Get “Gritty” With It: The Impact of Effort on Mathematical Achievement

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Abstract

Many students approach mathematics with the mindset that they simply “aren’t math people.” This fixed mindset has been shown to lead students to question their own mathematical ability, exert little effort in mathematics, and ultimately give up learning when faced with difficulties or challenges (Hwang et. al, 2016). However, Duckworth suggests that when it comes to a person’s achievement, “as much as talent counts, effort counts twice,” (Duckworth, 2016, p. 42). The purpose of this paper is to investigate the correlation between perceived effort and mathematical achievement, while also exploring ways in which teachers can encourage students to put forth effort in mathematics. Pre-Service Teachers (PSTs) in a southeastern university were given a pre-survey to investigate the PSTs’ mathematical background, their ideas about perceived effort and talent, and their own mathematical achievement. After being presented with a lesson involving strategies that are believed to encourage effort, the PSTs were given a post-survey to analyze how well these teaching strategies encouraged the PSTs to put forth effort.

Statement of the Problem

When considering what mathematics education topic to study, I realized that I really wanted to be able to research a topic that I predict could be very practical in a mathematics classroom. As a future teacher, one of my biggest goals is to help every child enjoy the process of learning mathematics and feel confident in their mathematical abilities.

As a mathematics tutor and a Supplemental Instructor for this university, some of the most common statements I hear in reference to mathematics is some variation of the following statements: “I’m just not a math person” or “I’m not good at math.” Even beyond the scope of the university, I hear these statements from adults, often after I’ve explained that my major in college is mathematics. In a 2005 poll of 1000 adults in the United States, 37% recollected that they “hated” math in school, and more than twice as many people said they hated math as any other subject (Willis, 2010, p. 5). Willis goes on to say that there are several misconceptions about mathematics that students tend to believe, including:

- You must be very intelligent to be good at math, and
- It is acceptable to be bad at math because most people are. (Willis, 2010, p. 6)

So why is it the case that students feel so defeated by mathematics? Why do students feel as though being good at math is a seemingly impossible task? These questions guided my leap into mathematics education research. I concluded that if my goal is to become an effective mathematics teacher, then I want to be able to not only understand the stigma against mathematics, but also learn how to dissuade my students from labeling themselves as “not a math person.”

According to Justicia-Galiano (2017), even students who are less competent in mathematics can “compensate for their impaired efficiency by making an extra effort.” (p. 575).

According to Duckworth (2016), effort is crucial in the journey towards achievement. In fact, although Duckworth acknowledges both effort and talent as being important components that lead to overall achievement, she states, “As much as talent counts, effort counts twice,” (Duckworth, 2016, p. 50). I began to wonder to what degree effort plays a role in a student’s mathematical achievement. If it does play a significant role, how can we as teachers help students to put forth more effort in the classroom? However, since effort is hard to measure empirically, I decided to consider perceived effort instead. In this research, I explored the significance of the role of perceived effort in a mathematics classroom. This research was guided by the following questions:

- Does perceived effort quadratically correlate to achievement in mathematics?
- How can teachers encourage students to increase the amount of effort they dedicate in mathematics?

Literature Review

Growth and Fixed Mindset

Dweck (2006) introduces two different mindsets that have a crucial impact on how we view and respond to everyday life: the fixed mindset and the growth mindset. A person with a fixed mindset views their knowledge and intelligence as unchanging and constant concepts (Dweck, 2006). In this person’s mind, there is nothing that he or she can do to increase their intelligence level; they are either “smart,” or they are not. However, a person who has a growth mindset believes that all of his learning experiences increase his intelligence (Dweck, 2006). This person sees intelligence as a process that is developed over time. When listening to people’s opinions of mathematics, determining the type of mindset a person has with respect to mathematics is quite apparent.

Students with a growth mindset tend to outperform students with a fixed mindset (Hwang et. al., 2016, p. 2). When students with a fixed mindset do not encounter success right away, they automatically assume that they are not going to understand or excel in the topic at hand because they are not “smart enough.” On the other hand, students with a growth mindset can “bunker down” when they do not understand something; they realize that the “struggle” or “challenge” is part of the process that leads to success.

Students’ mathematical performance is limited by their fixed mindsets. If students believe that they are incapable of being successful in mathematics, then the amount of time and effort they invest in mathematical endeavors diminishes. Fortunately, students are not simply born with one mindset or the other; they may develop either mindset over time. Teachers, knowingly or not, encourage students to develop either mindset through the approach they use to teach mathematics.

Grit

Teachers who teach mathematics in a manner that encourages a growth mindset will most likely, as a byproduct, teach their students the importance of grit in the classroom. According to Duckworth, grit is passion and perseverance for long term goals (Duckworth, 2016, p. 42). In order for students to develop a growth mindset, they must be presented with opportunities that they feel grow their intelligence. As in other areas of life, the development of learning and intelligence takes practice. The road to intelligence will most likely include “mistakes.” When students make “mistakes” or encounter very hard mathematics topics, then they must have grit to keep working toward mathematical achievement.

Duckworth (2016) states that people with high levels of grit are more likely to attain achievement in their endeavors than people without grit. She also suggests that effort is directly correlated with achievement. In her framework, she proposes that a person's skill is a direct product of his or her effort and talent. She further explains that a person's achievement is a direct product of his or her effort and skill. Thus, skill = talent * effort and achievement= effort * skill. When we compose these equations, we see that, according to Duckworth, a person's achievement can be determined by the square of a person's effort times talent; that is, achievement= talent * effort^2. This fact suggests that a person's effort is multiplicatively twice as influential as a person's talent when it comes to determining achievement. Thus, Duckworth's framework seems to suggest that those with a growth mindset and grit can achieve more than those with a fixed mindset. With this in mind, we can explore ways in which grit and a growth mindset can be developed in mathematics students to encourage mathematical achievement.

Productive Struggle & Cognitive Demand

One method that encourages students to develop a growth mindset is productive struggle. According to NCTM (2014), productive struggle is defined as students engaging in opportunities where they are "understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions" (p. 48). According to Townsend (2018), productive struggle can also "lead students toward mathematical resilience, encourage retention, and build growth mindsets" (p. 217). One cannot discuss productive struggle without introducing the concept of cognitive demand. Cognitive demand is "the level and type of thinking that a task has the potential to elicit." (Stein, 1998, p. 9). According to Stein, there are four levels of cognitive demand: memorization, procedures without connections, procedures with connections, and doing mathematics (see Figure 1). The first two levels, memorization and procedures without connections, represent lower levels of cognitive demand, while the second two levels, procedures with connections and doing mathematics, represent higher levels of cognitive demand.

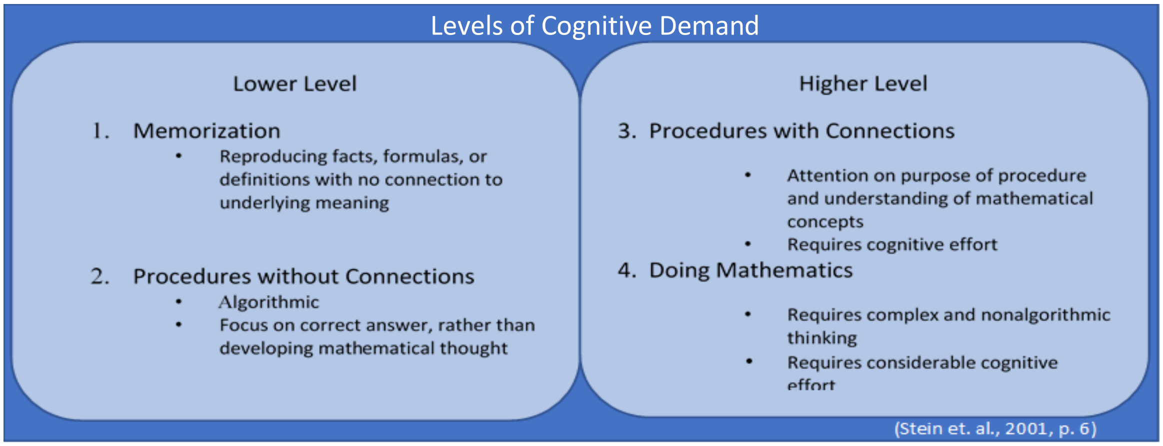


Figure 1. Levels of Cognitive Demand

Open & Closed Mathematics

When teachers encourage students to engage in productive struggle and develop growth mindsets, they are most likely also creating a classroom environment of open mathematics. Open mathematics places a higher emphasis on the process involved in doing mathematics. Most often this is achieved by allowing students to work on open-ended and practical scenarios that require them to make their own decisions about methods and procedures to use to solve problems (Boaler, 1998). Studies have shown that students who learn while engaging in open mathematics activities have higher levels of enjoyment and understanding (Boaler, 1998). When addressing the notion of open mathematics, we must also address and define closed mathematics. Closed mathematics, in contrast, emphasizes attaining the one correct answer, even sometimes at the expense of the mathematical process. Despite the research, in many mathematics classrooms, we still see students working exclusively on closed mathematics tasks, such as worksheets filled with exercises (Boaler, 1998).

Since open mathematics encourages students to make choices regarding problem-solving strategies, it is likely that students will come up with different ways in which to solve problems. It is also likely that at least some students will try a process that does not lead to the correct answer. In open mathematics, students who try to implement processes that are inconclusive or incorrect are encouraged to try again using a different process. In these situations, it is crucial for students to have grit.

Cooperative Learning

Another teaching strategy that encourages students to put forth effort is cooperative learning. Sheehy (2004) suggests that cooperative learning consists of “a small group of learners, who work together as a team to solve a problem, complete a task, or accomplish a common goal,” (p. 7). According to Sheehy, in order to effectively implement cooperative learning in a classroom, there are several assumptions under which the classroom must operate. One assumption is that students are able to learn from each other (Sheehy, 2004, p. 7). A teacher must also believe that under certain guidelines and with clear instructions students are able to govern themselves (Sheehy, 2004, p. 7). Another crucial assumption to cooperative learning is that the teacher in the classroom is not the only source of information (Sheehy, 2004, p.7). If students and teachers believe that the teacher is the only source of knowledge, then it will be very hard for the students to work together and solve the problem or task on their own.

Mathematical Proficiency

In order to effectively analyze mathematical achievement, we must also consider the notion of mathematical proficiency. The National Research Council (NRC) suggests five “strands” that work together to achieve mathematical proficiency (see Figure 1). One strand is conceptual understanding, which is the “comprehension of mathematical concepts, operations, and relations," (p. 5). Another strand is procedural fluency, which NRC has defined as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately," (p. 5). The next strand is strategic competence, which is defined as the "ability to formulate, represent, and solve mathematical problems," (p. 5). The fourth strand is adaptive reasoning, which is the "capacity for logical thought, reflection, explanation, and justification," (p. 5). The final strand is productive disposition, which is a "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (NRC, 2001, p. 5).

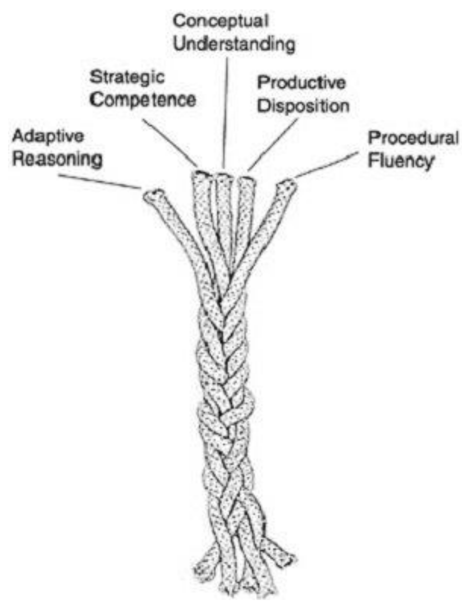


Figure 2. Five Strands of Mathematical Proficiency

Framework

Based on the previously mentioned literature, I have proposed that open mathematics, cooperative learning, productive struggle, and cognitive demand are each intertwined with each other. An effective productive struggle will most likely incorporate higher levels of cognitive demand. Higher cognitive demand will most likely require a productive struggle. Open mathematics sometimes works most effectively when students are allowed to work cooperatively with each other. Even more, each of these teaching strategies also individually lead to students developing a growth mindset and developing a sense of grit (Boaler, 2013, p. 143). Finally, students who have grit and a growth mindset also tend to have higher levels of mathematical achievement. As a result, I have proposed the following framework to from which I have constructed my research.

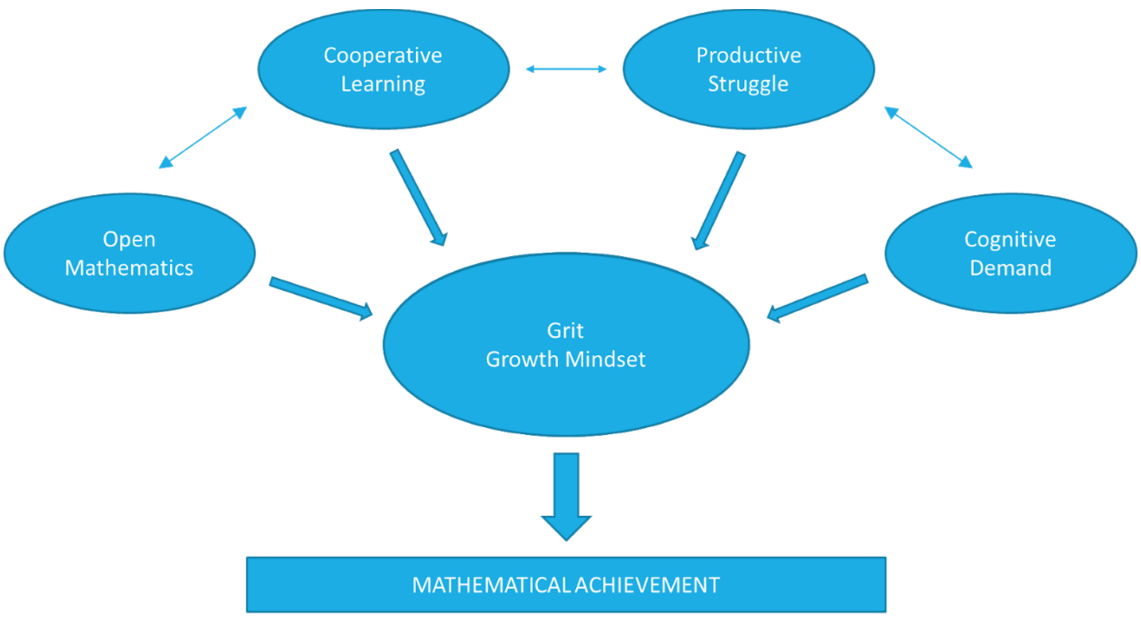


Figure 3. Research Framework

Methods

The Participants

The participants in this study were the pre-service teachers (PSTs) in two geometry courses from a southeastern university.

The Tools

In order to see how correlated perceived effort is to mathematical achievement, I tested Duckworth's achievement equation specifically as it relates to mathematical achievement; that is, I tested to see if a student's mathematical achievement is equal to the student's perceived talent times the square of the student's perceived effort in mathematics. To do this, I created a pre-survey (see Appendix A) for the PSTs to complete, which included sections related to their perceived mathematical talent, their perceived mathematical effort, and their mathematical achievement. The survey asked the PSTs to provide a list of the collegiate mathematics courses they had taken with the respective final letter grade for each course, answer a series of open-ended questions related to their mathematics courses and the notion of mathematical achievement, and to rate a series of statements on a scale of 1-5. This survey allowed me to determine a numerical value that represented the PST's perceived mathematical talent score, perceived mathematical effort score, and past achievement score. The perceived mathematical effort, perceived mathematical talent, and past mathematical achievement scores were each calculated out of possible 5 points.

In order to determine how teachers can encourage students to put forth more effort in the classroom, I developed a lesson plan (see Appendix B) to implement with the PSTs in the geometry courses that involved an open mathematical task (see Appendix C), which incorporated productive struggle, cognitive demand, open mathematics, and cooperative learning. This task was adapted from Zeybek's task (Zeybek, 2016, p. 400). After the lesson, I gave the students a post-survey (see Appendix D) that allowed for reflection upon the lesson, the amount of effort they put forth in the task, and their achievement. In this survey, I incorporated open-ended questions that required qualitative analysis. I also analyzed their work on the open-ended task during the lesson.

Analysis

After the PSTs completed the pre-survey, I took each PST's perceived mathematical talent score and multiplied it by the square of the perceived mathematical effort score. Because this calculation could range from 1-125, I transformed each score by taking the cube root in order to compare these calculated achievement scores with each student's past mathematical achievement score. Then, all the PSTs' individual scores were combined so that I could analyze the data quantitatively. I also qualitatively analyzed the open-ended questions on both the pre-survey and post-survey by looking for common themes that emerged within the responses of the PSTs. These themes were not intentionally prompted in the survey; rather, these emerged as a result of analyzing the open-ended questions through the lens of my framework.

Findings

Question 1: Does perceived effort quadratically correlate to achievement in mathematics?

I used the pre-survey responses to determine the correlation between perceived effort and achievement in mathematics. I first compared each PST's calculated and past achievement scores to determine how well they matched (see Figure 3).

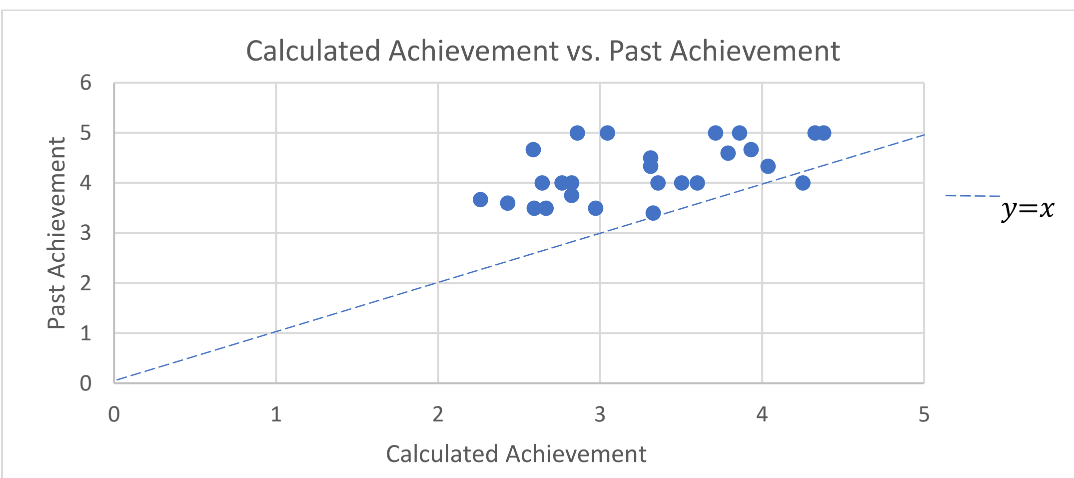


Figure 4. Calculated Achievement vs. Past Achievement

By the graph we see that most PSTs' past and calculated achievement scores were not precisely the same, as few of the points were located along the y=x line. However, it is interesting to note that two PSTs' calculated and past achievement scores are almost perfectly aligned. Overall, the

PSTs' past mathematical achievement scores are higher than their calculated mathematical achievement scores, which is based on their perceived effort and talent. Since our calculated achievement score accounts for perceived effort and perceived talent, with effort counting multiplicatively twice that of talent, it seems to be the case that these PSTs either underestimate their mathematical abilities or they do not perceive they put forth much effort in their past collegiate math courses, or both.

After analyzing the relationship between calculated achievement and past achievement, I analyzed the correlation between perceived effort scores and past achievement scores as well as the correlation between perceived talent scores and past achievement scores (see Figures 4 & 5).

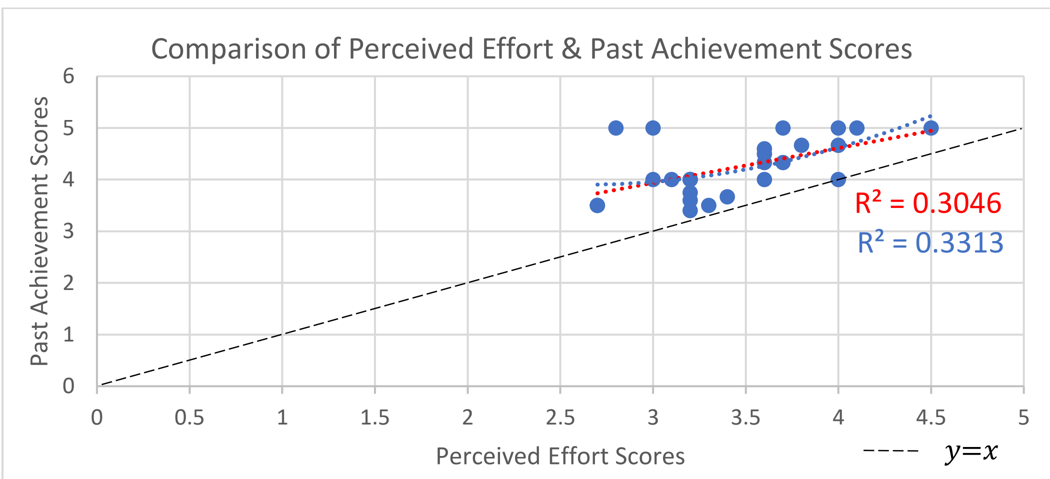


Figure 5. Comparison of Perceived Effort & Past Achievement Scores

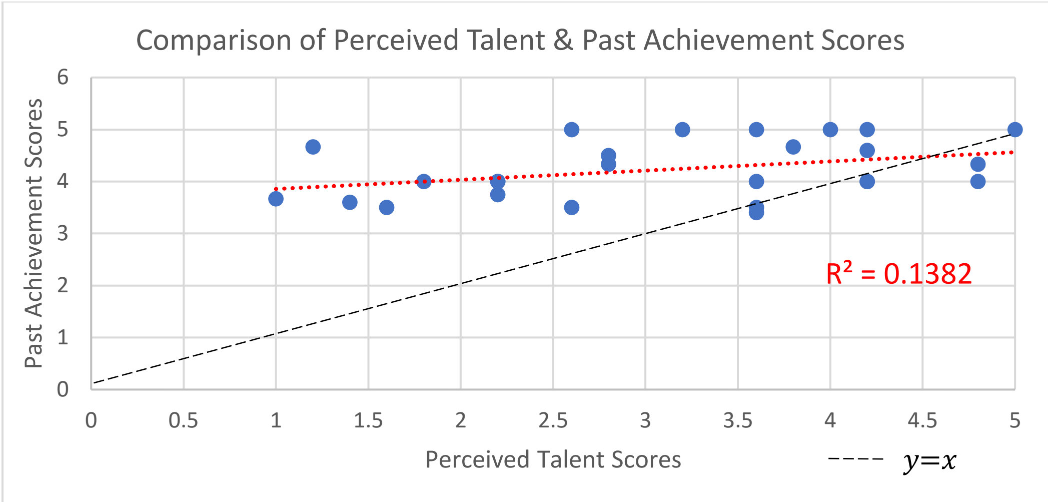


Figure 6. Comparison of Perceived Talent & Past Achievement Scores

The first research question asks if perceived effort is quadratically correlated with mathematical achievement. In Figure 4, the blue trendline represents the quadratic trendline, indicating how well the square of perceived effort correlates with past achievement scores for the students. The R-squared value for this trendline is 0.3313, which indicates a weak positive correlation. I also explored the linear correlation between these two values. The red trendline represents the least squares regression line between perceived effort and past achievement scores. The R-squared value for this regression line is 0.3046. By the scatter plot and R-squared, we can conclude that there is a weak positive correlation between perceived effort and past achievement. I also investigated the correlation between perceived talent and past achievement (Figure 5). The red trendline in Figure 5 represents the linear correlation between perceived talent and past achievement, with an R-squared value of 0.1382. From the R-squared value and the scatter plot, there exists a very weak positive correlation between perceived talent and past achievement. Although we cannot definitively say that perceived effort quadratically correlates to past achievement by the data, we can see that the correlation between perceived effort and past achievement is stronger than the correlation between perceived talent and past achievement.

Therefore, given the choice between perceived talent and perceived effort, we can conclude that perceived effort seems to be a better indicator of achievement than perceived talent. This conclusion directly supports the idea of growth mindset; if effort is more indicative of achievement than talent, then that must suggest that through effort, our intelligence can grow. Additionally, if perceived effort is better indicator of achievement than perceived talent, then this supports Duckworth's claim that effort counts more than talent in achievement.

Question 2: How can teachers encourage students to increase the amount of effort they dedicate in mathematics?

From my framework, I have suggested that open mathematics, cognitive demand, productive struggle, and cooperative learning among students will lead to the development of grit and a growth mindset, and therefore, increase the amount of effort put forth in the classroom. In order to test this claim, I conducted a lesson with the PSTs that incorporated all the above components. The lesson included the following task:

> Matt invited his best friends to celebrate his birthday. Matt and his two friends want to share a rectangle cake, which has a surface area of 81 square inches on top. The cake is frosted evenly on the four sides and the top.
>
> - How can Matt cut the cake so that each person receives an equal share of both cake and icing?
> - How can you justify that each person got the equal amount of cake and the icing? (Adapted from Zeybek's task (Zeybek, 2016, p. 400)).

In order to ensure this to be an open task, I gave very few explicit instructions on how to go about solving this problem to assure that the PSTs could solve this task in whatever manner they chose. In addition, there were several manipulatives readily available to the PSTs if they chose to use them, including grid paper, patty paper, Geometer's Sketch Pad®, Geoboards with rubber bands, and dry erase boards and markers. Several PSTs also made use of their iPads by using a Geoboard app. I also gave the PSTs the opportunity to work in groups.

This task required a high level of cognitive demand; solving this task involved complex and nonalgorithmic thinking, which is a key component of the "doing mathematics" level of cognitive demand (see Figure 1). I anticipated the students would "struggle" in this task in that they probably would not determine the correct answer in their first attempt. However, in order to keep the struggle "productive," I incorporated a "purposeful pause." This purposeful pause was an intentional time during the lesson, after they had been working for a while, where the PSTs came together as a class to discuss the strategies and ideas that the groups had tried as a means of solving the problem. In doing this, the PSTs were being held accountable in the amount of time they spent completing the task. In addition, and arguably more importantly, the PSTs were given an opportunity to see other potential as well as possibly ineffective ways to solve the task. This pause also allowed groups that were "stuck" an opportunity to see other ways of solving the problem, which could jumpstart them into trying to think about solving the task in a different way.

Examples of PSTs' solutions.

Categorization of PSTs' solutions.

As one might predict, there were several different ways in which the PSTs thought about this problem, and, thus, there were many different solutions presented. Zeybek (2016) suggested three categories into which students' solutions could be coded: Category 1, Category 2, and

Category 3. Solutions that were coded into Category 1 were correct solutions in which the cake was cut into more than three pieces (Zeybek, 2016, p. 409); that is, Matt and his two friends each received multiple pieces of cake in which, when combined, resulted in equal amounts of cake and icing among the three friends. All the PSTs' solutions in this study were coded into this category. Solutions that were coded into Category 2 were those in which the students were able to cut the cake into only three pieces based upon a grid system. That is, the students divided the cake by tracing their cuts along an outline of unit squares and were able to count the number of units within each section to determine that each of the three pieces cut contained equal amounts of cake and icing. The third and final category presented in Zeybek's article contained abstract solutions. These solutions required more than simply counting units of measure in order to justify the solutions; students whose solutions were coded into this category were able to reason and justify their solutions based on the notions of area and relationships between properties of shapes (Zeybek, 2016, p. 410). While a few of the solutions presented by the PSTs included abstract reasoning about relationships between shapes and properties, none included just three cuts.

Although we have already noted that all the PSTs' responses were coded into Category 1 with respect to Zeybek's article, these categories are somewhat less useful to us with respect to our framework. Since this is the case, I have created two categories to code PST responses, which now relate to the effort put into the task.

Categorization based on effort.

Recall that in the task prompt, the instructions were to cut a rectangle cake (with a surface area of 81 square inches on top) in such a way that Matt and his two friends are each able to receive equal amounts of cake and equal amounts of icing. In Figure 6, (see below) we see a solution presented by one group of PSTs who considered a specific size of rectangle that matches the surface area requirement presented in the problem: a 1 × 81 in. rectangle cake. When analyzing the PSTs' thinking processes and proposed solutions during and after the lesson, I have derived two categories in which I've decided to code each of the responses: trial and error or construction.

Category A: trial and error.

The first category in which I decided to code PST responses into is Category A: trial and error. When given a prompt or learning task, often, the first strategy proposed is the strategy of trial and error, when students can explore possible solutions that they think will work and are also able to slightly modify the idea if it does not work. Examples of the PSTs' proposed solutions obtained by utilizing trial and error are given below (Figures 6, 7, 8).

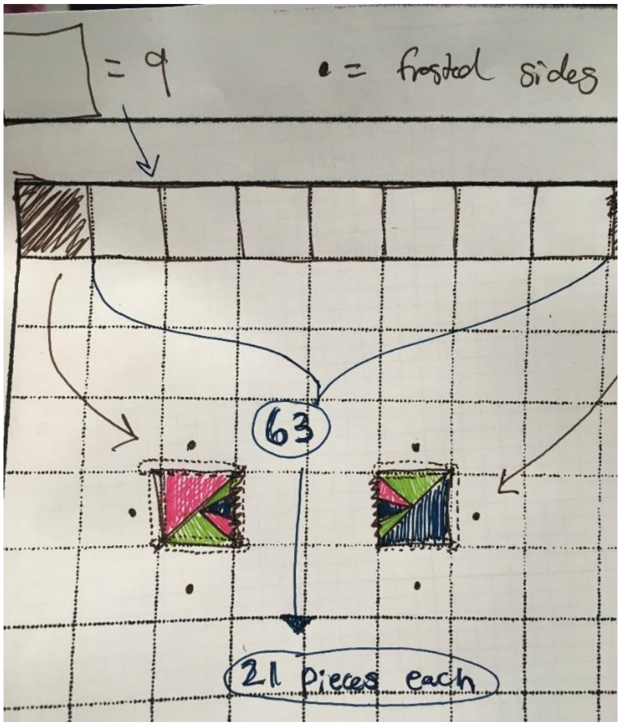


Figure 7. Example 1 of Category A Solution

In this solution, the PSTs' manipulative of choice was graph paper. This group of PSTs began by drawing the rectangle cake to be 9 inches × 9 inches. However, this group of PSTs noticed that the extra sides of icing were going to be more difficult to equally divide among the friends. They reasoned that drawing the cake as a rectangle (i.e., a cake with a length shorter than its width) would minimize the extra icing they would have to divide among the three friends. They further decided to make the cake into a rectangle with a very small length (1 inch) and a very long width (81 inches). Their proposed cake in the above picture is drawn as a 1 unit × 9 units rectangle, which they intended to represent a 1 inch × 81 inches cake. The width of the cake, which is denoted as nine units on the graph paper, is scaled so that one unit of width represents 9 inches. On the other hand, the length of the cake, which is denoted as one unit on the graph paper, also represents one inch of length. This group of PSTs then proceeded to divide the bulk of the cake (the 64 inches in the middle of the cake) into thirds yielding a 1 inch × 21 inches rectangle of cake, with equal area and icing among these pieces to give to each of the three friends. However, an issue arose when the PSTs divided the remaining endpieces without regard to the scaling inconsistency. Since these PSTs scaled this cake differently on its length measure (a scale factor of 1) and on its width measure (a scale factor of 1\9), the resulting endpieces do not yield equal amounts of icing and equal amounts of cake. Although the endpieces look like they are squares, they are actually 1 inch × 9 inches rectangles. Thus, the resulting shaded area regions are not equal among the three friends.

Although these PSTs did not ultimately propose a completely correct solution, their solution indicates that they put forth a good deal of effort in their solution. They tried several strategies of representing the rectangle cake, first representing the cake as a square, then an arbitrary rectangle, and finally deciding to represent the cake as a specific rectangle. Then, even after deciding on a certain rectangle, the PSTs continued to refine their cuts until they reasoned that they would achieve equal amounts of cake and icing for each of the three friends.

Figure 7 shows another solution presented by different group of PSTs that involved a process of trial and error. In this solution, the manipulative of choice was a gridded white board. This group’s first strategy was to equally divide the cake into four quadrants. These PSTs easily saw that they could give each of the three friends one of the quadrants and justify that each of the pieces given contains equal amounts of both cake and icing. However, the “struggle” started when deciding how to divvy up the remaining quadrant of cake. At first, the PSTs tried to cut the remaining quadrant into 6 pieces in a 2 × 3 array. The PSTs noticed that this cutting method wouldn’t allow for equal amounts of icing to be distributed to each of the three friends. Their next step was to subdivide the remaining quadrant into 81 units in a 9 × 9 array.

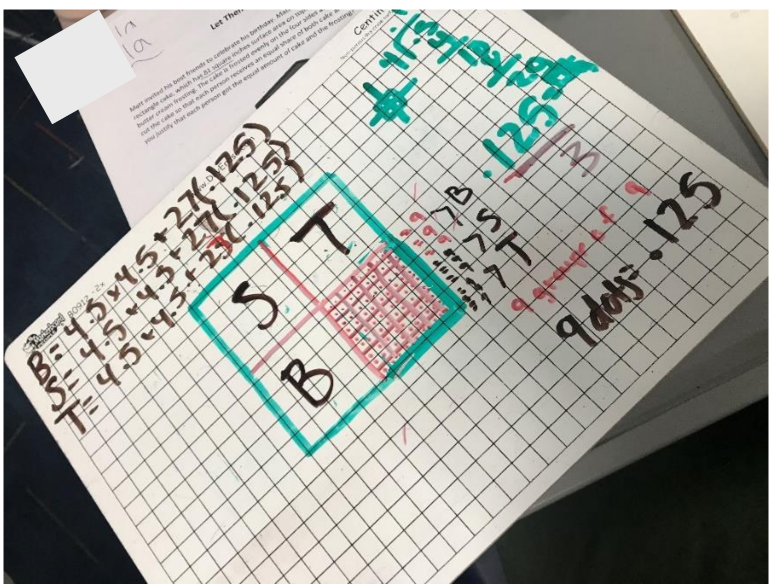


Figure 8. Example 2 of Category A Solution

Another example of a solution proposed by a group of PSTs that was coded into Category A is given below in Figure 8.

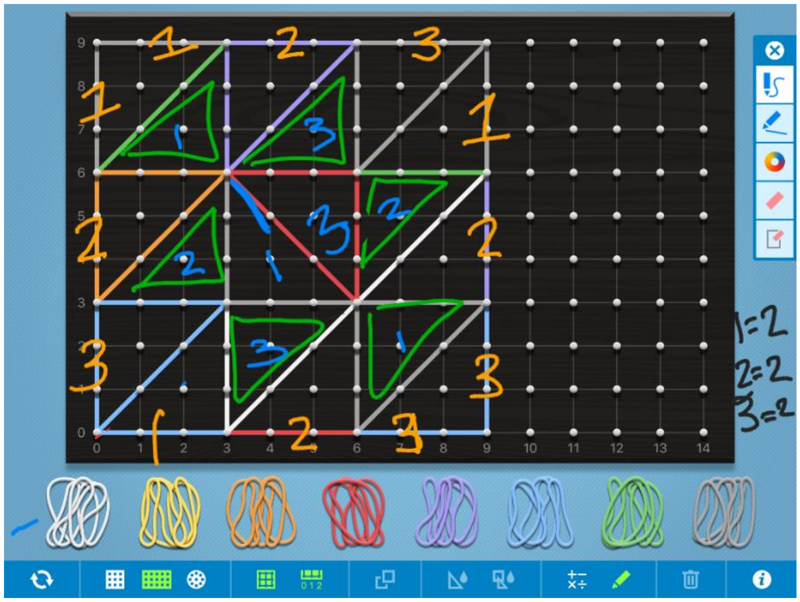
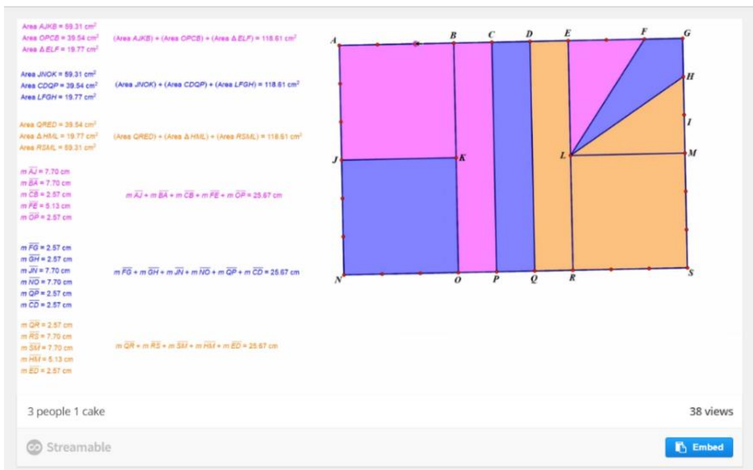


Figure 9. Example 3 of Category A Solution

In this representation, the manipulative of choice was an electronic geoboard that the PSTs were able to use on an iPad. This group of PSTs, like the previous group, decided to consider the case of a rectangular cake that was a square that was 9 inches × 9 inches. This group, however, decided to consider right triangle units instead of only squares. The orange numbers along the outside edge of the square denote the assignment of "icing sides" given to each of Matt and his two friends, which this group denoted as Person "1", Person "2", and Person "3". The PSTs divided the cake in such a way that there are 12 side pieces with extra icing, so each of the three friends were able to receive four side pieces with extra icing. However, some of these pieces with extra icing were iced on two of the three sides while other pieces with extra icing were only iced on one of the three sides. In order to compensate for this, the PSTs distributed the middle pieces with no extra icing to the people who had two of the three triangle sides iced so that they could increase the amount of area the person received while not adding to the amount of icing. Since the PSTs divided the cake into 18 total triangular pieces with 12 of the pieces containing extra icing, the PSTs were able to achieve equal amounts of both cake and icing among Matt and his two friends.

Category B: Construction

While most of the groups of PSTs achieved their solutions by implementing trial and error, one PST reasoned more abstractly to arrive at his solution. Figure 9 shows this PST's proposed solution (see below).



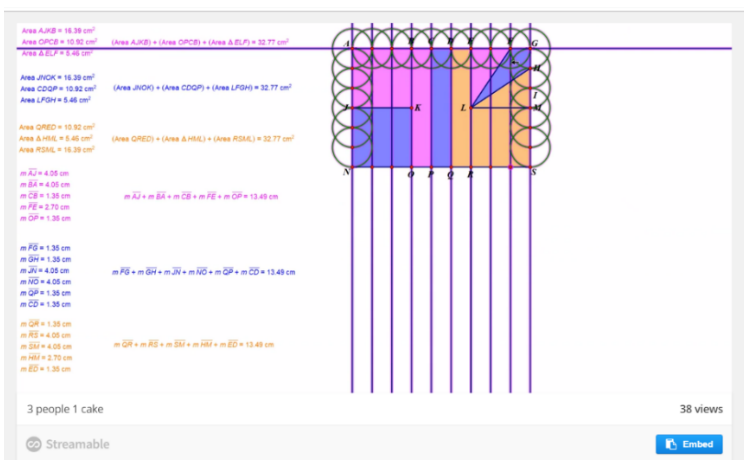


Figure 10. Example of Category B Solution

This PST developed a similar, hand-drawn solution during the class period when the lesson was implemented. However, in an email with the subject line reading “My mind won’t stop…” this PST sent in this solution, in which he not only recreated his solution in Geometer’s Sketch Pad, but he also was able to justify and show that his solution worked for any rectangle similar to the one he originally constructed, not just a rectangle with 81 square inches of surface area on top. This PST’s actions demonstrate a high level of grit. However, it was interesting to note that his self-reported grit (effort) score was considerably low, while his achievement and perceived talent scores were high. In fact, when asked on the pre-survey to evaluate how much effort he put forth in a mathematics classroom on a scale of one to five (where 1 is very little effort and 5 is maximum effort), he indicated that his amount of effort put forth was 1. His perceived talent score was a 5, and his achievement score was a 5. This seems to contradict our claim that effort counts twice as much as talent in mathematical achievement. However, his continuous thought on this task, after it was completed, solutions were presented, and the lesson was over, indicates a substantial amount of prolonged effort. We speculate that his seemingly contradictory actions and perceived effort score result from the fact that he does not consider this further thought as “effort” because it just comes naturally, and he enjoys it. This begs the question, how synonymous are perceived effort and objective effort?

Post-Survey Analysis of the Lesson

After I presented the lesson to the PSTs, I gave them a post-survey (Appendix D) that allowed them to both reflect upon the lesson and the amount of effort they put forth in the lesson.

Post-Survey Question: What Aspects of the Lesson Inspired You to Put Forth Effort?

One of the questions that I asked in the post-survey was as follows: “What aspects of the task inspired you to put forth effort?” Although this question was a free-response question, when

I analyzed the responses, I was able to find common themes that specifically correlated to my framework. When highlighting the aspects of the lesson that inspired the PSTs to put forth effort, several responses included phrases like "there were so many different ways we could solve the problem" and "using different manipulatives," which highlights that the open mathematics aspect of the lesson was interesting enough to inspire effort. Other students highlighted that the cognitive demand aspect of the lesson inspired them to put forth more effort by responding with phrases like "very challenging task" and "really made me think." There were also phrases among the PSTs' responses that included "I kept having to try to figure it out" and "I didn't want to give up," which indicated that some of the PSTs thought that the productive struggle aspect of the lesson encouraged them to put forth effort. Still other responses included phrases like "I enjoyed working with my group," which indicated that some of the PSTs were inspired to put forth effort by the cooperative learning aspect. The following graph (Figure 6) shows how the PSTs' responses highlighted the different themes of my framework.

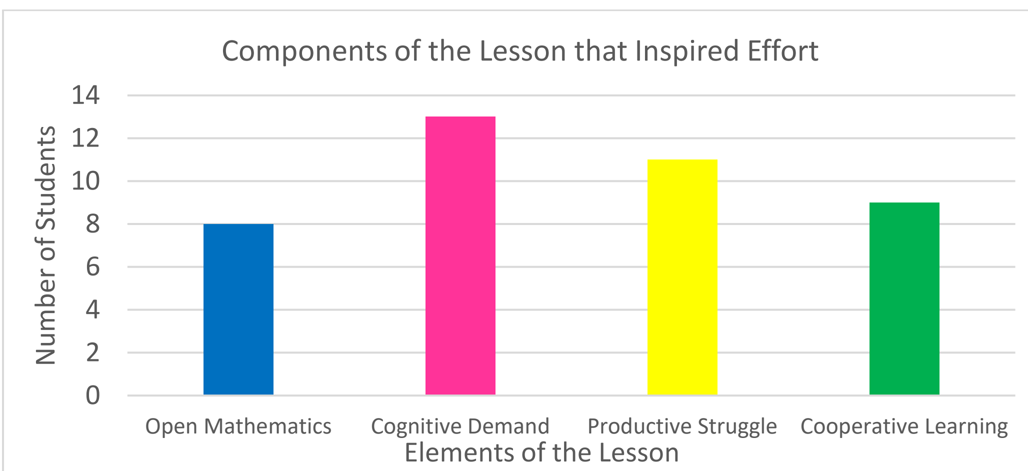


Figure 11. Components of the Lesson That Inspired Effort

From the graph, we can see that the PSTs were most inspired to put forth effort by the cognitive demand aspect. However, it is also important to note that most PSTs included more than one component in their response; that is, the number of students who highlighted one of the aspects could overlap with the number of students who highlighted a different aspect. However, Figure 7 shows how each PST's response relates to and beyond my framework.

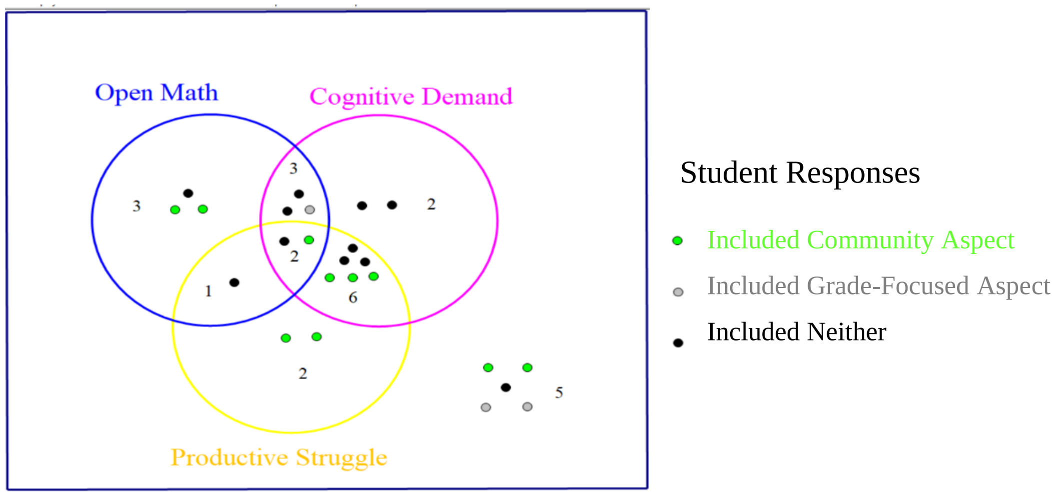


Figure 12. Effort Motivators for PSTs

Each of the points in the Venn diagram represents a PST's response to the post-survey question regarding the aspects of the lesson that motivated effort. One interesting thing to note is that if the PSTs noted one of the components in my framework as an effort motivator, then they also tended to acknowledge more than one effort motivator. This might suggest that these effort motivators work together to encourage effort. Additionally, Figure 7 indicates that cognitive demand is the highest motivator of effort among PSTs. However, we can further note that if a PST was inspired to put forth effort by the cognitive demand aspect of the lesson, then they were more likely to include the productive struggle aspect as an effort motivator, as well. For example, several of the PSTs wrote that the task was a "challenge" or a "puzzle" that they were determined to "figure out." This connection between productive struggle and cognitive demand among the PSTs seems to suggest that a large percentage of the PSTs were motivated to put forth effort as a result of the challenge of the task.

The green points represent an PST's response that also included an aspect of cooperative learning. In my original framework, I considered only cognitive demand, productive struggle, and open mathematics as key components of my lesson. However, after analyzing the responses in the post-survey, a theme of cooperative learning emerged as an effort motivator as well. The PSTs identified this cooperative learning component in saying that their effort increased when they were able to work in groups. By the Venn diagram, we can see that the PSTs who noted that the cooperative learning aspect motivated them to put forth more effort were also more likely to note that they were inspired to put forth effort by the productive struggle aspect of the lesson. One group noted, "using group work encouraged me to keep working," while another group said, "wanting to get the answer forced us to work together and figure it out." This seems to suggest that the PSTs were more likely to welcome a productive struggle when they were able to work in groups.

In contrast with Figure 6, Figure 7 shows how some PSTs responded outside of the scope of my framework. As evidenced by the gray points in Figure 7, a few of the PSTs said they would be more motivated to put forth effort if the task has been for a grade. Stein et. al. (2000), suggest that one factor that is associated with the decline of high level of cognitive demand is when students are not held accountable for their efforts. This can occur when "students are given the impression that their work will not 'count' towards a grade" (p. 27). Although each of the groups of PSTs began working diligently on the task, there came a moment for each group where their productive struggle was on the verge of turning unproductive. For some PSTs, this point came early on, as the only effort motivator for a few was getting a grade on the assignment. Because this assignment wasn't being taken for a grade, these PSTs didn't put forth much effort into the task.

While the PSTs were working on the task, several asked whether this task was going to be graded. They were told that while it was not going to be graded, they were expected to be ready to present to the class twice throughout the lesson. They would present once during the "purposeful pause" moment in the lesson, where they would begin to discuss what strategies they were using and why they chose to use those strategies. They would also present their final solutions at the end of the lesson. One PST indicated that she was motivated to put forth effort in the task because she would have to present her solution to her peers.

Although each of the components of lesson that I tested seemed to inspire the PSTs to put forth effort in the task, none of these components alone were enough to inspire prolonged effort. The PSTs were eager to work together and try different strategies at first, but as time went on and as more strategies proved unsuccessful, the students lost motivation to continue to put forth effort in the task. This point was the moment when the PSTs' productive struggle turned "unproductive." At this moment, it was crucial to intervene in some way to prevent the PSTs from giving up. Two ways in which I tried to combat this unproductive struggle was through questioning strategies and scaffolding. As the PSTs were working, I was constantly circulated from group to group, asking about what strategies they were using and why they believed they would work. When I came to a group who had tried a strategy that wasn't working, I asked questions like, "Why did this strategy not work?" and "Do you think you could modify the strategy in some way now that you know why it didn't work?" When the PSTs stopped to think about why a strategy didn't work, they were more able to modify that strategy and try again. If I felt the PSTs were getting discouraged, I also tried to encourage them in their efforts and provide suggestions to help reroute their thinking. After this guidance and reassurance, the PSTs seemed more eager to reattempt the problem and increase the effort they put forth.

Post-Survey Question: Amount of Effort in this Lesson vs. Typical Lesson

Since my motivation for presenting this lesson was to explore ways to inspire students to put forth more effort in the classroom, I decided to have the PSTs rate the amount of effort they put forth, both in a typical classroom setting and specifically in the lesson I presented to them. Figure 8 shows the comparison between the amount of effort the PSTs typically put forth in a classroom and the amount of effort they put forth in the lesson I presented.

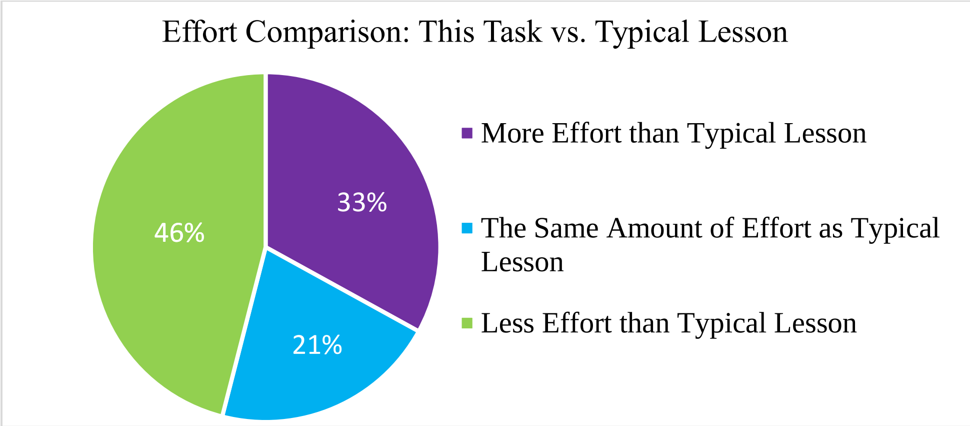


Figure 13. Effort Comparison: Typical Class vs. This Lesson

From the graph, we can see that the almost half of the PSTs stated that they put forth less effort than in a typical lesson. However, we also can see that over half of the PSTs put forth the same amount of effort or more in this lesson.

Additional Exploration: Describing Mathematical Achievement

In addition to the two research questions that guided my research from the beginning, I became curious about the concept of mathematical achievement. In particular, I asked the question:

- How do the PSTs describe mathematical achievement?

Although mathematical proficiency was not involved in my original framework, when I asked the PSTs to define mathematical achievement, their descriptions revealed themes related to mathematical proficiency (see Figure 9).

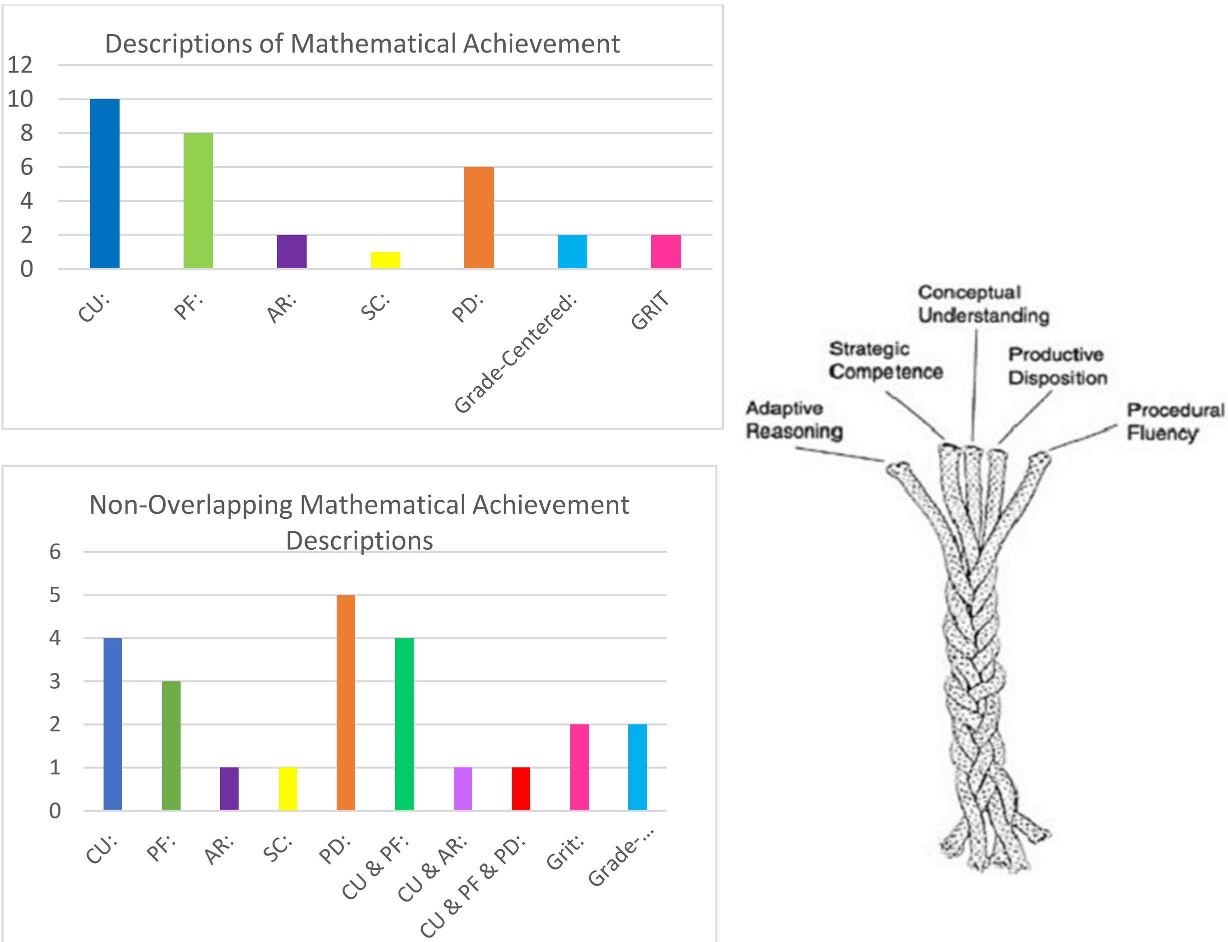


Figure 14. Descriptions of Mathematical Achievement & Mathematical Proficiency

The top bar graph shows the frequencies of the different themes the PSTs highlighted as a part of mathematical achievement. It is important to note that some of the PSTs highlighted more than one theme in their description of mathematical achievement, which explains why the sum of the frequencies is greater than the PST sample size. The second bar graph accounts for how each PST defined mathematical achievement, including descriptions that highlighted more than one component of mathematical proficiency. As evident from the second bar graph, the PSTs most commonly described mathematical achievement as either a combination of conceptual understanding and procedural fluency or with respect to a productive disposition. That is, the PSTs generally thought about mathematical achievement as being able to conceptually and procedurally understand mathematical concepts or thought about mathematical achievement as a combination of mathematical confidence (self-efficacy) and using mathematical concepts and ideas in the “real world” (seeing mathematics as worthwhile and useful). Interestingly, one PST identified three of the five mathematical proficiency strands—conceptual understanding, procedural fluency, and productive disposition—by saying mathematical achievement was “understanding and being able to apply mathematical concepts in the real world.” Understanding mathematical concepts is evident of conceptual understanding while applying mathematical concepts is evident of procedural fluency. Applying these concepts “in the real world” suggests that the PST thinks that mathematical concepts are useful in everyday situations which highlights the productive disposition strand of mathematical proficiency.

Another noteworthy observation from the second bar graph is that some of the PSTs thought about mathematical achievement without mention of any of the mathematical proficiency strands. Rather, these PSTs thought about mathematical achievement as either solely “grade-focused” (getting an A in the class) or as being “gritty” (constantly working through/practicing math problems) in the classroom. When comparing the pre-survey and post-survey responses, I noticed that the PSTs who thought about mathematical achievement as being solely grade-focused were the same students who indicated on their post-survey that they were not inspired to put forth effort on the task because it wasn’t being taken for a grade. This reveals that grades should not be the only source of accountability for students.

Implications

There are several conclusions and implications that become evident as a result of this research. First, it is important to consider how the results of pre-survey effort questionnaire among the PSTs relates to my first research question: Does perceived effort quadratically correlate to mathematical achievement? I have concluded that perceived effort is a better predictor of mathematical achievement than perceived talent. Consequently, this result validates the claim that the aforementioned statements—*it is acceptable to be bad at math because most people are,* and *you must be intelligent to be good at math*—are indeed myths. People who believe these myths are operating under a fixed mindset when it comes to mathematics. However, the results of this study seem to suggest that having a growth mindset and putting forth effort in a mathematics classroom, regardless of how "talented" one naturally feels in mathematics, will more likely lead to mathematical achievement. Therefore, it seems that one appropriate response to these findings would be to make students aware of this realization, and to continue to promote the importance of a growth mindset to students.

This study also investigated the difference in the amount of effort this lesson inspired in contrast to a what the PSTs considered to be a typical lesson. I have shown that 46 percent of the PSTs indicated that they put forth less effort in this lesson than they did in a typical classroom setting. One potential explanation for this discrepancy is the way in which students think about effort. Suppose some of the PSTs viewed effort as a negative construct. For example, suppose students feel like effort is a way to "make up for" a lack of understanding. In this case, the PSTs may not want to have to put forth a lot of effort in a lesson because, in their minds, more effort is required as a consequence of low understanding. In this scenario, putting forth less effort could actually be indicative of the PST having a good understanding of the concepts. If PSTs think of effort with a negative connotation, it could also be the case that the PSTs who were engaged in the task didn’t consider the work being done as “effort.” The PST mentioned above whose “mind wouldn’t stop” is an example of how a student could be putting forth effort without really feeling like they are. Since the PSTs seemed to be engaged during most of the lesson, it could be that these PSTs didn’t consider their work done on this task to be effort. Unfortunately, the PSTs were not asked to define “effort” on their pre-survey. However, in the future, it would be interesting to see not only how students define the word “effort” but also to see whether they see effort in a positive or negative light. It would also be interesting to further investigate how perceived effort is related to a PST’s mathematical proficiency as opposed to a PST’s past mathematical achievement.

This study also analyzed potential ways in which effort could be encouraged in the classroom, specifically with respect to cognitive demand, productive struggle, open mathematics, and cooperative learning. When analyzing the findings of this research with respect to the original framework, each of cognitive demand, productive struggle, open mathematics, and cooperative learning inspired at least some of the PSTs to put forth effort in the lesson. One interesting result of this research was the connection that emerged between cognitive demand and productive struggle. From Figure 12, it was seen that only three students who identified productive struggle as an effort motivator did not also identify cognitive demand as an effort motivator. Similarly, only two people that identified cognitive demand as an effort motivator did not also identify productive struggle as an effort motivator. This begs the question, “Are productive struggle and cognitive demand synonymous?” At first glance, they may seem like two sides of the same coin. After all, productive struggle is necessary to complete tasks that require a high level of cognitive demand. When considering the converse, though, it is evident that productive struggle isn't a necessary consequence of high levels of cognitive demand. Students may not "rise to the occasion" and implement productive struggle when presented with a task that requires a high level of cognitive demand. There was evidence of both of these instances in the PSTs' responses on the post-survey. Some PSTs responded to the high level of cognitive demand with almost no productive struggle. It would be interesting to further study potential ways in which we can encourage students to respond to high levels of cognitive demand with a productive struggle.

Despite the effort that each of these motivators inspired, none of these components alone was not enough to inspire prolonged effort. In order to prolong the amount of effort put forth by students, there are other motivators that need to be put into place. Some examples of these additional motivators are scaffolding (providing hints and suggestions when the PSTs were "stuck"), encouragement, and questioning to make sure the PSTs were understanding the reasoning behind the strategies. These additional motivators worked as effective ways to keep the students both engaged in the task and encouraged them to continue working to solve the problem. It was also shown during the lesson that the PSTs are more willing to put forth effort when they know that they will be held accountable. Some of the PSTs worked hard on the lesson when they realized that they were going to have to present their solutions to the class. Others noted that they would have worked harder if they were going to be held accountable for their work by receiving a grade for the assignment. Either way, it seems students want the work they do to "'count' for something" (Stein et. al., 2000, p. 27).

With the discussion of these motivators, however, one must also note the difficulty of incorporating these prolonged effort motivators in a classroom full of students. Even when students are working in groups, it is sometimes hard for a teacher to circulate around to all the groups and effectively scaffold and question the students. Liljedahl (2015) suggests that the use of vertical, non-permanent surfaces (VNPS) as workspaces for students can help alleviate some of the pressure on teachers when having students work in groups (p. 16). According to Liljedahl, teachers who implement the VNPS can see the work that students are engaged in without being directly in front of the students and looking over their shoulders (p. 16). Additionally, the VNPS allow each of the groups of students to see each other's work in the task, which allows them to "check" their work while also keeping them accountable to the other students in the class. Another positive aspect of the VNPS is allowing all the groups of students to see how their peers are also "struggling" to solve the task. Helping the students to see that "everyone is in the same boat" allows for a more prolonged, productive struggle.

As mathematics teachers, our ultimate goal is to lead our students toward mathematical achievement. As we have eluded to in this research, having students put forth effort in the classroom seems to be a factor necessary for a developing growth mindset and grit in mathematics, which eventually lead to mathematical achievement. Convincing students to put forth effort in the classroom through different teaching techniques can sometimes be difficult both for the teacher and the student, but these struggles ultimately lead to a great reward for the students. One PST made the perfect comparison in one of her post-survey responses; she compared productive struggle to an amusement park ride. She noted that having students complete tasks with no struggle "is boring and leaves no room for growth." However, this PST said a good task involves a little struggle and is like a roller coaster; "it has ups and downs but, in the end, you still enjoy the ride."

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Appendix A

Impact of Effort on Mathematical Achievement Survey

1. In the chart below, include the course names of all the mathematics classes you've taken at Georgia College and your respective grades for each.

GCSU Mathematics Class Course Title	Semester Taken (i.e. Fall 2016)	Grade Earned (A, B, C, D, F)	Were you satisfied with your grade? (Yes or No)

2. How many hours per week did you study for your last mathematics course?

3. For each of the courses mentioned above, please briefly explain why you were or were not satisfied with your final grade.

4. How much effort did you put into your last math class, on a scale from 1 to 5, where 5 is maximum effort and 1 is no effort?

5. How would you define mathematical achievement?

6. How would you rank your mathematical achievement on a scale of 1-5 (where 1 is very little to no achievement and 5 is exceptional achievement)? Explain your reasoning.

7. Read each of the statements below. Circle the number in the column that corresponds to your response to the statement.

	Not at all like me	Not much like me	Somewhat like me	Mostly like me	Very much like me
1. I am not a math person.	5	4	3	2	1
2. I think I have a natural ability in math.	1	2	3	4	5
3. I pick up on mathematical procedures quickly	1	2	3	4	5
4. I pick up on mathematical concepts quickly.	1	2	3	4	5
5. Math comes easier to me than it does to most people.	1	2	3	4	5
6. New mathematical topics sometimes distract me from mastering previous ones.	5	4	3	2	1
7. When I realize that I've completed a problem incorrectly, I don't get discouraged.	1	2	3	4	5
8. I often set a goal but later choose to pursue a different one.	5	4	3	2	1
9. I work hard in my math classes.	1	2	3	4	5
10. I have difficulty maintaining my focus on math problems that take more than a few minutes to complete.	5	4	3	2	1
11. I finish whatever I begin.	1	2	3	4	5
12. My interest in mathematics changes from year to year.	5	4	3	2	1
13. I am diligent. I never give up.	1	2	3	4	5
14. I have been obsessed with a certain mathematical idea for a short time, but later lost interest.	5	4	3	2	1
15. I have overcome setbacks to become more successful in mathematics.	1	2	3	4	5

Appendix B

Lesson Plan

Georgia Standard of Excellence:

MGSE4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area

MGSE4.MD.8 Recognize area as additive. Find areas of rectilinear figures by decomposing them into non -overlapping rectangles and adding the areas of the non - overlapping parts, applying this technique

I. Goals:
- The Preservice teachers (PSTs) will productively struggle through a learning task in order to develop grit
- The PSTs will see the value of letting their students engage in productive struggle

II. Objectives
- PSTs will develop an understanding of how to partition rectangles into sections that have equal area and perimeter.
- Students will make use of properties of squares and rectangles in order to draw or construct sections of equal area and perimeter.
- Students will also make use of sheers to construct shapes of equal area based on their base and height.

III. Materials and Resources
- Each PST will be given the task sheet.
- Each PST will have many choices of materials to use in order to solve the learning task, including geoboards and rubber bands, patty paper, grid paper, and Geometer’s Sketch Pad.

IV. Motivation
- The PSTs will be given the opportunity to explore connections between area and perimeter using the tools and technology of their choice while developing mathematical reasoning.

V. Lesson Procedures
1. Show PSTs a video of kids arguing over the size of their respective pieces of cake and the amount of icing on the pieces of cake.
2. Ask PSTs, “Have any of you ever been in this situation before? Today, we will make sure that you will never be forced to endure eating a slice of cake with insufficient icing again!”
3. Hand out the task sheet to each PST.
4. Have PSTs read over and begin to think about the task individually (about 2 minutes).

5. Ask PSTs to pair up and begin discussing possible strategies and devising a plan of action to come up with a solution. PSTs can use any of the resources available to them as mentioned above.
6. Stella will begin to ask each pair to elaborate on their strategies for solving this task. This discussion may include probing and prompting questions.
7. (PURPOSEFUL PAUSE) After 10 minutes, come together as a class and have the pairs give a status update. This may include pairs sharing some strategies that seem to be working and strategies that didn't pan out.
8. Give the PSTs more time to work together in their groups. (about 10 more minutes)
9. Come together as a class and have pairs present their strategies (20 minutes)
10. Present the three extension questions and have PSTs make initial predictions about at least one of the questions. The PSTs can use these extension questions in their write-up.
11. Have PSTs complete the post-survey about their experience during this lesson.

VI. Closure

- Have PSTs share and make connections between a variety of problem-solving strategies and solutions.

VII. Extension

- After the PSTs are able to come up with solutions to the original task given, the PSTs will be asked to come up with a solution to one of the following questions:
    i. If your solution involved the cake being a square, how would your solution change if you began with the cake being a rectangle that is not a square?
    ii. How would the problem change if the cake wasn't rectangular (i.e. a different shape)?
    iii. How would the problem change if Matt invited five friends over instead of two?

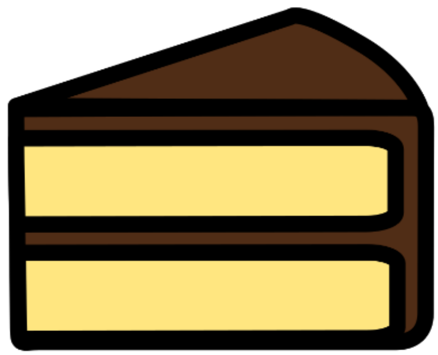
VIII. Assessment

- The PSTs will be assessed throughout the activity when Stella is able to go around and ask the PSTs about their thoughts and plans to find the solution.
- The PSTs' solutions will also be assessed based on the rubric provided in the paper

Appendix C

Lesson Task

Let Them Eat Cake!



Matt invited his best friends to celebrate his birthday. Matt and his two friends want to share a rectangle cake, which has 81 square inches surface area on top. It is a chocolate cake with buttercream frosting. The cake is frosted evenly on the four sides and the top. How can Matt cut the cake so that each person receives an equal share of both cake and frosting? How can you justify that each person got the equal amount of cake and the frosting?

Appendix D

Post-Lesson Survey: Impact of Effort on Mathematical Achievement

1. What were your initial thoughts when you were presented with the learning task?

2. Did you find the task engaging/interesting? Why or why not?

3. Which parts of the lesson, if any, inspired you to put forth effort into the task? Why do you think these parts encouraged you to put forth effort?

4. What parts of the lesson, if any, could be changed in order to encourage you to put forth more effort into the task?

5. On a scale of 1-10, where one is minimal effort and 10 is maximum effort, how much effort do you put forth in a typical classroom setting?

6. On a scale of 1-10, where one is minimal effort and 10 is maximum effort, how much effort did you put forth during this lesson?

7. Did you feel like you had made an achievement by the end of the lesson? Why or why not?

8. Did you find this task to be challenging enough for you to "struggle" to come up with a solution? Explain why or why not.

9. Did you find this task to be too challenging for you to "struggle" productively? Explain why or why not.

10. Was there any time throughout the lesson where you hit a mental "road block," and you felt like you couldn't solve the problem and wanted to give up? If yes, why did you feel that way? What was your response after you hit the "road block"? If no, why do you think that was?

11. After participating in this lesson, have your views on incorporating productive struggle in your own future classroom changed? Explain.