Increasing Students' Understanding of Multiplying Binomials through Manipulatives

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Abstract

The National Council of Teachers of Mathematics (2000) suggests that the use of Manipulatives in the mathematics classroom can improve a student's Conceptual Understanding of a mathematical concept. Kelly defines a manipulative as "any tangible object, tool, model, or mechanism that may be used to clearly demonstrate a depth of Understanding, while problem solving, about a specified mathematical topic or topics" (2006, p. 184). The purpose of my study is to see if the use of manipulatives gives students a Conceptual Understanding of multiplying binomials. Eighth grade students from the Early College and students of the Middle Grades Cohort at Georgia College were given a pre-assessment and a post-assessment to assess the improvement of their Understanding of the FOIL method after a lesson using Algebra Tiles.

INTRODUCTION

As a mathematics major with a teaching concentration taking mathematics education courses, I am studying new ways of Understanding mathematics topics through diverse tools, with one being manipulatives. As someone who has always been strong in Mathematics, I have developed a deeper Understanding of numerous topics just through the use of manipulatives. I was interested to find if students were able to gain a Conceptual Understanding of mathematics topics through the use of manipulatives in a mathematics lesson. I was also interested to see if students who already obtained Procedural Fluency were able to gain a Conceptual Understanding. Lastly, I was eager to find if using manipulatives in a mathematics classroom can improve the students' attitudes towards mathematics.

Students are becoming less interested in mathematics because they are not making connections between the world around them and the mathematics classroom. The mathematics classroom has become a series of rules or repeated steps to find the answer, rather than Understanding the topics at hand. With the change in curriculum and positive outcomes from the use of hands on learning, today's classrooms are beginning to gradually shift towards using manipulatives in the classroom (Golafshani, 2013). With the use of manipulatives, students will be able to see connections in real life and find more appreciation of the subject of mathematics. Student will be able to find the connections through different representations of abstract mathematical ideas through the use of manipulatives. The National Council of Teachers of Mathematics (2000) suggests that the use of manipulatives in the mathematics classroom can improve a student's Conceptual Understanding of a mathematical concept. Highly valued learning theorists, such as Piaget, Dienes, and Bruner, believe that "using concrete tools is an

important stage for learners as they develop an Understanding of new concepts" (Cope, 2015, p.14).

LITERATURE REVIEW

Mathematical Proficiency

"In the 1950s and 1960s, the new math movement defined successful mathematics learning primarily in terms of Understanding the structure of mathematics together with its unifying ideas, and not just as computational skill" (National Research Council, 2001, p. 115). Education today has come to a series of repeated steps that students follow to come to the correct answer without knowing its reasoning or how it may be used in our daily lives. The National Research Council (2001) believes that these are the five aspects of learning mathematics that allow students to be Mathematically Proficient: Conceptual Understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning, and Productive Disposition. These five strands work interdependent, but they must all be present for a student to be Mathematically Proficient (National Research Council, 2001). Bautista (2013) believes that Mathematical Proficiency begins with Understanding the problem, learning strategies and procedures to problem solve, along with having the reasoning justified. For my research, I focused on the strands of Procedural Fluency and Conceptual Understanding of the FOIL Method through the use of manipulatives.

Procedural Fluency

The National Research Council defined Procedural Fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (2001, p.116). If students are able to perform the procedure for a math problem correctly, they are likely to remember the pattern and steps to complete the problem. However, if students are dependent upon the wrong

procedure for a problem, they are likely to continue to use the wrong procedure in the future. This demonstrates the importance of including the other four strands of Mathematical Proficiency so students are able to make the connections and understand why they are implementing a certain procedure and when it is appropriate. The National Research Council states that "without sufficient Procedural Fluency, students have trouble deepening their Understanding of mathematical ideas or solving mathematics problems" (2001, p.122). Understanding where a theorem or procedure stems from and how it can be executed allows students to make connections between other areas of mathematics or other concepts. It is important to understand the concepts in Mathematics since the concepts are building blocks leading to new discoveries and ideas. Although it is important for students to remember certain procedures to complete a problem, the other five strands must be present for students to expand their mathematical problem solving.

Conceptual Understanding

Conceptual Understanding is defined by the National Research Council as "an integrated and functional grasp of mathematical ideas" (2001, p. 118). When students are able to connect different representation of a problem or make connections with other mathematical concepts, they are believed to have a Conceptual Understanding. Students who are able to organize their thoughts are able to form new ideas by making connections from what they have previously learned (National Research Council, 2001). Conceptual Understanding allows for a deeper Understanding of mathematical ideas that reduces the number of errors a student may have on a problem. If a student is only fluent in procedures, then they are not able to come across new ideas and are not able to realize certain connections. Conceptual Understanding allows students to explain their reasoning for connecting ideas and are able to back their reasoning with

arguments and logical connections (National Research Council, 2001). There is no Conceptual Understanding without making connections and using a variety of representations to prove ones Understanding in the mathematics classroom.

Connections. When students are making connections in the mathematics classroom, they are able to create new insight and find new ways of Understanding previously learned material (National Council of Teachers of Mathematics, 2000). It is important for students to have a strong foundation of each concept of mathematics, since the topics build from one another. Without a strong foundation of certain topics, students will not be able to easily find the connections between new ideas presented to them with material they have learned previously which they need to succeed in the classroom. NCTM believes that it is a teacher's responsibility to "encourage students to think, question, solve problems, and discuss their ideas, strategies, and solutions" (2001, p. 18). A generous amount of wait-time or independent thought should be allowed for students to reflect upon their previously learned knowledge and the new concepts presented to them. "When students work hard to solve a difficult problem or to understand a complex idea, they experience a very special feeling of accomplishment, which in turn leads to a willingness to continue and extend their engagement with mathematics" (NCTM, 2000, p.21). When students are able to solve a difficult problem, classroom discussions can be used to allow students to hear other explanations of their problems which could allow for more connections and Understanding from a classmate's point of view. NCTM believes that "Procedural Fluency and Conceptual Understanding can be developed through problem solving, reasoning, and argumentation" through this form of classroom setting (2001, p.21).

Representations. There are multiple representations of mathematical concepts that can be offered to students in the classroom. "Representations, which can be used in mathematics education, include physical (concrete), pictoral (static visual), and virtual (dynamic electronic) representations" (Cope, 2015, p.11). Physical representation or such as manipulatives allow students to bridge the gap between the concrete Understanding of mathematics to the abstract Understanding (Protheroe, 2007). Physical manipulatives can involve blocks, paper, or any easy to access supply that would allow students to move the objects and better understand a mathematics concepts. Mathematical topics can be given by pictoral representations which help students envision certain ideas giving them an explanation or reasoning for the use of the concept. Lastly, students can use virtual representations to understand mathematical topics. Virtual representations are used via a computer program which are interactive allowing students to interact with the virtual manipulatives. For my research, I used the physical manipulative of algebra tiles to allow students to gain a Conceptual Understanding of FOILing polynomials.

Manipulatives

Catherine Kelly defines a manipulative as "any tangible object, tool, model, or mechanism that may be used to clearly demonstrate a depth of Understanding, while problem solving, about a specified mathematical topic or topics" (2006, p. 184). Manipulatives in the classroom can range from any tangible object that students are able to hold and move around to allow students to explore the problem presented to them. Some concrete manipulatives that are used in the classroom include pattern blocks, algebra tiles or blocks, and Geoboards. Concrete manipulatives allow for an engaging, interactive, and meaningful lesson that students will be able to recall after the lesson. When using manipulatives, students should be allowed a wait-time. During this time, students are participating in trial-and-error as they try to find the solution.

Students are engaging in critical thinking and problem solving so that when they have completed the problem correctly, they feel accomplished. This creates a morale booster in the mathematics classroom which is important for students to be engaged in the subject.

The Research on the Benefits of Manipulatives explains the three stages for learning a new mathematical concept in the order of concrete stage, representational stage, and abstract stage (Hand2Mind, 2017). The concrete stage comes first which introduces the math concepts with the use of manipulatives in a lesson (Hand2Mind, 2017). Manipulatives come first in the lesson so that students are able to think through their previously learned knowledge to make connections with a different representation of a mathematical concept. Next is the representational stage. This stage allows for a visual model of the concept where students are able to communicate their ideas and Understanding (Hand2Mind, 2017). Reflecting over the knowledge gained from the manipulatives is an important part of the process because students are able to hear the reasoning behind the manipulatives. Lastly, the abstract stage implements the use of symbols and mathematical notation to explain the concept (Hand2Mind, 2017). The abstract stage pulls the knowledge from the manipulatives and gives the information in mathematical notation.

Pros of Using Manipulatives

Throughout literature, there are many pros that are found through the use of manipulatives. Manipulatives allows for students to gain a Conceptual Understanding of the mathematical concepts. “The concrete objects engage kinesthetic learners and lead students to develop a Conceptual Understanding of how the different components of an algebraic expression or equation can be combined” (Bruins, 2014, p. 3). When new material is presented to students in the form of notes, PowerPoints, or worksheets, students who are kinesthetic learners have a

difficult time grasping the information. Manipulatives allow for all visual, auditory, and kinesthetic learners to understand the material since they are hands-on, visual, and allow for verbal explanations to deepen the Understanding for all students. Manipulatives are designed to represent explicitly and concretely abstract mathematical ideas (Moyer, 2004, p. 2). It is important that students who are learning mathematics are able to see the diverse ways of Understanding the material. Using manipulatives allows for students to think mathematically giving them a tool for problem solving. "Manufacturers advertise manipulatives as materials that will make the teaching and learning of mathematics 'fun' and promote their products as catalysts for engaging students in mathematical learning" (Moyer, 2001, p. 176). Since using manipulatives in the classroom is nontraditional, students are engaged and interested to learn using tangible objects. When students find a correct solution, they become more interested in the learning mathematics because they feel accomplished.

Cons of Using Manipulatives

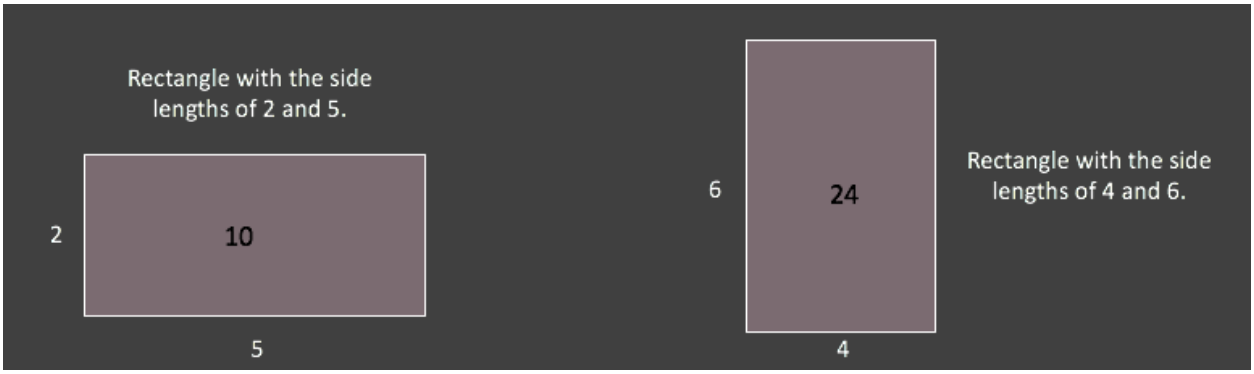
Throughout literature there are a few cons that are constant throughout. Teachers play an important role in creating mathematics environments that provide students with representations that enhance their thinking (Moyer, 2001, p. 178). Teachers must give students explicit instructions allowing students to understand how they can use the manipulatives to model the mathematical idea. Without showing the students how they are used, teachers must ask the students guiding questions and open-ended questions allowing students to think through the problems individually without assistance. "The process of effectively making the connection between hands-on activities and abstract algebra concepts takes skillful planning on the part of the teacher and a larger investment of instructional time than traditional instructional methods" (Bruins, 2014, p. 8). Teachers must make it clear to the students that the manipulatives are used

for Understanding purposes only. Many teachers believe that students will take the manipulatives as toys rather than a tool used for Understanding (Bruins, 2014). They also believe that using manipulatives is too time consuming when there is already a lack of time to cover all the standards (Witzel, Smith, & Brownell, 2001). Since there is wait-time required for students to use the manipulatives and grasp the Understanding, teachers believe that it involves too much time for the amount of time they are allowed to cover all of the standards.

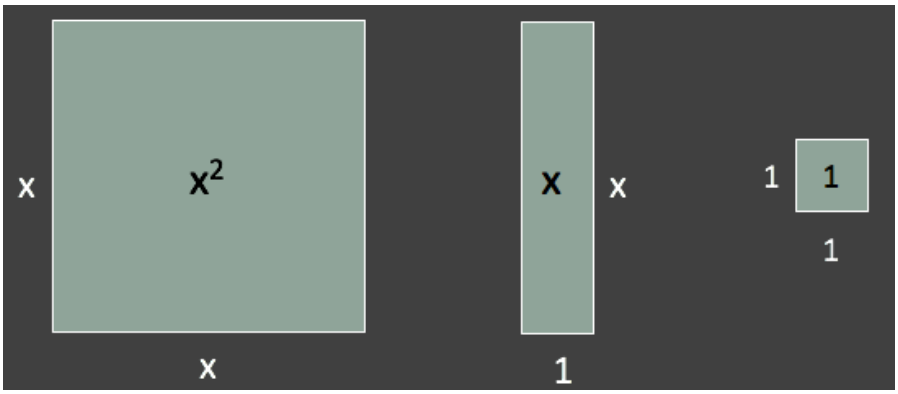
METHODOLOGY

My research was conducted in Dr. Santarone's Math Education course for the Middle Grades Cohort and in Mrs. Claxton's Eighth Grade math course at the Early College. I started my lesson by giving both classes a pre-assessment over FOILing two binomials (See Appendices A and B). The pre-assessment asked the students to use the FOIL method on the given binomials. They were asked to explain the steps they used to foil the equation. Lastly, they were asked to model the equation by drawing a picture. After the pre-assessment, I began my lesson.

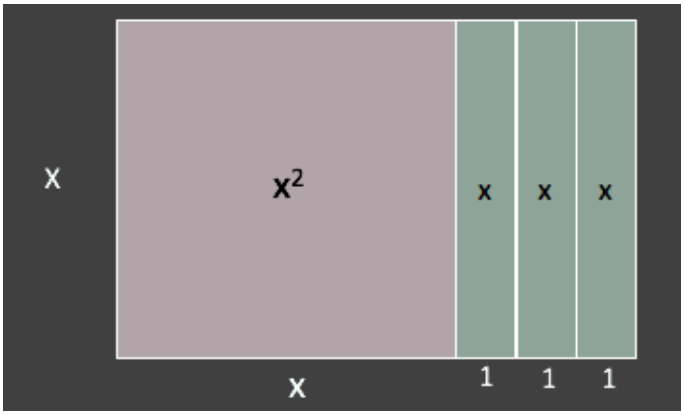
I started my lesson by allowing students to review upon finding the area of a rectangle when given two numbers that represent the side-lengths. Here are a couple examples of problems where I asked students to find the area of a rectangle with side measures of 2 and 5 along with the measures of 6 and 4.



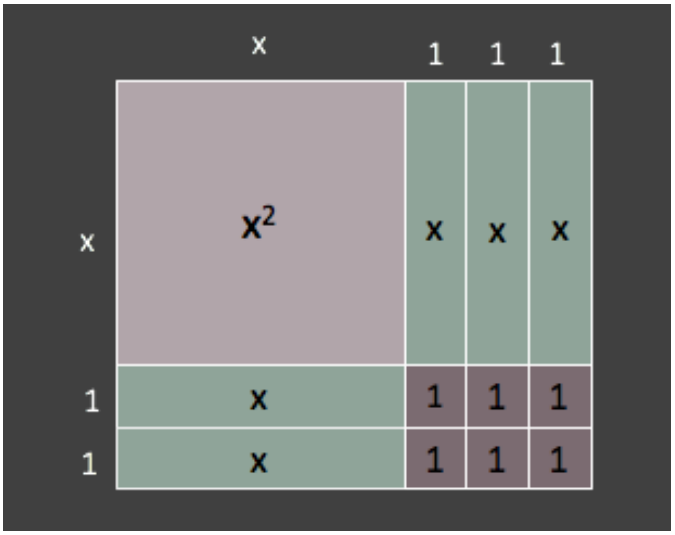
Next, we moved to finding the area of a rectangle if the side lengths were given by variables. Students were then given the manipulatives which is when I explained their purpose. There were three different types of shapes that were used in the demonstration. Below is a picture that explains how the manipulatives were represented.



The first shape was a square with side length of 1 unit, thus giving us the area of 1unit ^{2}. Students were asked to pick up the bar or rectangular shape allowing them to notice that the shorter side is the same length as the square previously explained, making one side length 1 unit. The longer side of the rectangle was represented by the length of x, hence giving us the area of x units ^{2}. The last shape was a larger rectangle whose side length was equal to the side length of x from the rectangle, thus giving us the area of x^2 units with each side length represented by x. After explaining the manipulatives, I asked students to give a rectangle with the side-measures of x and x+a with a being an integer. Below is an example of where students were asked to form a rectangle with the side measures of x and x+3.



Students spent time thinking through the process of forming a rectangle with the given side-measures shown. They were asked to draw the shape along with labeling the side-measures of the overall shape, the individual pieces and the area of each piece. Lastly, students were asked to form the rectangle given by two binomials. Below is an example of where students were asked to form the rectangle and find the area of the rectangle given by (x+2) and (x+3).

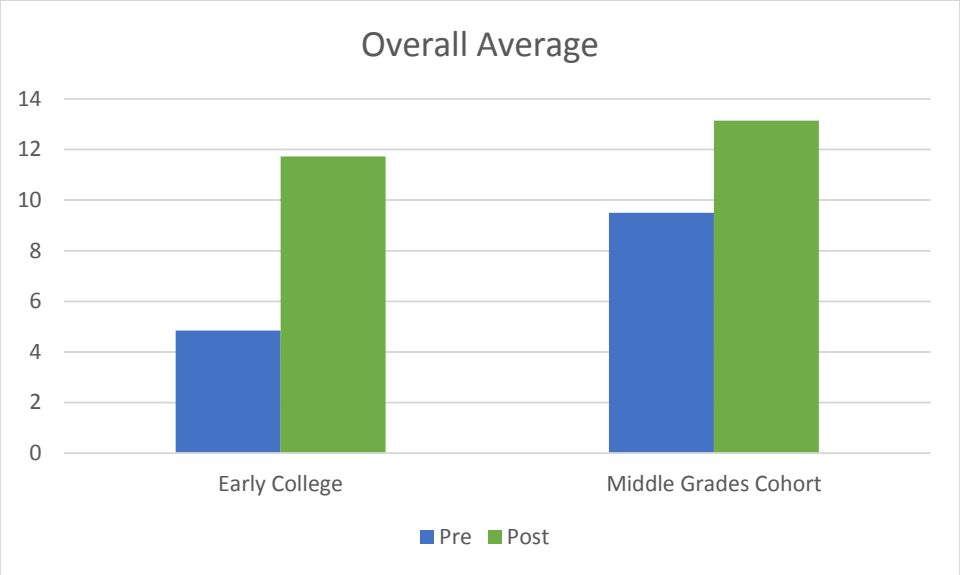


This portion of the lesson was the most time-consuming since students were allowed thinking time to figure out how to form a rectangle. After they were able to form the rectangle, they were asked to draw the rectangle on paper and label all of the side lengths of each piece along with filling in the area of each individual piece. Once their pictures were drawn, we came together as a class and wrote out the Area as the sum of all the individual areas of the shapes used within the

rectangle. We continued with a couple more examples. After students were comfortable with representing the area of the rectangle with the manipulatives, I gave them the meaning of FOIL as it stands for First, Outer, Inner, Last. Then, students used the FOIL method on binomials that were previously used with the manipulatives where they were able to compare their answers given by the manipulatives and the FOIL method. Lastly, they were given the post-assessments (See Appendices C and D) with the same questions as the pre-assessment, which included using the FOIL method on given binomials, explaining the steps used, and representing the equation by a picture. Students had their Pre-assessments and Post-assessments graded by the following rubric on a scale of a zero to a five evaluating each of the three parts: FOILing the equation, Explaining the steps of FOILing the equation, and drawing a picture that represents the equation (See Appendix E).

FINDINGS

After collecting the data from my Research, I have found that students have a more Conceptual Understanding of the FOIL method after the use of manipulatives in the classroom. There was an overall individual improvement in the Early College and the Middle Grades with 97.3% of students who increased their overall score from pre-assessment to post-assessment. The overall average is the average of scores from the assessments out of fifteen with each of the three questions being out of five total points. The Early College showed a 142% increase of their overall scores from pre-assessment to post-assessment with an average score of 4.839 on the pre-assessment and 11.725 on the post-assessment. The Middle Grades increased their scores from pre-assessment to post-assessment by 38% with the overall average score of 9.5 on the pre-assessment and 13.136 on the post-assessment. The graph represents the average total scores on the pre-assessments and post-assessments for both the Early College and Middle Grades Cohort.

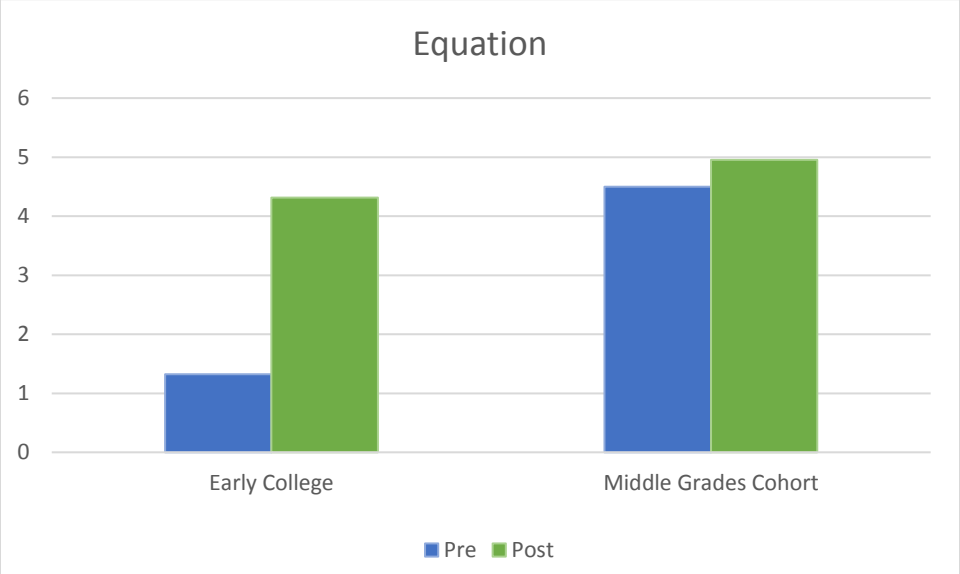


Both the median and mode for the score of the Early College's foil the equation question increased from a 4 and 5 to a 12 in both the median and mode. While both assessments for the Middle Grades Cohort had a median and mode of a 10.5 and 11 which increased to a 13 for both the median and mode on the overall average. We can see from the graph and statistics that the Middle Grades Cohort came into the research with more knowledge of the FOIL method than the students of the Early College. This is due to the fact the Middle Grades Cohort had Procedural Fluency of the foil method because of previous mathematics classes, which we can find from the data received from the Equation portion of the assessments.

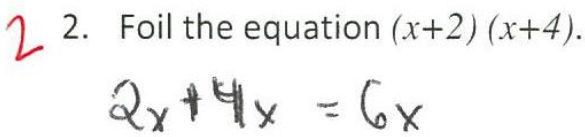
Question 1: Equation

The first question of both the pre-Assessment and post-Assessment asked students to perform the FOIL method on two binomials, such as (x+2) and (x+4). Between both the Early College and the Middle Grades, there were 91.8% of students who increased their scores from pre-assessment to post-assessment on the Equation portion of the assessments. This graph represents the average scores on FOILing the equation on both the pre-assessments and post-

assessments for both the Early College and the Middle Grades Cohort which was out of a total of five points.

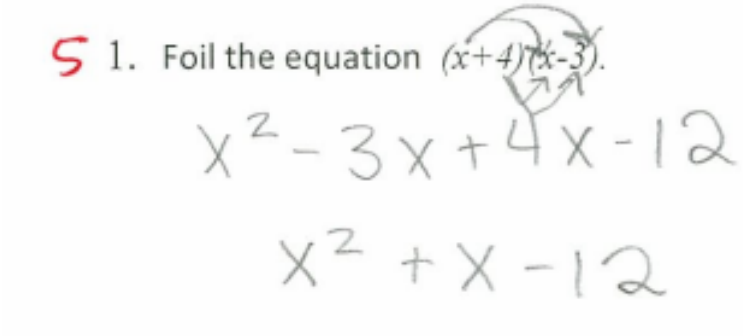


The students of the Early College at Georgia College had an average score of 1.323 on this portion of the pre-assessment. A typical answer for the Early College students on this portion of the pre-assessment is shown in the picture below.



This sample scored a two because their answer written had mathematical logic, but no connection with the FOIL method. While the students of the Early College had a low average on this portion of the exam, the students of the Middle Grades Cohort had an average score of a 4.5. Both the median and mode for the score of the Early College's foil the equation question increased from a 1 to a 5. While both assessments for the Middle Grades Cohort had a median and mode of a 5. This is an indication that the students in the Middle Grades Cohort have

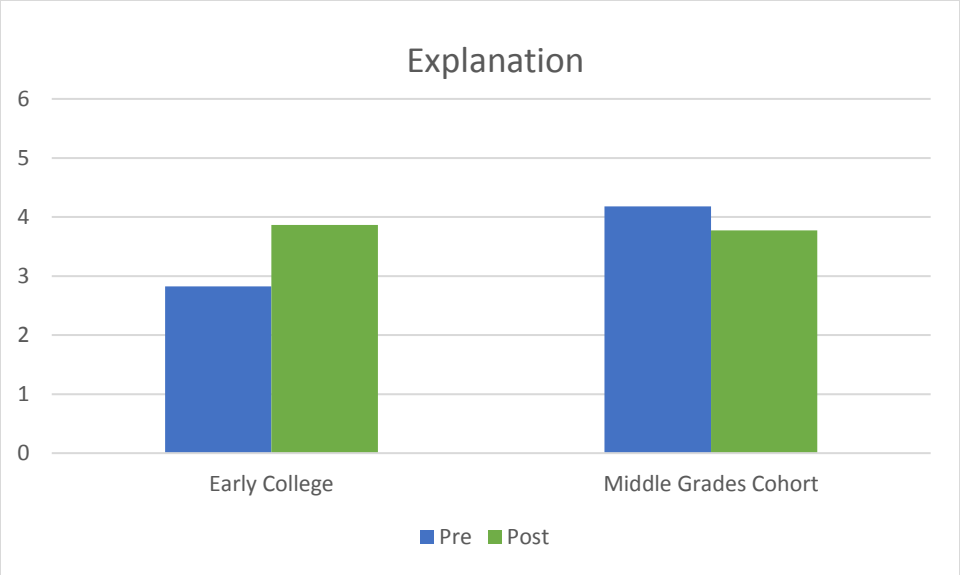
Procedural Fluency because they are able to accurately carry out the steps of the FOIL method. This is likely because the students of the Middle Grades Cohort have previously learned the FOIL procedure in high school, whereas the students of the Early College had never been exposed to the FOIL method. There was a 226% increase from pre-assessment to post-assessment for the Early College with the average post-assessment score of 4.314 on the equation portion of the assessment. Also increasing the average score on the equation question from pre-assessment to post-assessment, is the Middle Grades Cohort with the increase of 10% with the average post-assessment grade of a 4.955. A sample of a student who showed Procedural Fluency, receiving a score of a 5 on the assessments is shown below.



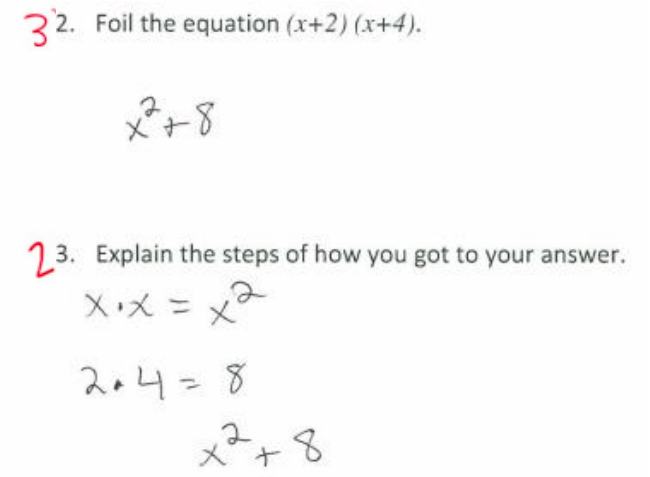
The student scored a five since they were able to multiply the binomials accurately using the FOIL method. I found that 79.5% of students in both the Early College and the Middle Grades Cohort showed Procedural Fluency on FOILing the Equation on the post-assessments all receiving the highest available points after the FOILing with Manipulatives Lesson.

Question 2: Explanation

The next question on the assessments asked student to explain the steps of how they got their answer in relation to the FOIL method. The Graph shows the difference in the pre-assessment and post-assessment average for the explanation for both the Early College and Middle Grades.

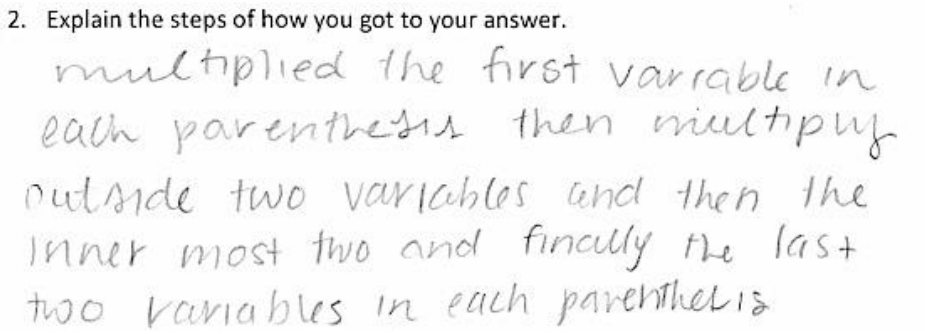


The average score for the Early College students on the explanation portion of the pre-assessment was a 2.823. A typical explanation for the students of the Early College is shown in the sample below. The student scored a three on the FOIL the equation portion because what they wrote had mathematical logic, but no connection with the FOIL method.



This student scored a two on the explanation portion because they were able to show how they got their answer from the FOIL the equation portion, however used no words to explain. Students of the Early College were not familiar with the FOIL method previous to the lesson so

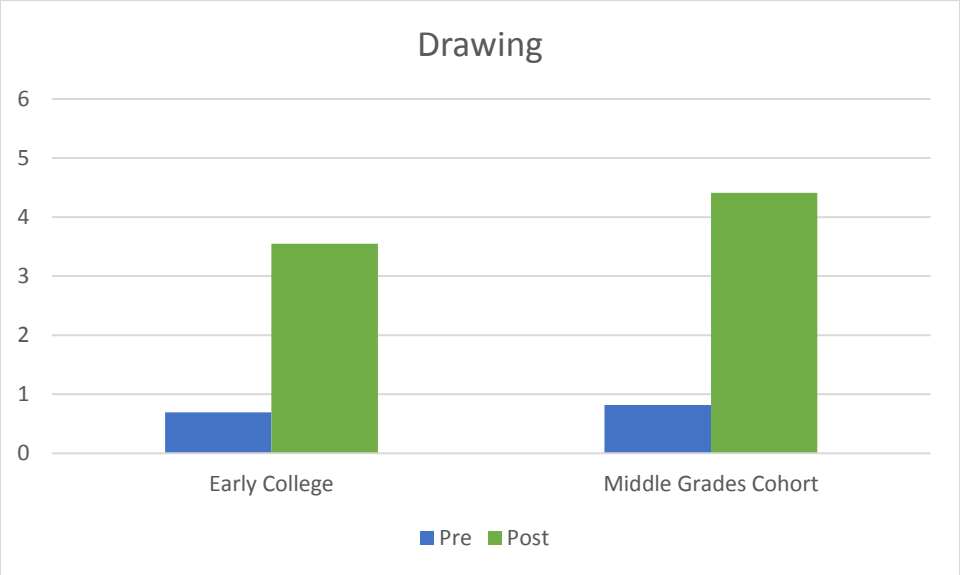
their explanations consisted of them explaining the wrong procedure which is reflected by their low average score. The average score for the Middle Grades on the explanation portion of the pre-assessment was 4.182. Since they were familiar with the foil method, their explanations reflected the correct explanation which is why their score was so much higher. A typical answer for the Middle Grades is shown below.


2. Explain the steps of how you got to your answer.
multiplied the first varrable in each parenthesis then multiply outside two variables and then the inner most two and finally the last two variables in each parenthesis

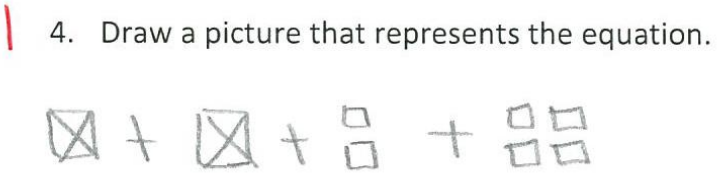
There was a 36% increase in the explanation portion of the Early College from pre-assessment to post-assessment with the average score of a 3.86. There was a 9% decrease in the score on this portion for the Middle Grade's post-assessments with the average score of a 3.773. The Early College had a median and mode of a three on the pre-assessment with a mode of a three on the post-assessment and a median of a four. The Middle Grades Cohort had a pre-assessment median and mode of a five which decreased to a three on the post-assessment for both the median and the mode. After students were reminded what FOIL stands for, most of the students were only using the terms First, Outer, Inner, Last giving them the score of a three but they were not clarifying which of the variables were being multiplied together, thus causing the decrease in their scores.

Question 3: Drawing

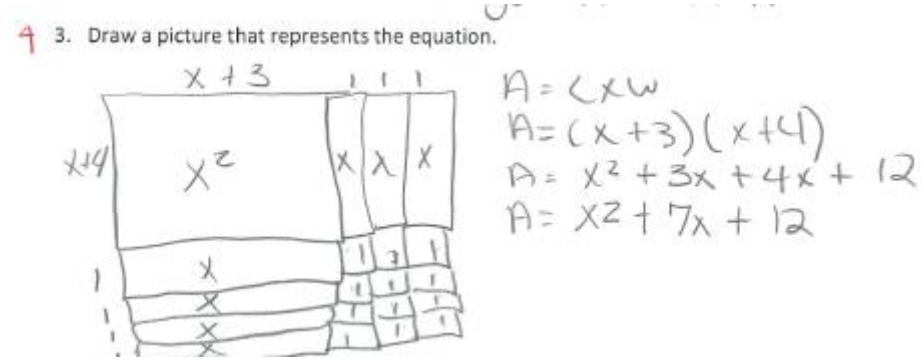
Lastly, students were evaluated on their ability to draw a picture representing the multiplication equation and its product as well as making connections between their picture and the FOIL method. Students who are able to connect the equation with their drawings show a Conceptual Understanding of the FOIL method. This portion of the pre-assessments score was the lowest overall. Here is a graph that shows the difference in the average on pre-assessments and post-assessments for both the Early College and the Middle Grades Cohort.



The average score on the pre-assessments for the Early College students on the drawing question was a 0.69 while the average for the students of the Middle Grades Cohort was a 0.82. These drawings consisted of a blank page, question marks or a picture like the one below.

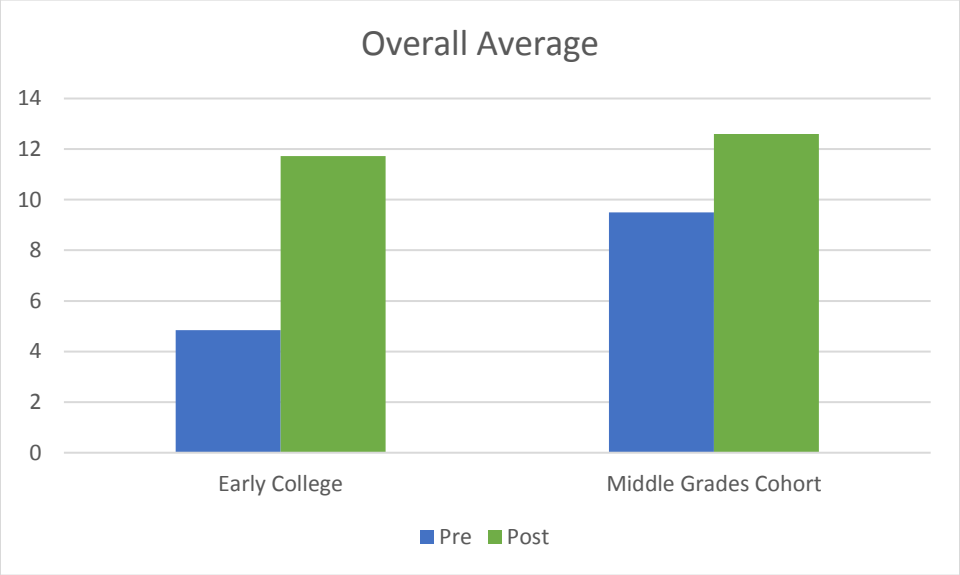


The drawing portion scored the lowest on the pre-assessments for both sets of students creating the largest percent of increase from pre-assessments to post-assessments. The Early College students had a 411% increase from the average score of 0.69 to a 3.55 on the post-assessments. The Middle Grades students had a 439% increase from the average score of a 0.82 to a 4.41 on the post-assessments. Both the Early College and Middle Grades Cohort had an increase from one to four in their median and mode from pre-assessment to post-assessment. A student who was able to draw a rectangle labeling all of the sides showing they understand the areas of the rectangles are found within the FOIL method shows Conceptual Understanding. The high percentage of increase shows that students in both the Early College and the Middle Grades Cohort gained a Conceptual Understanding of the FOIL method because they were able to make connection between the algebra and geometric representation. Below is a picture of a student who shows Conceptual Understanding.



Since the student did not label the sides correctly, they did not receive a perfect score. However, from this sample, we can observe that the student was using the Area Formula *A*=LW to show how to find the area of the rectangle. We can see the labels of each rectangle and square's areas which were added together under the Area formula.

Recall the overall average graph below.

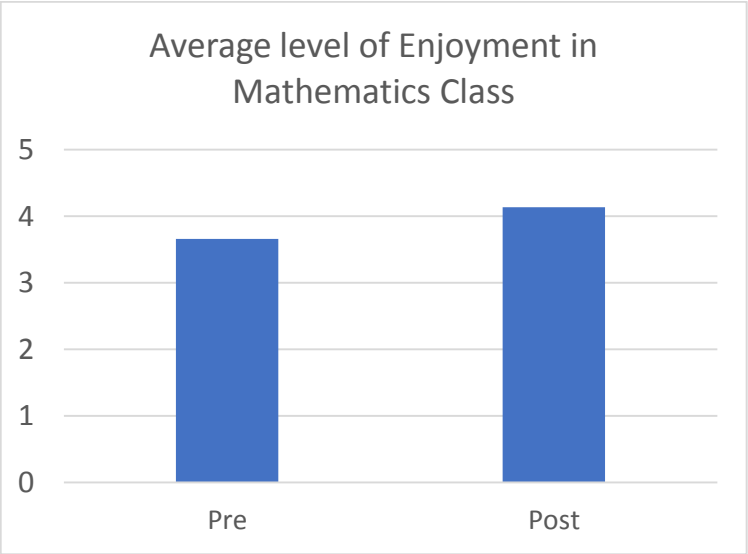


This graph shows that there is a large difference between the pre-assessment average scores of the Early College and the Middle Grades Cohort. The Middle Grades Cohort showed Procedural Fluency on the equation portion of the pre-assessment since they have taken a course previously where they were required to use the FOIL method. The Early College learned the FOIL method for the first time during my lesson. Both groups obtained Conceptual Understanding of the FOIL method because majority of the students were able to make the connections between the algebraic and geometric representation of multiplying binomials. Even though the Middle Grades Cohort had procedural fluency from their previous use of the FOIL method, they were able to attain conceptual understanding of the FOIL method. This is important for pre-service teachers because they come into the program knowing the mathematical concepts Procedurally, but they are still able to gain conceptual understanding from manipulatives.

Additional Finding

Students' Attitudes Towards Mathematics

On the pre-assessments and post-assessments for the Early College students, I asked the students on a scale of 0 to 5 how much do they enjoy math class with a score of a zero meaning not at all and a score of a five-meaning math is fun. Here is the graph of the data.



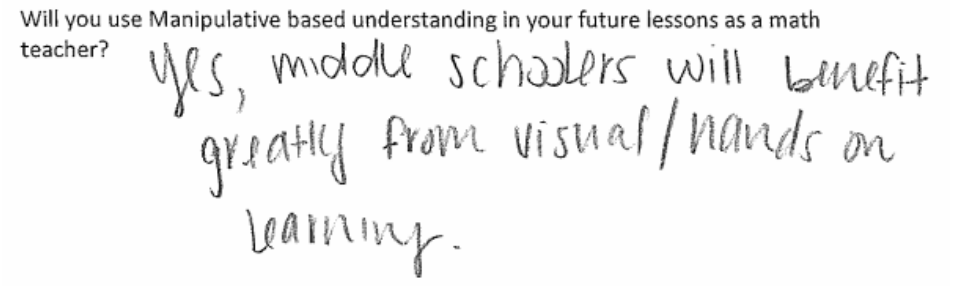
There was an average score of a 3.66 before the lesson using manipulatives to explain the FOIL method. However, the average score after the lesson was 4.14 which increased by 13%. Through my research I have found that students enjoy mathematics class better when they are using manipulatives to learn new mathematical concepts. This shows that just using manipulatives in one mathematics lesson improved the attitudes of the students.

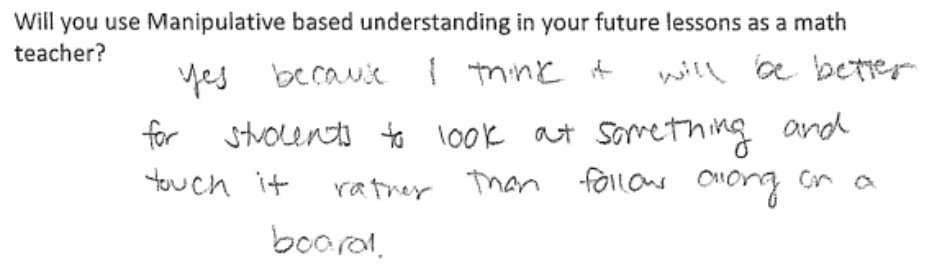
Pre-Service Teachers' Use of Manipulatives

On the pre-assessments, students of the Middle Grades Cohort were asked "how often did you use Manipulatives when learning Mathematics?" They rated their response on a scale of 0 to 5 where 0 meant they never used manipulatives in their lessons and a 5 meant they used manipulatives often in their previous mathematics classrooms. I found that there was an average

of a 2.25 and a mode and median of a 2 on the scale of 0 to 5. In conclusion, there was not a consistent use of manipulatives in their previous mathematics classrooms.

Students of the Middle Grades Cohort were asked if they will use manipulatives based Understanding in their future lessons as a math teacher. All of the students were in favor of using manipulatives in their lessons creating a positive outcome from the lesson. Here are some the responses of the Middle Grades Cohort.





The Middle Grades Cohort consists of students who are studying to be Middle school teachers. Since teachers are likely to teach how they are taught, it is important for these students to learn through a Conceptual based Understanding such as through the use of concrete manipulatives.

CONCLUSION

Throughout my research I found that students gain a Conceptual Understanding through the use of manipulatives in the classroom when learning how to Multiply Binomials through the FOIL Method. I found that most of the Pros and Cons of previous research to be relevant in my study. Students gained Conceptual Understanding as they were able to make connections

between the algebraic and geometric representations of the FOIL method. I found that a lesson with manipulatives can improve students' attitudes in the classroom. They were also to see abstract mathematical ideas through visual representation. In my research, I found that a pro for the use of manipulatives is that students can gain Conceptual Understanding even when having Procedural Fluency. I also found that using manipulatives can be time consuming to use when teaching a new mathematical concept. When teaching the Early College, I spent two days teaching the lesson when planning to teach it during one class period.

Through my research, I hope that future mathematics teachers will use manipulative based Understanding in their future mathematics classrooms. Although it may be time consuming, students gain Conceptual Understanding, improve their attitudes towards mathematics, and see visual representations of abstract mathematical ideas. Mathematics Curriculum should encourage lessons to be taught through manipulatives. There should also be opportunities for teacher to be trained on this form of teaching, so that all teachers are aware of the positive effects and how these lessons should take place.

A major negative to using manipulatives in a mathematics classroom is that the lessons are too time-consuming for the amount of information needed to teach all of the standards. However, these lessons consist of days of repetitive practice of steps and procedure. For future research, I am interested to see if taking this valued time to teach a lesson through a Conceptual Understanding with using manipulatives is worthwhile if students are going to have a deeper Understanding of the material along with knowing how to implement procedures and steps. I am also interested to see if students who learn a mathematics concept Conceptually are likely to remember the content longer. If this were the case, then the time-consuming aspect would not matter because there would be less time needed to review at the end of the year.

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APPENDIX A

EARLY COLLEGE PRE-ASSESSMENT

Multiply with Manipulatives Pre-assessment

Do you enjoy Math class? Circle one of the following (0-not at all, 5-math is fun).

0 1 2 3 4 5

1. Foil the equation $(x+2)(x+4)$.

2. Explain the steps of how you got to your answer.

3. Draw a picture that represents the equation.

APPENDIX B

MIDDLE GRADES COHORT PRE-ASSESSMENT

Multiplying Polynomials with Manipulatives Pre-assessment

1. Foil the equation *(x-2) (x+5)*.

2. Explain the steps of how you got to your answer.

3. Draw a picture that represents the equation.

4. How often did you use Manipulatives when learning math? (0: Never, 5: Often)
   0 1 2 3 4 5

5. Have you ever used Algebra tiles/Algeblocks?

APPENDIX C

EARLY COLLEGE POST-ASSESSMENT

Multiply with Manipulatives Post-assessment

1. Foil the equation $(x+3)(x+5)$.

2. Explain the steps of how you got to your answer.

3. Draw a picture that represents the equation.

4. Did using Manipulatives help you to better understand FOILing?

5. Did you enjoy Math class today? Circle one of the following. (0-not at all, 5-math is fun).

0 1 2 3 4 5

APPENDIX D

MIDDLE GRADES COHORT POST-ASSESSMENT

Multiplying Polynomials with Manipulatives Post-assessment

1. Foil the equation *(x+4)(x-3).*

2. Explain the steps of how you got to your answer.

3. Draw a picture that represents the equation.

4. Did using Manipulatives help you to better understand Foil?

5. Will you use Manipulative based Understanding in your future lessons as a math teacher?

APPENDIX E

GRADING RUBRIC

	0	1	2	3	4	5
Foil the Equation	No Answer or attempt to answer the question.	Attempted to solve the equation, but there is no mathematical logic.	Used mathematical logic but there is no connection with the FOIL method.	Used a form of the Distributive Property but not the FOIL method.	There was partial use of the FOIL method with few errors.	No errors in implementing the FOIL method. No errors.
Explanation of steps to find equation	No Answer or attempt to answer the question	No words, only numbers and variables used. No connection with the algebra.	No words but connections were made with the algebra.	Used words to explain but no connection with algebra or wrong procedure.	Words used to explain matched algebra but lack of explanation or errors in the procedure.	Explanation with words matched the correct use of FOIL method. No errors.
Draw a picture to represent the equation	No Answer or attempt to answer the question	Attempted to draw a picture but not a picture or no mathematical connection.	Rectangle drawn with dimensions not connected to area or not completed rectangle.	Rectangle drawn with overall dimensions correct, but individual pieces of FOIL not drawn.	Rectangle with each piece of FOIL drawn, but incorrect labeling or incomplete labeling.	Rectangle drawn with correct dimensions and labels of each piece of the FOIL method. No errors.