Investigating the Content Knowledge Needed to be an Effective Mathematics Teacher

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Abstract

There is a major reform movement happening in the world of mathematics education. Not only does this change require teachers to do more than lecture at the front of the classroom, but it also requires teachers to know and understand mathematics more deeply. Through a survey given to high school mathematics teachers identified as effective, I looked for themes that may help to identify what teachers need to know in order to teach mathematics effectively. I will present findings on how these teachers draw on previously taken college courses as well as their own teaching experiences to inform their practice.

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Introduction

In the past, teaching mathematics involved more of a lecture-style approach with the teacher standing in front of the classroom going over the previous day's homework followed by a short lesson introducing new material. The class would end with the teacher assigning homework pertaining to that day's lesson. More often than not, the teacher repeats this teaching process daily. While this style of teaching may have worked in the past, it is less effective and less likely to survive in today's mathematics classrooms. Today, mathematics teachers not only have to teach different mathematics content, some content being more complex than in the past, but they also have to change the way they teach it to be effective. This means that mathematics teachers need to know and understand the content they are teaching inside and out as well as how to convey their knowledge and understanding to their students so that their students walk away with a good mathematics education. As a result, many problems arise; the most critical one being that teachers are not equipped with the right amount content knowledge needed to teach effectively. Moreover, a challenge arises in how to prepare future mathematics educators to teach effectively.

The purpose of this research is to explore the possible solutions to these challenges. It is to answer the questions, what do teachers need to know in order to teach mathematics effectively and how can we prepare teachers to do so? I have always had a love for mathematics and I have thought about being a math teacher since the beginning of high school. I had many good teachers, but I also had teachers that could have been better prepared. For example, while learning about matrices in a high school math class, it was easy to see that most of the students, including myself, had no idea what the teacher was trying to explain to us. My teacher was visually flustered and reached a point of giving up. She threw her hands up in the air and said, “I don’t know how else to explain it.” She ended up telling the few students who did understand the concept to help the students who did not or those who did not understand could just read the book for an understanding. This example has always stuck out to me as an example of what not to do as a math teacher. This has also affected me in that I have always been concerned that I may not fully understand a certain concept well enough to able to convey my knowledge to my students in a way that they can understand. This can be related to Li Ping Ma’s idea of PUFM, or profound understanding of fundamental mathematics; that is, an understanding of fundamental mathematics that is “deep, broad, and thorough” (1999, p. 118-120). In other words, if I do not have a profound understanding of fundamental mathematics, then I will find it more difficult to provide the knowledge my students need to build on the concepts they learn. Moreover, if I am not fully equipped with the right amount of subject knowledge, I may not be able to answer certain questions my students have for me.

Consequently, I am interested in this research so that I can better prepare myself with the content knowledge needed to be an effective mathematics teacher. This research will help me to teach my students mathematics in a way that they can grasp, understand, and appreciate. However, this research will not only benefit myself, but it will also benefit the community. As a result of this research, local schools could create programs that equip local math teachers with the content knowledge needed to teach mathematics effectively. It may also influence local colleges when it comes to preparing future mathematics educators. Though it may take time, teachers all over the country could be better prepared to teach mathematics, resulting in a better mathematics education for all students.

Literature Review

In the past, researchers have given more attention to how to improve the act of teaching rather than what kind of content knowledge teachers need to know in order to teach effectively (Ball & McDiarmid 1990). Lee Shulman, however, has done much research on the content knowledge teachers should know. In his article, Those Who Understand: Knowledge Growth in Teaching, Shulman (1986) divides his idea of content knowledge into three different categories: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge (p. 9). He defines subject matter knowledge to be "the amount and organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p. 9). Not only does this entail what the teacher knows about the subject they teach, but also how deeply they understand the subject matter they teach. Shulman (1986) goes on to say that,

> Teachers must not only be capable of defining for students the accepted truths in a domain. They must also be able to explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions, bot within the discipline and without, bot in theory and in practice (p. 9).

What Shulman is saying is that a teacher having the right amount of subject matter content knowledge means knowing about the subject they teach inside and out and also knowing why what they teach is true.

Another category of content knowledge is pedagogical content knowledge, which Shulman (1986) says, "goes beyond knowledge per se to the dimension of subject matter knowledge *for teaching*" (p. 9). He expands this idea by stating,

> Within the category of pedagogical content knowledge I include, for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations-

> in a word, the ways of representing and formulating the subject that make it comprehensible to others (p. 9).

In other words, a teacher uses their pedagogical content knowledge when they are able to blend together what they know about the subject they teach with what they know about the practice of teaching. This also includes being able to explain concepts to their students using a variety of teaching methods in a way that is understandable to their students. Moreover, pedagogical content knowledge involves the teacher's understanding of common misconceptions student have going about a certain topic (Shulman, 1986, p. 9). This is beneficial because understanding common students' misconceptions allows the teacher to be better prepared when it comes to teaching a difficult topic that students typically mess up on.

In another article by the name of Knowledge and Teaching: Foundations of the New Reform, Shulman (1987) once again talks about subject matter content knowledge as well as pedagogical content knowledge. He lists the categories of knowledge teachers should have in order to be effective. Once again, it seems the most important takeaways from this list of

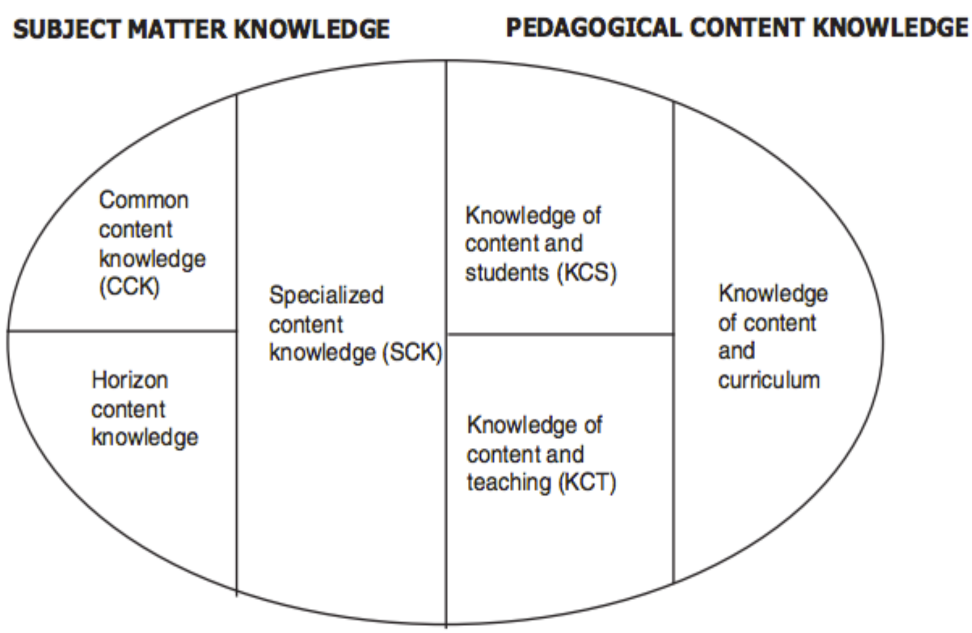
- General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter
- Knowledge of learners and their characteristics
- Knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures
- Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds
- Content knowledge
- Curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade" for teachers
- Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding

(Shulman, 1987, p.8)

categories for effective teaching are the ideas of "pedagogical content knowledge" and "subject matter content knowledge." In this article, Shulman (1987) explains the idea of pedagogical content knowledge in a different way by stating that teachers "must have not only the depth of understanding with respect to the particular subjects taught, but also a broad liberal education that serves as a framework for old learning and as a facilitator for new understanding." That is, teachers should deeply understand the subject they are teaching and be able to blend their understanding together with what they know about teaching, resulting in a more effective teacher. After all, as Shulman (1987) suggests, "Mere content knowledge is likely to be as useless pedagogically as content free skill."

Around the same time that Shulman published his research, Deborah Ball and G. McDiarmid published their article, The Subject Matter Preparation of Teachers, which focuses on Shulman's idea of subject matter content knowledge (1990). The authors are quick to point out that "if teaching entails helping others learn, then understanding what is to be taught is a central requirement of teaching" (Ball & McDiarmid, 1990). That is to say, if teachers do not understand the subject matter they are teaching, not only will it be very difficult to pass on their knowledge to their students but they may end up passing on wrong information to their students. The authors also make sure to call attention to the fact that knowledge of a subject is not the only essential quality a teacher should have. They say, "Good teaching demands that teachers know a lot of other things- for example, about learning, about their students, and about the cultural, social, and political contexts within which they work" (Ball & McDiarmid, 1990, p. 4). This idea is related to Shulman's idea of pedagogical content knowledge in that "good teaching" involves both knowledge of subject matter and knowledge of teaching, learning, and students' background.

Deborah Ball, Mark Thames, and Geoffrey Phelps (2008) take Shulman's two categories of pedagogical content knowledge and subject matter knowledge even farther by splitting each category into more subcategories (p. 403). Under subject matter knowledge the subcategories are common content knowledge (CCK), horizon content knowledge (HCK), and specialized content knowledge (SCK) while pedagogical content knowledge includes knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC) (Ball, Thames, & Phelps, 2008, p. 403). The authors go on to define each of



these subcategories as follows:

- Common content knowledge - the knowledge that any educated adult should know.
- Horizon content knowledge - "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum"

- Specialized content knowledge – knowledge that goes beyond what any educated adult should know, but does not require knowledge about students or teaching either
- Knowledge of content and students – knowing about students and knowing about mathematics
- Knowledge of content and teaching – knowing about teaching and knowing about mathematics
- Knowledge of content and curriculum – knowing about mathematics and knowing about the curriculum for which you teach (Ball, Thames, & Phelps, 2008, p. 399-403).

The authors continue the discussion of these subcategories by explaining further and giving examples of each type of knowledge. The first three examples are for the subcategories of subject matter knowledge. An example of a teacher displaying common content knowledge is being able to identify any incorrect answers the students give as well as noticing errors in the textbook (Ball, Thames, & Phelps, 2008, p. 399). Teachers knowing how the concepts in the mathematics they teach in a certain grade relate to the concepts in the mathematics that will be taught in the next grade is an example of horizon content knowledge (Ball, Thames, & Phelps, 2008, p. 400). Specialized content knowledge involves more of an understanding of mathematics that only teachers would need. Ball, Thames, and Phelps (2008) describe the following when it comes to specialized content knowledge:

> In looking for patterns in student errors or in sizing up whether a nonstandard approach would work in general, as in our subtraction example, teachers have to do a kind of mathematical work that others do not. This work involves an uncanny kind of unpacking of mathematics that is not needed— or even desirable—in settings other than teaching. Many of the everyday tasks of teaching are distinctive to this special work (p. 400).

The next three explanations delve deeper into the subcategories of pedagogical content knowledge. Knowledge of content and teaching includes organizing lessons in a way that allows students to learn by connecting old ideas to new ideas in conjunction with choosing specific and advantageous ways to teach a concept so that students get a better understanding of that concept (Ball, Thames, & Phelps, 2008, p. 401). For example, "KCT would be knowing different instructionally viable models for place value, knowing what each can reveal about the subtraction algorithm, and knowing how to deploy them effectively" (p. 402). When it comes to knowledge of content and curriculum, also known as curricular knowledge, the authors (2008) rely on Shulman's definition; that is,

> Curricular knowledge is represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances (p. 398).

Finally, knowing what students mess up on or find confusing as well as knowing how students think about mathematics constitutes only some of what knowledge of content and students entails. It also involves the teacher's own understanding of the concepts that students mess up on and "deciding which of several errors students are most likely to make" (Ball, Thames, & Phelps, 2008, p. 401).

Methods

As an attempt to answer our research question, I adapted the 2000 National Survey of Science and Mathematics Education to create my own survey (2001). The survey focuses mostly on teacher preparation. Some of the questions asked include current teachers' majors in college, whether or not teachers took any mathematics education classes in college, which upper level mathematics courses the teachers completed in college, as well as how qualified the teachers feel they are when it comes to teaching the courses they teach among many others. The survey I created contained some of the same questions as those of the Horizon Study (2001) but I added a few questions as well. One question I added was how the mathematics courses they took in college affect the way they teach today as well as to give a specific example of how. I did the same kind of question relating to the mathematic education courses the teachers took in college. I decided to survey several teachers that are considered to be “good” mathematics teachers. I only wanted to survey good teachers in order to find similarities between how the teachers were prepared to teach mathematics. Being able to find similarities on how these teachers were prepared will help with the preparation of all future mathematics teachers. The group of teachers I surveyed were recommended to me by way of recommendation from professors and administrators at Georgia College and State University. Once I obtained a list of teachers, I emailed them an online survey through email. Additionally, I used the snowball effect to get more responses by asking all the teachers I emailed to encourage any other teachers they know to be effective to take the survey The results of the survey will be qualitative with some quantitative analysis. The results of the survey are available to me only by way of logging into my account on the survey website. Additionally, the results of the survey are and will remain anonymous.

Analysis

The purpose of this survey was to find similarities between the teachers’ responses so that I could find possible solutions in how to prepare future mathematics educators to teach effectively. The questions I asked in my survey are as follows:

1) What was your major in college?

2) Which of the following college level mathematics courses did you take in college? Select all that apply.

- Linear Algebra
- Probability and Statistics
- Abstract Algebra
- Number Theory
- Geometry
- History of Mathematics
- Mathematical Analysis

3) Please list any other college level mathematics courses you took in college that are not in the list above.

4) Of the college courses you took, which courses do you draw on the most for teaching?

5) Of the classes you put for question 4, please describe how these courses influence your teaching. If possible, give specific examples.

6) During college, did you take any general education courses? If so, which courses did you take?

7) During college, did you take any mathematics education courses? If so, which courses did you take?

8) Out of the classes you put for questions 6 and 7, please describe how these courses influence your teaching. If possible give specific examples.

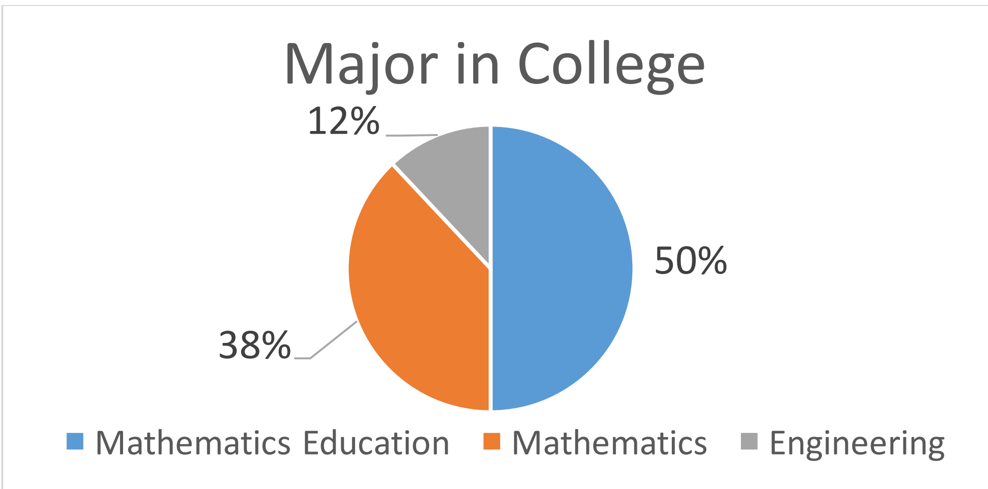
9) What high school mathematics courses do you currently teach?

10) What high school mathematics have you taught in the past?

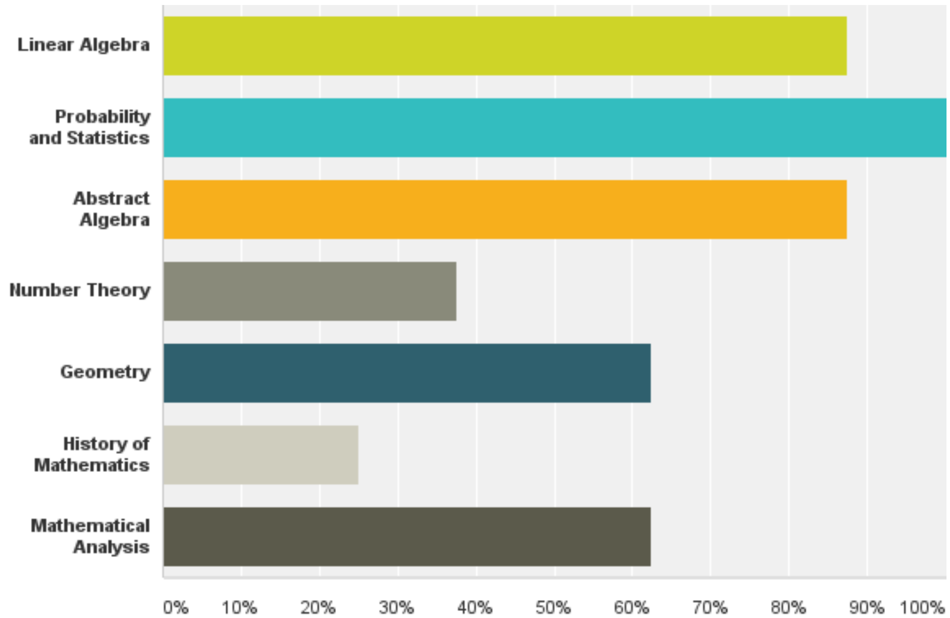
11) On a scale from 1 to 5, where 1 represents "not well qualified," 3 represents

"adequately qualified" and 5 represents "very well qualified," please rate how qualified you feel you are to teach each class you put for questions 10 and 11. Example: Calculus-4.

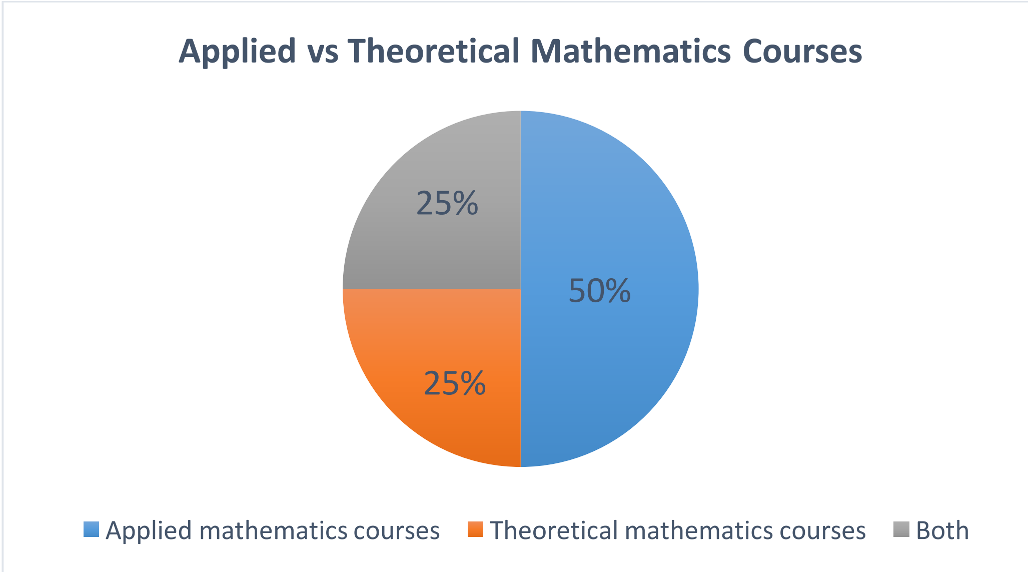
I will now share the results of each question asks. The first question asked for the teachers' majors in college. There were three answers for this question: 50% of the respondents said they majored in mathematics educations, 38% said they majored in mathematics, and 12% said they majored in engineering.



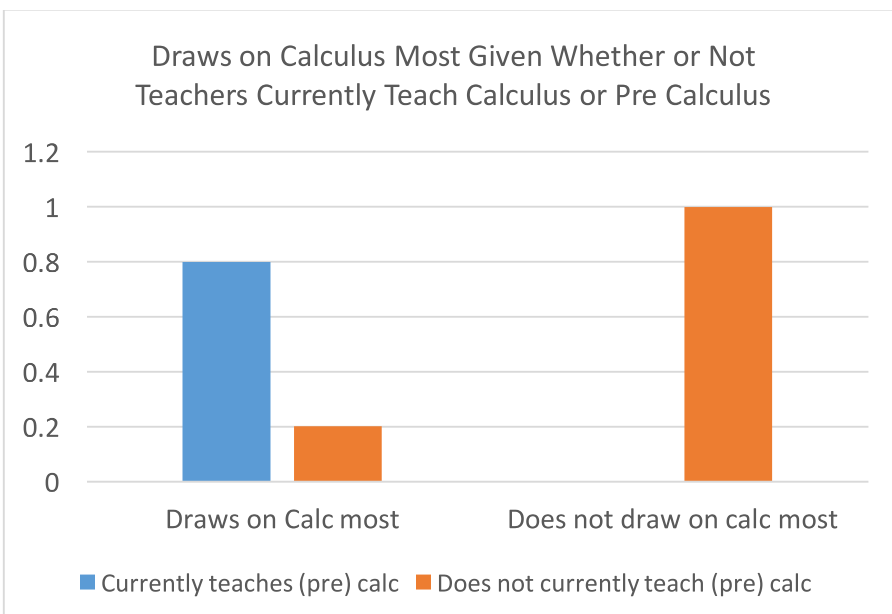
The second question was about which college level mathematics courses the teachers took out of the list given. All the respondents took probability and statistics, 87.5% of respondents took abstract algebra and linear algebra, 62% of respondents took geometry and mathematical analysis, while less than 50% of respondents took number theory and history of mathematics, 37.5% and 25% respectively. The results of this question can also be seen in the bar graph at the top of the next page. Question three asks what other classes the teachers took in college. Some of the answers include calculus I, II, and III, statistical analysis, differential equations, discreet mathematics, numerical analysis, into to higher level math, decision theory, set theory, and matrix theory to name a few.

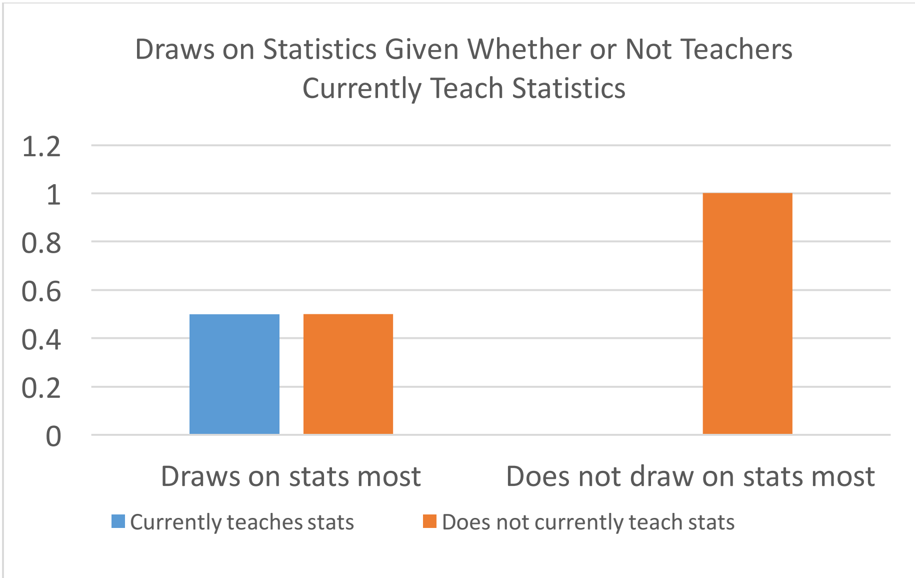


Questions four asks the teachers out of the courses they took, which ones do they draw on most for teaching. For this question, I decided to categorize their responses into applied mathematics courses and theoretical mathematics courses. Applied mathematics courses include linear algebra, probability and statistics, and geometry, while theoretical mathematics courses include abstract algebra, number theory, history of mathematics, and mathematical analysis. I found that 50% of respondents draw on applied mathematics courses most, 25% draw on theoretical mathematics courses the most, and 25% draw on both applied and theoretical mathematics courses for their teaching as show below on the top of the next page. Since statistics and probability and calculus were the most common answers to this question, I created a side by side bar graph that shows whether or not teachers currently teach calculus or pre-calculus given that they draw on calculus. Similarly, the other graph shows whether or not teachers currently teach statistics and probability given whether or not they draw on statistics and probability. The results show that given teachers draw on calculus the most, 80% currently teach calculus or



pre-calculus and 20% do not currently teach calculus/pre-calculus while given teachers do not draw on calculus, 100% do not currently teach calculus/pre-calculus. On the other hand, given that teachers draw on statistics and probability the most, 50% currently teach statistics and probability and 50% do not currently teach statistics and probability while given that teachers do not draw on statistics and probability the most, 100% do not currently teach statistics and probability. The graphs are given below.





The next question asked was how the mathematics courses they took influence their teaching. I will give exact quotes from some of the teachers and relate each quote to Deborah Ball's idea of mathematical knowledge for teaching. One teacher said:

> They taught me how to think about math and strategies for analyzing problems and situations; thus, I try to not only teach the course content but more generally how to think about math.

We can relate this back to knowledge of content and teaching. Another teacher writes that

> The flavor of those courses is to prove everything possible. In teaching, we should never settle for mechanics when we can get to an understanding of why things work the way they do.

Once again, we can relate this back to knowledge of content and teaching. The last two quotes are both about the content knowledge learned through the courses they took. One teacher says, "They provided the content knowledge needed for me to be able to teach AP math classes," while the another teacher simply wrote, "Deep knowledge of math!" The way the mathematics courses both of these teachers took influence their teaching can go back to specialized content knowledge.

Questions six and seven ask whether or not the respondents took general education courses and whether or not the respondents took mathematics education courses. The results show that 75% of respondents took at least one general education course and 75% of respondents took at least one mathematics education course. This question was followed by which general/mathematics education courses did the teachers draw on for teaching. The answers were all so different so I will just list a few responses. Some general education courses the respondents draw on for teaching include educational psychology, special education and the exceptional child classes, and principle/foundations of education, while some mathematics education courses the respondents draw on for teaching include methods of secondary mathematics teaching, mathematics curriculum, and practicum/student teaching classes.

Once again, I asked how the general education courses and the mathematics education courses influence the respondents' teaching. I will give exact quotes relating to general education courses first and relate them back to Ball's idea of mathematics knowledge for teaching. One teacher said that they "let me know why the learner learns, the mechanisms, the downfalls." We can relate this back to the idea of knowledge of content and students. Another teacher responded with more about what to expect by saying, "Much reading and classroom role play was used to assist in the learning of what to expect (the worst) and how to properly and professionally deal with it." Since this mostly deals with knowing about students, we can attribute this to knowledge of content and students. Lastly, there was one teacher whose response involved standards, lesson plans and observation. This teacher said

> We discussed questioning techniques, what the new Georgia standards looked like, how to plan units and lessons given specific standards, observing and working in schools, and having to experience problem solving tasks in class.

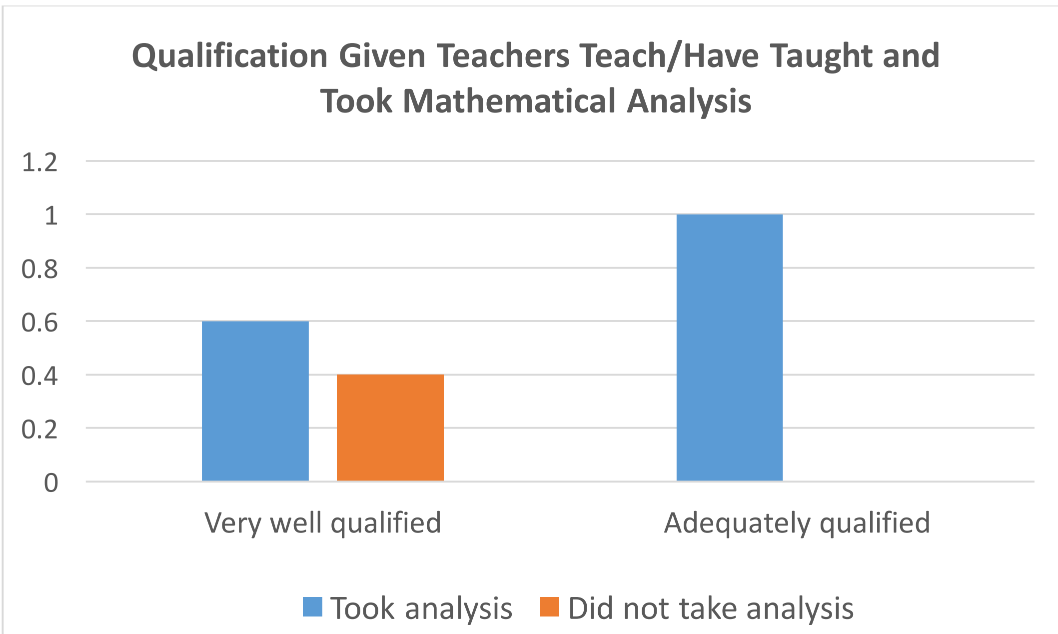
This teacher's quote involves both knowledge of content and teaching and knowledge of content and curriculum.

Next are the quotes that show how mathematics education courses influence the respondents' teaching. The first quote I will show is simple: "It helped to refine techniques of questioning and problem solving." The way this teacher was influenced shows a knowledge of content and teaching. The second quote from a teacher is

> Guiding us as learners to understand that to teach math well we needed to see the forest instead of the trees (shoot for the big over arching concepts instead of individual skills).

We can attribute this influence to the idea of knowledge of content and curriculum. The last two quotes are less related to mathematical knowledge for teaching and more about the advantages of taking mathematics education courses. One respondent states that, "these had a massive influence on my teaching, especially the teaching secondary math courses." The other respondent felt that "these courses offered at the university I went to gave me an advantage when it came to teaching in a high school math setting today."

Finally, the last question asked was how qualified the respondents feel they are to teach the classes they currently teach as well as the classes they have taught in the past. The results show that 62.5% of respondents feel very well qualified to teach **all** the classes they teach/have taught, 25% of teachers feel adequately qualified to teach statistics, 12% of teachers feel adequately qualified to teach AP Calculus, and 12% feel they are adequately qualified to teach AP Physics. Because calculus was the most common answer when it comes to classes the respondents currently teach, I made a side by side bar graph that applies only to the teachers who teach calculus. It compares whether or not the respondents took mathematical analysis given whether or not they feel very well qualified to teach calculus. The graph shows that given that the respondents feel very well qualified to teach calculus, 60% took analysis while 40% did not take analysis. On the other hand, given that the respondents feel adequately qualified to teach calculus, 100% took analysis. The graph is shown below.



Conclusions

For my conclusions, I will briefly recap the data analysis. The following are the major conclusions:

- The majority of respondents majored in Mathematics, followed by Math education.
- 75% of respondents draw on applied mathematics courses for their teaching, especially statistics and calculus
- Mathematics courses provide subject matter knowledge and enhances higher level mathematical thinking and problem solving.

- The majority of respondents took both general and mathematics education courses.
- General education courses address how students learn, what to expect as a teacher, teaching techniques, how to prepare a lesson, etc.
- Mathematics education provide knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum.
- All teachers feel very well qualified to teach non-AP classes; some feel adequately qualified to teach AP calculus and/or AP Statistics.

Implications

Once again, I will write exact quotes from teachers the tie in nicely with my implications. This first quote is about learning to teach from your previous teachers. The respondent says

> I think some educational courses could've helped, but I think for me it was important to see what good teachers did and try to generalize those things for effective educational methods and for my own teaching.

This teacher suggests that learning from teachers is important. While this can be a good thing, it mostly poses a problem in that mathematics education will not change for the better if we do not learn from and teach differently than our previous teachers (Ball, 1990). John Dewey (1998) reinforces this idea by saying, “Experience and education cannot be directly equated to each other. For some experiences are miseducative. Any experience is miseducative that has the effect of arresting or distorting the growth of further experience.” Deborah Ball (1990) clarifies John Dewey’s quote by stating, “Prospective teachers, equipped with vivid images to guide their actions, are inclined to teach just as they were taught.”

This next implication goes beyond undergraduate school. One respondent suggests that the biggest influence on his or her teaching is experience. He or she says

> BY FAR the biggest influence on my teaching is experience - seeing what concepts the kids always mess up, understanding their misconceptions and trying to correct them, and discussing with respected colleagues these ideas and how to address them.

The idea of learning through experience implies that teachers must reflect on their own teaching experiences. That is to say, teachers need to make it intentional to reflect on what works, what does not work, what they can do better, etc. This quote also alludes to understanding misconceptions that students. It shows that understanding common student misconceptions is beneficial so that teachers learn how to teach certain concepts more effectively. I think that adding a mathematics educations course devoted to addressing common student misconceptions or at least adding this topic to current mathematics education courses would help future mathematics educators. This teacher points out yet another important implication; that is, communication with colleagues. Just like in any profession, it is important to talk to your colleagues in an effort to better your own practice.

The last implication once again goes beyond undergraduate school and involves a quote from the same teacher in the previous quote. He or she says

> The only aspect from graduate school that I can clearly recall that is definitely an aspect of my teaching is the idea of teaching through questioning and the notion of wait time. Sometimes my students get uncomfortable when I don't just give them the answers, but they eventually realize that they will have to at least try to figure something out before I'm willing to either nudge them forward or explain a solution. They know that they will have to think instead of just being told the answers.

This teacher makes reference to the idea of teaching through questioning and the notion of wait time being learned in graduate school. While this is true and good, it may be beneficial to make the techniques explicit in undergraduate courses. Once again, these methods could be added into a current mathematics education course. It is also important to note that a teacher can learn different teaching techniques through experience. If this is the case, I must reinforce the idea of intentionally reflecting on your own practice so that you are able to learn from your own experiences.

I would also like to mention the limitations and implications for future research. For one, I have a small number of respondents. I do believe a larger number of participants could have resulted in discovering more themes, but it is also possible that no themes come from a larger number of participants. Secondly, surveys are less specific. In the future, I would do case studies on a few teachers as follow up research.

To conclude, I will say that there may not be one right way to change the world of mathematics education for the better, but others have definitely begun to find some solutions. According to the survey I conducted, it is obvious that there are many ways to become an effective mathematics teacher. I think the results of my survey could help the mathematics education community. More importantly, I know it will certainly help me when it comes to my own experiences as a teacher.

References

Ball, D. L.. (1990). Breaking with Experience in Learning to Teach Mathematics: The Role of a Preservice Methods Course. *For the Learning of Mathematics*, *10*(2), 10–16. Retrieved from http://www.jstor.org/stable/40247978

Ball, D. L., & McDiarmid, G. W. (1990). The Subject Matter Preparation of Teachers. In W. R. Houston, M. Haberman, & J. Sikula (Eds.). *Handbook of Research on Teacher Education*, 437-449.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special?. *Journal Of Teacher Education*, *59*(5), 389-407.

Dewey, J. (1938). *Experience and Education*. New York: Macmillan.

Horizon Research. (2001). Report of the 2000 National Survey of Science and Mathematics Education. Chapel Hill, NC: Author. Available at http://2000survey.horizon-research.com/.

Ma, L. (1999). *Knowing and Teaching Elementary Mathematics*: *Teachers' Understanding of Fundamental Mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15 (2), 4-14.

Shulman, L. (1987). Knowledge and Teaching: Foundations of the New Reform. *Harvard Educational Review*, 1-23.