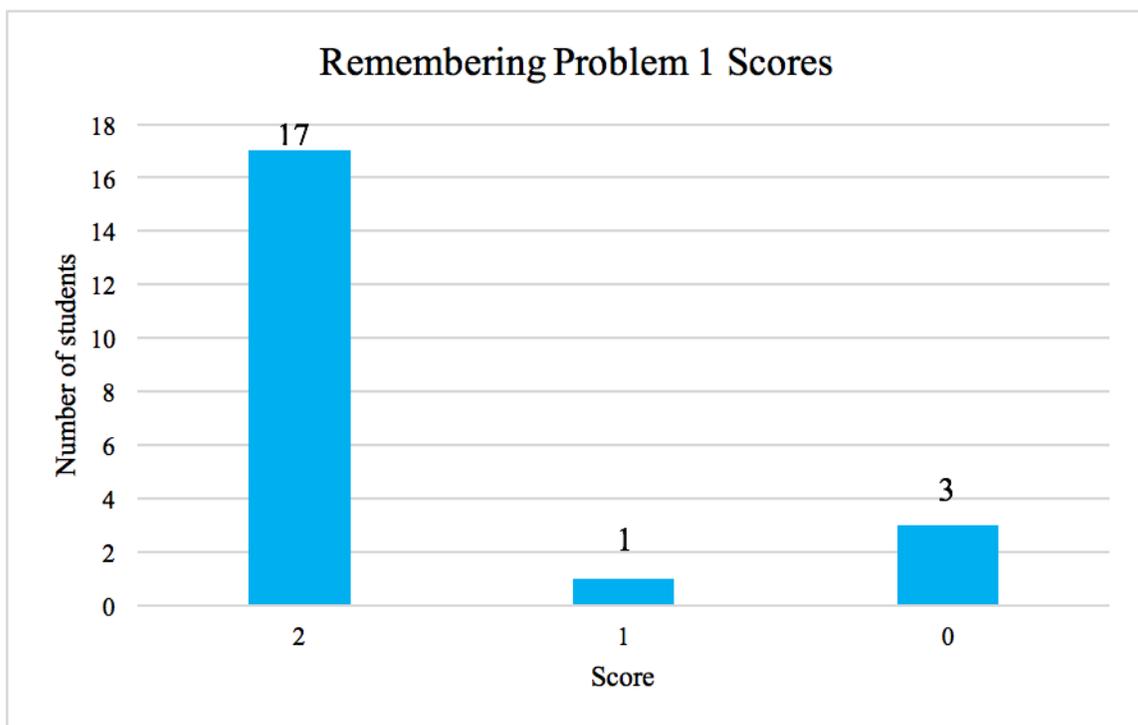


Figure 42

The last assignment I used to measure retention required the students to apply their knowledge even further. They had to draw a triangle with the same area of a given quadrilateral. Many students from the MAED 3100 class could not solve this problem. Nine (or 60%) of the students received a 0. Two students got a 1 and four students got 2's. This means (26.7%) of the

class could apply the lesson on their own. In the MAED 3119 class, two students were unable to solve the problem, one student received a 1, and three students were able to solve the application problem. That is 50% of this class could apply the lesson on their own. The average score of this assignment is 0.8 or 40%. Note that the average for the MAED 3119 class is 1.2 compared to the average for the MEAD 3100 class of 0.7. This shows that the MAED 3119 class was better able to apply the concept we learned in the original lesson.

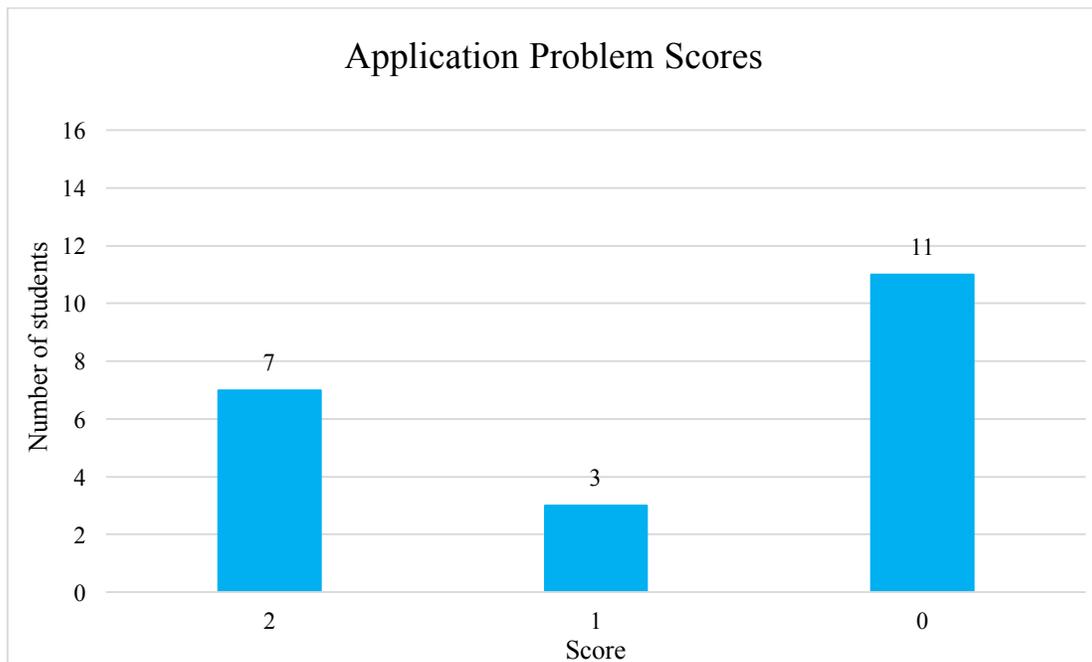


Figure 43

The following images exemplify a common misconception that students came across when solving the application problem. A few students thought you could alter the quadrilateral as a whole in the same way we altered the triangle in the previous problems.

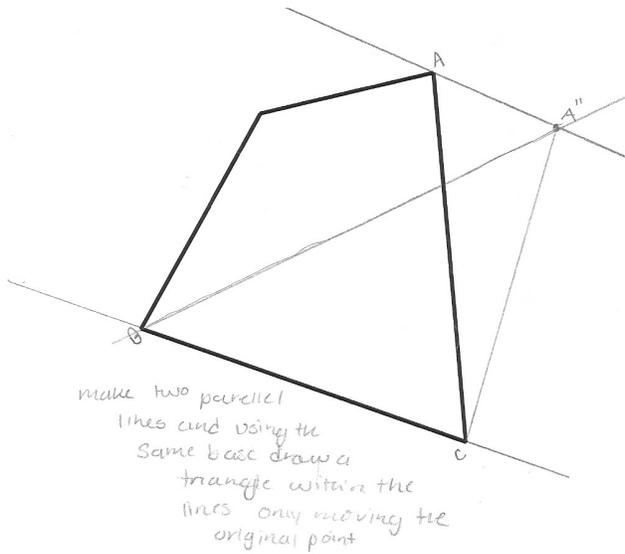


Figure 44

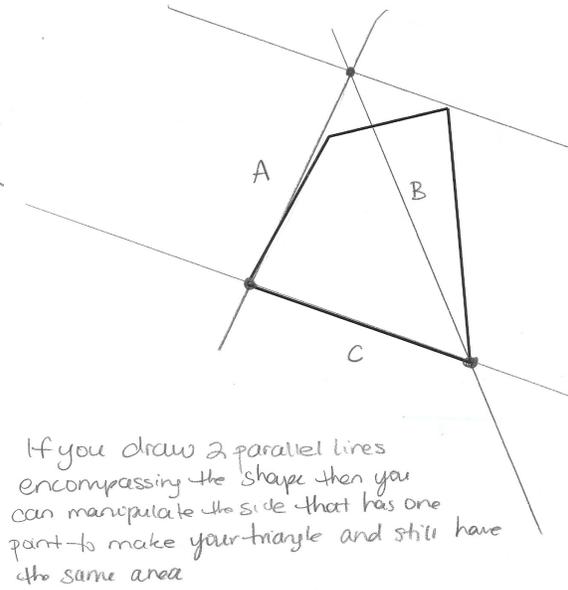


Figure 45

The next two images are good examples of correct solutions to the application problem. One of which includes a formal proof.

Since  $\overline{EF} \parallel \overline{AC}$ , we know by similar thinking from the cake problem,  $\triangle ABC$  area is invariant as point  $B$  goes across  $\overline{EF}$ . Since  $\triangle ABC$  and  $\triangle ACD$  make the quadrilateral  $ABCD$  and the area of  $\triangle ACD$  is invariant also as point  $B$  goes across  $\overline{EF}$ , we know that the area of  $\triangle ECD$  is and the area of  $\triangle AFD$  is equal to the area of the quadrilateral  $ABCD$ . Furthermore, we can do this to all the vertices of the quadrilateral  $ABCD$  to come up with 8 different triangles that has the same area of the quadrilateral  $ABCD$ . ☺

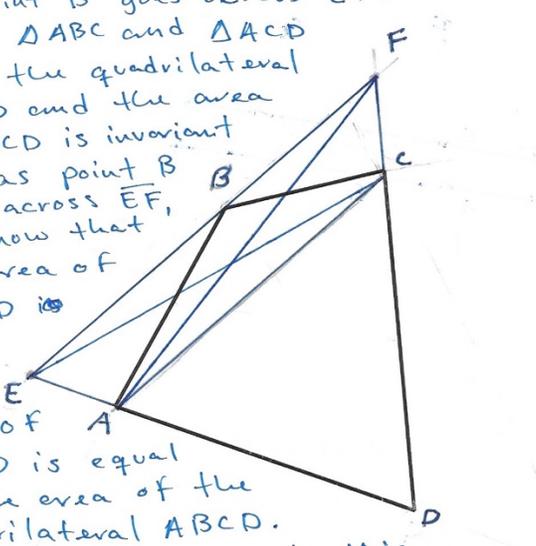
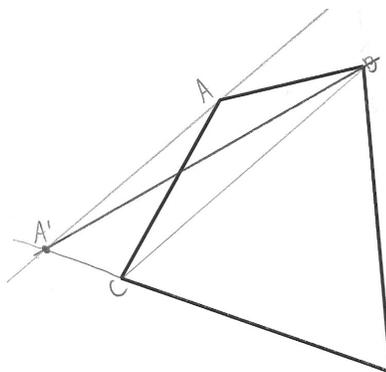


Figure 46



This is very similar to the cake problem, in that point "A" can move anywhere on the line. The difference is that, no matter where "A" was put on the line to create a triangle, "A" would need to be connected to the rest of the quadrilateral.

Figure 47

The average score of the homework assignment is 1.6. The average score was slightly higher for the assignment where the students had to redo the problem we explored through the lesson.

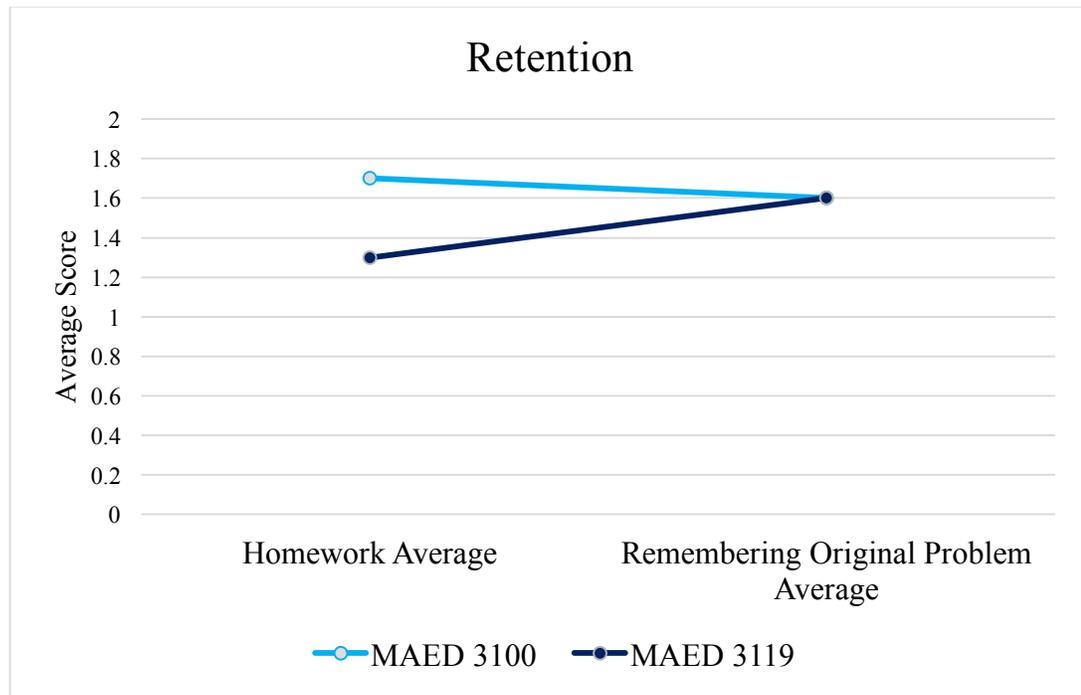


Figure 48

This slight increase tells me a few things. First of all, it means the students could retain the concept three weeks later with accurately. Secondly, I believe the slight increase in score is due to the homework problem being a different problem where the other assignment was the exact same problem we had been over. The average score of the application problem is a 0.8. The average score decreased by 0.8 from the homework assignment to the last application assignment. However, if we look at just the MAED 3119 class' application assignment, the average score decreased by only 0.4. This may be due to the MAED 3119 more extensive knowledge and experience with upper level mathematics. Overall, I expected to see a decrease in average due to the difficult nature of this problem. However, I speculate that the seven students

that could completely this problem may not have been able to do it if taught through step-by-step instruction.

### **Echoing the Literature**

I believe my findings echo the evidence from my literary review. The lesson I taught required the students to challenge themselves cognitively in a way they were not used to. They communicated through the lesson and struggled to come to a solution together. I believe most of the students gained conceptual understanding from my lesson. I was also able to see the level at which students were challenged through high school. This mimicked the idea that Japanese students are more successful because they discover concepts on their own rather than being told how to solve problems

### **Summary and Conclusions**

According the Third International Mathematics and Science Study Japanese students outrank U.S. students in mathematics. The average mathematics score of 8<sup>th</sup> grade students was a 586 compared to the United States average score of 518 (National Center for Education Statistics, 2015). I performed a study comparing the mathematics education of Japan and the United States. I looked at the levels of communication, productive struggle, and cognitive demand of a typical Japanese lesson and compared these to those of a typical United States lesson. I also evaluated the level of retention from this lesson. Communication and productive struggle are a large part of conceptual understanding. Conceptual understanding is what leads to students being successful in mathematics. Cognitive demand refers to the level at which students are challenged cognitively in the lesson. Students must be challenged in order to grow. I have found that a typical Japanese lesson uses problem solving based learning rather than step-by-step instruction. I created my lesson to mimic this type of lesson. I taught two groups of students, the

MAED 3100 class and the MAED 3119 class. I taught them a problem solving lesson and gave them a survey to give feedback on the lesson. I also conducted a few assessments to test the level of retention. I concluded from my lesson that many students saw the benefit of this type of learning. They struggled to come to solutions but remembered them well when they did. Some could apply their knowledge further to similar concepts. I also concluded that the average U.S. students is not challenged in this way during their middle school and high school careers. This shows me that Japanese education is more beneficial in making students futures more successful.

One major implication of my conclusions is curriculum development. Japanese curriculum covers about 10 topics per year. This allows the students and teacher to have time to explore each concept and formulate their own procedures. The teachers are not rushed to get through material. This also allows for better conceptual understanding and thus better retention of curriculum. Another implication of my conclusions is that teachers in the U.S. should strive to teach conceptually rather than procedurally. This gives students understanding in a way that allows them to make further connections.

Future research could be done to enhance on my conclusions. I see the benefit of completing another study with a greater population of students. Half of the students could be taught one lesson “American style” and the other half taught the same lesson “Japanese style”. The conceptual understanding, cognitive demand, and retention could be recorded and compared. This would allow researchers to see the difference of academic success based solely on the style of the lesson taught. However, one limitation that we could not overcome is the cultural expectation of Japanese students. All American students would not experience authentic Japanese education due to this cultural difference. If I could change or redo my lesson, I would

strive to include more students in my lesson. I would teach one lesson using step-by-step instruction and one using problem solving based learning then compare the two.

My research echoed many discoveries from the TIMSS. The level of communication and cognitive demand were high in my lesson. Each student experienced productive struggle and they retained the concepts three weeks later. When compared to the personal experiences of my students, they were not nearly as challenged through mathematics. Overall I believe my findings confirm that Japanese style education yields better success for students in the future.

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