Examining the use and Effectiveness of Questioning in a Mathematics Classroom

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Lecture is defined as the transfer of information from the notes of the lecturer to the notes of the students without passing through the minds of either (Small 6). So why is lecture one of the primary teaching methods that is used in mathematics classrooms today? The teaching of mathematics needs to move away from traditional teaching methods and into methods that require students to think on a higher level. The method of questioning in mathematics could require students to form their own conjectures and ideas about concepts. Good questioning can help students move from simply memorizing and taking notes on mathematics to doing mathematics. Using questioning can be effective in helping students to think critically about mathematics and therefore understand mathematics better.

The National Council of Teachers of Mathematics, NCTM, state in their Principles and Standards that “Mathematical thinking and reasoning skills, including making conjectures and developing sound deductive arguments, are important because they serve as a basis for developing new insights and promoting further study” (NCTM 15). Teachers should use questioning that requires students to use what they already know to form ideas and conjectures about something that they do not know. This helps students create connections between different concepts within math and help them to see a bigger picture. The process also helps students to “do mathematics” as opposed to just memorizing and repeating. “Doing Mathematics means generating strategies for solving problems, applying these approaches, seeing if they lead to solutions, and checking to see if your answers make sense.” (Van De Walle 13). In creating a classroom environment where students "do mathematics" one can avoid memorization and drill that can be implemented with a lack of understanding. "You may consider analysis questions to be the start of inquiry or problem-solving process, and the beginning of a change from direct to indirect instructional strategies" (Borich 251). Higher level critical thinking questioning is good because it can help a teacher take a lesson plan from a simple lecture into students forming their own conjectures about certain concepts. By posing questions teachers encourage students to participate in "doing mathematics."

There are different types of questions that a teacher can ask for different purposes in allowing a student to "do mathematics." Teachers can ask questions in order to test students' knowledge, encouraging higher level thought process, and structure and redirect learning, which build onto what the students already knows (Borich 239). For our purposes we will be focusing on encouraging higher level thought process questioning.

Questions can be classified as either convergent or divergent questions. A convergent question limits an answer to a single or small number of responses where as a divergent question encourages general and open responses. Convergent questions can be classified as either knowledge, comprehension, or application questions. A knowledge question, as defined by Bloom's Taxonomy, is a question that requires a student to exhibit memory of what was previously learned by the recall of facts, terms, and basic concepts and answers. In a mathematics classroom this type of question would look like "How many degrees are in a triangle." The second level of questioning, comprehension questions, are questions that require a student to demonstrate their understanding of ideas and facts by comparing, organizing, and stating main ideas. "What is the best answer" or "compare and contrast the following" are examples of a comprehension question. The third level of questioning contained in convergent questions is application questioning. This form of questioning requires students to solve problems by applying acquired knowledge, facts, and techniques. "How should we approach this problem" is an example of how an application question would be used in the classroom. This type of question requires students to use what they have previously learned.

Divergent questioning, or open questioning, encourages higher level thought process and critical thinking. "Most rationales for using higher-level, divergent type questions include promotion of thinking, formation of concepts and abstractions, encouragement of analysis-synthesis-evaluation, and so on" (Borich 241). This is important for questioning in mathematics. Whenever a question is posed, it should be used to get the student thinking about the concept and help guide them towards forming a conjecture. Marian Small states that open questions should meet the goal of differentiation in order to meet the needs of all students (Small 6). Small continued, saying that in order for this to be achieved the teacher needs to create or develop a single question that is inclusive, allowing students to approach the problem using different approaches and methods. The divergent question posed should allow for students at different stages of mathematical development to benefit from the problem that is solved, because students can solve it the way that makes sense to them (Small 6). Not all problems require the same steps to solve.

Analysis, synthesis, and evaluation questions all fall under the category of divergent questions. Analysis questioning calls for students to examine and break information into parts by identifying causes as well as finding evidence to support generalizations. Analytical questions require student interpretation of concepts from inductive reasoning to deductive reasoning. Inductive reasoning uses specific examples to form a general idea and deductive reasoning takes a general idea and applies it to specific examples. Analysis questioning allows students to begin forming their own conjectures about concepts. “What conclusions can you draw” requires students to take what they know with certain examples and apply to a concept, inductive reasoning. The next level of questioning in Bloom’s Taxonomy, synthesis, requires students to develop patterns and alternative solutions. Students can to approach the problem different ways, and in mathematics there is often more than one method to solving a problem. The final level of divergent questions is evaluation questioning. This form of questioning calls for students to present and defend their findings and verify validity of results. Evaluation questioning in mathematics guides students to explain how and why they got their solution. This allows for the teacher to confirm whether or not a student actually understands a concept.

Both convergent and divergent questioning have benefit in a classroom, but divergent questions can be more effective in leading students to think critically and move from memorizing mathematics to understanding mathematics. In order to do this the teacher must make sure that the divergent question that is posed does not exceed the students’ zone of proximal development, the distance between the actual development level of the student and the level of potential development. “Instruction within the zone of proximal development allows students, whether through guidance from the teacher or through working with other students, to access new ideas that are close enough to what they already know to make the access feasible” (Small 3). The teacher needs to pose a divergent question that does not exceed the students’ zone of proximal development while also making sure to not pose a question that is below their development level in order to ensure that the question is effective in broadening the students' understanding and knowledge of a concept.

A way to help students reach a new plateau of understanding on a subject after the initial divergent question has been posed is by using a probe. A probe is a question that immediately follows a student's response to a question in order to elicit clarification of the student's response, solicit new information to extend or build on the student's response, or to redirect/reconstruct the student's response in a more productive direction (Borich 254). A probe can be used to further students' critical thinking on a concept and help them reach a new level of understanding. Eliciting probes are also used to have students clarify or rephrase what they have stated. "Could you say that in a different way?" Doing this helps the student show more of what they know and helps a teacher know the level of student understanding (Borich 255). This helps the teacher to know how the student is thinking and what the students understands.

The students understanding of how they arrived at their conclusion can be more valuable than the solution itself. If a student is arriving at the correct response, but does not understand why or how they got their response then the student is not doing mathematics. "Use probes to solicit new information following a response that is at least partially correct or that indicates an acceptable level of understanding" (Borich 255). Probes lead the student by pushing the students' response to a more complex level. "Failing to follow up a student's answer with 'Why' Or 'How did you get that answer' or 'Can you explain your thinking' is another often missed opportunity I observe in classroom after classroom" (Leinwand 69). When asking a question it could be a missed opportunity if you hear the answer and then move immediately on to another. The teacher should ask how and why the student has that as their answer.

"[A probe that solicits] builds higher and higher plateaus of understanding by using the previous response as a steppingstone to greater expectations and more complete responses. This involves treating incomplete responses as part of the next higher-level-response-not as a wrong answer. The key to probing is to make your follow-up question only a small extension of your previous question; otherwise, the leap will be too great and the learner will be stymied by what appears to be an entirely new question" (Borich 255). Teachers need to make sure that they not do move too fast when probing a student. The temptation is for teachers to quickly connect relating concepts, but this could confuse the students more because students do not make the connection for themselves. "Use probes to redirect the flow of ideas instead of using the awkward and often punishing responses such as 'You're on the wrong track,' 'That's not relevant,' or 'You're not getting the idea'" (Borich 255). Doing this shuts down the students thinking and willingness to respond. A teacher needs to encourage student thinking and make a safe classroom environment within which to share their ideas.

The National Council of Teaching Mathematics stresses it is essential to create an environment that fosters critical mathematics thinking skills of forming conjectures, experimenting with different approaches to problems, and constructing mathematical arguments (NCTM 18). Both Larry Lesser and Steven Reinhart believe in creating a classroom environment in which effective questioning is used to help further student understanding. In Larry Lesser's presentation at the GCTM Conference in October 2012, *Setting the Tone: Establishing a culture of Engagement from Day One,* he discussed the importance of setting up a classroom structure that encourages students to discuss and share their findings for problems posed (Lesser). Students need to be able to fell safe and comfortable sharing their answers and Lesser's ideal classroom is effective in creating this environment for students. Lesser stressed beginning the first day of class with the classroom structure that the teacher wants for the entirety of the school year. This classroom structure starts with the presentation of a good question being posed and is followed by individual reflection of the problem, group discussion of conjectures made, and finally a class discussion. In this classroom structure a divergent question is posed to the students in order to help them think critically and after working on the problem by themselves they are then able to work in pairs or groups. This discussion is valuable to the students' learning because it allows students to discuss what they have discovered and compare their discoveries to other students' work. By allowing time for self discovery and group comparison the classroom discussion of the concept is made much richer. Students are more willing to share the conjectures they have made in the safe environment that has been fostered through these techniques (Lesser).

Steven Reinhart, a middle grades math educator, also believes in creating a classroom to further student understanding of concepts. In his article *Never Say anything a Kid Can Say,* Reinhart discusses the importance of the classroom structure and the critical importance of effective questioning. "As I moved from traditional methods of instruction to a more student-centered, problem-based approach, many of my students enjoyed my classes more." (Reinhart 478). Reinhart discovered that his students enjoyed being able to discuss their conjectures they found to problems he posed. He also states that by applying strategies that require students to participate he has been able to create a classroom in which students are actively engaged in learning mathematics as well as feeling comfortable enough to share their ideas and conjectures about concepts (Reinhart 479).

Reinhart uses several questioning strategies and techniques to help further his students' education. Asking good question that require more than a recall of a fact or the reproducing of a skill is one of these. "I encourage students to think about, and reflect on, the mathematics they are learning." (Reinhart 480). Reinhart is encouraging his students to "do mathematics" as opposed to the memorization of facts or skills. Asking divergent questions that require students to reflect, analyze, and explain their thinking and reasoning is another method that Reinhart uses to increase the learning of mathematics in his classroom (Reinhart 480). Reinhart makes sure that he is replacing lecture, a traditional teaching method, with sets of divergent questions and a classroom structure for a positive classroom environment.

Reinhart also stresses the importance of allowing enough wait time in between the asking of divergent questions (Reinhart 480). Students need to be allowed time to process and develop a conjecture about the concept. Teachers need to plan questions well in order to pose effectively to their class and they need to allow time for students to ponder and think through the questions. Divergent questions cause students to break problems into parts and determine relationships between them (Borich 250). This is useful for connecting the many different concepts of mathematics.

By posing a divergent question to students the teacher can receive a variety of answers that can create a discussion among the students on how to go about solving a certain problem or concept. Reinhart stresses the importance of being prepared for this variety of responses. The broader ranges of responses that occur are at an analysis level and this is how the students are able to form their own ideas about the problem on which they are working (Borich 251). Teachers should be prepared for the different types of responses that students may have when posing an analysis question. “Questions at the synthesis level ask the student to produce something unique or original-to design a solution, compose a response, or predict an outcome of a problem for which the student had never before seen, read, or heard a response.” (Borich 251).

“Accept all reasonable answers, even though your own solutions may be limited to one or two efficient, practical, accurate solutions. Efficiency, practicality, and accuracy might be built from student responses, but you cannot initially expect it of them” (Borich 252). Allowing students to talk with each other and form their own ideas helps the students to “do mathematics.” Even if there is a wrong answer, before telling the student that they are incorrect one should let the students correct each other. Another student might disagree with the incorrect answer and the students can discuss why they arrived at different conclusions. In doing so, the students can help one another arrive at the correct conjecture for the concept being discussed.

One other note that Reinhart makes about asking questions in the classroom is that teachers need to remain nonjudgmental about a response or comment that a student makes. If a teacher responds to a student in a very positive way, highly praising them for their contribution, then other students will be less willing and confident in sharing their response to the question. The same can also happen if a teacher responds very negatively to a student’s response. Many students do not have the confidence to share their answer when the student before them has been chastised for their poor and incorrect response.

Reinhart states that teachers should instead encourage more discussion among students. "Allowing students to listen to fellow classmates is a far more positive way to deal with misconceptions than announcing to the class that an answer is incorrect." (Reinhart 481). A classroom where discussion of concept is fostered allows for students to correct their classmates' conjectures and strengthen the entire classes understanding of the concept.

In a classroom study of an American mathematics classroom conducted in 1995 by Trends in International Mathematics and Science Study, TIMSS, the teacher of the classroom appears to use traditional teaching methods to conduct a lesson. In this classroom the teacher leads checkups and warm-ups, homework review, seatwork, checking more homework, and learning new material. During this process there appeared to be little active learning. The teacher would pose a question and when it was not answered in a timely manner he would answer it himself. Steven Reinhart warns against this type of teaching: "Answering my own questions only confuses students because it requires them to guess which questions I really want them to think about, and I want them to think about all my questions" (Reinhart 483). By answering the questions themselves teachers are training students that they do not need to think about the question at all, that is they wait long enough the teacher will answer the question for them.

The teacher moved into a review of the students' homework where he used more convergent questioning about the problems they had completed. Recall that convergent questioning limits an answer to a single or small number of responses. After the review the teacher directed individual seat work. He began explaining the examples on the worksheet by asking questions, such as: "Can you tell me about angle 2 and 3?" A student answered, "They're vertical." The teacher responded saying, "No. They are not vertical." Not only did the teacher pose a convergent lower-level thinking question, but instead of asking the student why or how they had come up with their answer he responded immediately saying that they were incorrect. This reaction is opposite of what Steven Reinhart uses in his classroom. Reinhart remains nonjudgmental about his students' responses whether they are correct or incorrect in order to encourage responses. Responding to a student response either positively or negatively may discourage students to respond.

Teachers can miss an opportunity to probe the student and guide them through critical thinking to the correct response. The teacher could have allowed for class discussion here and questioned the entire class so that the other students might explain why the angles were not vertical.

After new material was presented, the teacher wrote on the board the formula for the sum of angles in a polygon. The teacher worked several examples using this new formula. Instead of giving the students the formula, the teacher could have given the students different polygons with their angle sums and had them try to find the pattern and develop the equation themselves. This would have allowed the students to think critically and form their own conjectures about this new concept instead of just memorizing the equation that was given to them.

Throughout this entire lesson, few critical thinking questions were being used by the teacher. Instead the lesson was mainly lecture based and questions asked were simple convergent questions at the knowledge level. There was no discussion among the students. Interaction was between the teacher and student when the teacher asked convergent questions.

In the TIMSS study conducted in 1999 of a Japanese classroom the teacher of the classrooms approach looked vey different from the approach used in the American mathematics classroom. After the students have filed into the classroom the teacher begins the class by showing what they did the previous class. He directed their attention to a television monitor and then proceeded to show what they had already gone over concerning triangles that share the same base and height. After this, the teacher posed a problem to the class that required critical thinking.

After allowing students to discuss their predictions about the solution to the problem, he allowed the students to think individually for 3 minutes. After this amount of time students were allowed to either continue individual work, work with other students, use hint cards, or discuss their findings with a second teacher in the classroom. During this time the teacher walked around and probed students that were struggling or needed to clarification. These methods of questioning helped focus the students' thinking so that they can form conjectures about the concept on which the students are working.

Two students presented their findings to the entire class. The students discussed what they had discovered while working on the problem and showed their fellow classmates how they solved the problem. During the presentations the students' classmates would comment and ask questions on how the student arrived at their answer.

The amount of learning and understanding that went on in this classroom appeared to be greater then the American classroom, even though the teacher only posed one question to the class. This Japanese lesson is a an example of a classroom where questioning is used and is used effectively. The teacher posed one question and yet all the students appeared to be actively engaged, learning, and doing mathematics. The students seemed to have an understanding of the concepts they were learning and the students appeared to be able to connect it with the previous lesson. This teacher fostered a classroom where "doing mathematics" is key as well as through the use of effective questioning in order to help further the students' knowledge.

So which classroom is better for allowing students to "do mathematics" and acquire mathematical concepts? According to the Average Scores on TIMSS 1995 and TIMMS 1999 of eighth grade mathematics classrooms, Japan scored an average of 83 points higher than the United Sates (TIMMS Video Research Group 769). Why is this? The TIMMS Video Mathematics Research Group found that although 17% of the problem presented in the United States classrooms focused on making connections, the way these problems were discussed by teachers did not adequately explain the mathematical connections to the students (TIMMS Video Research Group 773). This percentage of connection making problems is in the range of many of the other higher-achieving countries, but the questions were not presented in U.S. classrooms in a manner that allows for students to make connections among concepts. So while not all American classrooms focus on lecture based lessons, those that attempt using good questions might not discuss the problems in a way that makes the mathematical connections visible to the students or uses the potential within the problem presented

In Japan the connections between concepts is discussed very thoroughly and this shows in the higher scores that were achieved in testing. The Japanese lesson focused on making a connection between what the students had learned the previous lesson and the new concept on which they were focusing. The teacher did this through posing a divergent higher-level thinking question that called for students to create their own conjectures and discover a new mathematical concept.

The Japanese lesson followed the structure that both Larry Lesser and Steven Reinhart suggest. The teacher began by posing a divergent question, allowed for students to think individually, then allowed for students to work in groups or ask for help and hints, and then finished with a classroom discussion of what discoveries the students made. This structure with the use of questioning appeared to be effective as every student was actively engaged in working on the problem and all appeared to have a clear understanding of the conjecture made during the class discussion. The students also understood how the concepts were connected, which is an important part of "doing mathematics."

This is a very different structure from what was used in the American classroom. The lesson was predominately a lecture. There was little discussion between the students and between the students and teacher (Other than answering when called on). This set up created an environment where students did not have to be active in learning. The students could rely on the teacher to answer all of the convergent lower-level questions for them; there was no need to think critically. Based on the TIMMS 1995 and 1999 mathematic assessment scores in mathematics this form of teaching is not as effective as questioning. The American assessment scores are significantly lower than the scores of Japan.

It appears that overall questioning is more effective then the traditional method of lecturing. Using questioning in the mathematics classroom allows students to form their own ideas and make discoveries about concepts as well as connections among concepts.

NCTM states, “Learning mathematics involves accumulating ideas and building successively deeper and more refined understanding.” This deeper understanding of mathematics can be achieved through effective questioning. Students move from inductive reasoning to deductive reasoning through the use of questions. Students are “doing mathematics” as they have to discover how to solve problems for themselves instead of the alternative of being given the steps or formula. Giving students a particular method for solving a problem forces students to reproduce the procedure with a no or little understanding of why the procedure works. Using divergent questions allows for students to think critically on a higher level and allows students to make connections and develop their own pathway to a solution. Ideally students will move from mimicking procedures and memorizing rules to “doing mathematics.”

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