Children's Understanding of the Equal Sign

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Abstract

This paper compares research which was conducted by Falkner, Levi and Carpenter (1999) with data collected from a local school. Informal research had suggested a similar misconception among the general public. With my research, I was motivated to determine whether the intervening sixteen years had seen any improvement among second, fourth, and sixth graders when asked to solve a relatively simple equation. The recent research also incorporated a survey that asked the students to list their favorite subject, define the equal sign, and rate themselves as math students. These answers were used to ascertain any possible correlations between the survey answers and the equation answers. The answers from the different grades were compared to discern any possible trends. Possibly the conclusions can be useful to educators looking to improve the abilities of their pupils.

Children's Understanding of the Equal Sign

Researchers have analyzed students who struggle with the transition from arithmetic to algebra and have found one of the causes to be a misunderstanding of the meaning of the equal symbol. In their study, Falkner, Levi, and Carpenter (1999) revealed an alarmingly high number of elementary and young middle school students who possessed this misconception. In reviewing this study, I found it difficult to believe that over ninety percent of the students in the second, fourth, and sixth grades could offer an incorrect answer to a seemingly simple equation. However, other in-class research confirmed the study by illuminating a very real disconnect between students' perception of the equal sign and its actual function. Allowing students to blindly continue in a system of education which fails to accurately teach the necessary meaning would remove the opportunity for current students to improve over older students. As these young scholars progress towards algebra, the misunderstanding of the sign will continue to grow and impede a successful passage from elementary to upper levels of math.

As society pours billions of dollars into education, it would only make sense to analyze instructional methods to determine what is helpful and what is not. The changes necessary to improve children's understanding of the equal sign are relatively inexpensive and have the potential to provide a much stronger foundation as those children learn the concepts of algebra.

Literature Review

The most accurate, basic definition of the equal sign should be founded on the concept of balance. The equal symbol balances the two sides. Additionally, the notion of comparison is helpful with the meaning. Ideally, students should be taught to view an equation as the right side is compared to the left side or the right side is the same as the left side. However, according to

Falkner, Levi, and Carpenter (1999) note the desired concept of seeing the equal sign as demonstrating the idea of 'the same as' is not possessed by the large majority of students. Instead, the equal sign is seen as an indicator that the answer is next. Kieran (1981) believes that left to continue without intentional efforts to clarify children's understanding of the equal sign, students will progress to high school with the notion the symbol means 'to do something'. Even a majority of early teens when asked about the equal sign incorporated 'answer' in their response. According to Falkner, et al. the goal is to have students "see the equals sign as a symbol describing a relationship rather than" a command to do something.

While Carpenter, Franke, Levi (2003) found that researchers agree there is a definite disconnect as described above, there is not much agreement as to the cause of it. Because students, even young ones, possess a vast amount of informal understanding of mathematics, Kieran (1981) found that early instruction didn't circumvent the students acquiring the wrong notion and opines the natural tendency could be tied to the idea of 'running total'. Once children are old enough to collect objects, such as toys in a bag, they will keep a running total in their heads or out loud. It is possible these types of activities are the start of viewing what later will be symbolized by the equal sign. Among children studied by Carpenter (2003) there is an inclination to solve problems in a series of steps. It has to be noted that almost all activities are sequential. It seems reasonable that this ordering tendency would influence mathematical reasoning. If this natural bent is encouraged at all by early educational jargon such as "two plus two equals…," one can readily see the student is sent on a course of believing the view that the equal sign is a command signal with an expected answer to follow. Since kindergarteners were found to "have enduring misconceptions about the meaning of the equals" symbol, it is logical to think the proclivities are more innate than school related according to Falkner, et al. This would

support the argument for repetition of varied equations so as to retrain the natural concept that the equal sign means to do something.

Carpenter, et al. introduces the possibility that calculators serve to reinforce the misconceptions of the equal sign as a calculation indicator. It is not a stretch to believe that students are actually thinking "this equals" as they hit the equal sign button. However, elementary students struggled with understanding and applying the balancing aspect of the equal symbol long before the prevalence of calculators. They may not have helped, but calculators did not cause the disconnect most students possess.

Kieran (1999) may give the key to this disconnect by defining equality as "an attribute which does not change" and equivalence as a concept that allows one value "to replace" another. This could explain why young students have such a difficult struggle in the two sided equation problem. To them, the equal sign means one answer that is not influenced or changeable. When one sees the disturbed looks when students notice the 5 at the end, it is reasonable to accept Kieran's definitions as accurate and beneficial in influencing a resolution to how students mistake the work of the equal sign. Gavin and Sheffield (2015) determined the balancing concept of the equal symbol does not seem to be a natural inclination as "research has shown that students need help in constructing meaning for equality". As such, students must be taught the concept of equivalence in relationship to all the other parts of the equation. After being taught, students will need frequent reinforcement of the knowledge.

As referenced in their article, Falkner, et al. (1999) note that even though students grapple with finding the correct number for the box in the equation, when given blocks in piles, the same students who miss the equation will correctly demonstrate the same concept with the blocks. In one activity, students answered the equation incorrectly, modeled the answer correctly,

but returned to the equation still convinced the incorrect answer was right. The struggle for students seems to be in the transition from objects to symbols. Knowing how many blocks are needed to make two piles even does not seem to translate into knowing what number makes the two sides of an equation balance. Carpenter (2003) agrees this indicates the misconception is with the actual equal symbol not the concept of equality. Gavin and Sheffield (2015) found that "giving students an opportunity to solve problems in different contexts helped them develop a deeper understanding of equations, the equal symbol, and the concept of equality". For those students who wrestle with tying the model to the equation, they are helped by instructors who will utilize as many resources as necessary to improve correct conceptualization of the equal symbol.

Even though Kieran (1981) laments that students will continue to view the equal sign as separating "a problem and its answer", Falkner, et al. (1999) found that with intentional efforts to teach the relationship aspect of the equal sign, by the "end of the year, fourteen out of sixteen" first and second graders answered the problem correctly. It has to be noted that haphazard teaching won't overcome the natural inclinations of students, nor will the correct concept be retained without intentional efforts. It is alarming to Carpenter (2003) to note that "even after students develop an appropriate conception of the meaning of the equal sign, they may revert to previous incorrect conceptions if they only see number sentences with the answer after the equal sign for an extended period of time". This regression demonstrates the diligence necessary from educators to prevent it. Great energy has to be expended insuring that lessons consistently provide variations in equation structure. Heightened awareness can hopefully eliminate lessons and curriculum which fail to maintain the needed variations of problems. While there seems to

be a current push away from drill and repetitious activities, one is wise to recognize that understanding has to be practiced to be retained.

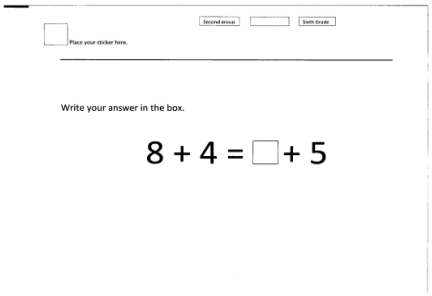
While there exists real misconceptions by most young students concerning the equal sign, Falkner, et al. found that there is similar difficulty in educators to recognize what are appropriate algebra concepts to introduce in the elementary years. Carpenter (2003) believes because educators and curriculum often present arithmetic in a disconnected fashion from other concepts, steps must be taken to secure a smooth transition from the math of elementary grades to the pre-algebra of middle school. With algebra defined as a generalization of arithmetic, one can see how important it is to ensure basic concepts are understood so as to benefit and not handicap students.

The balance concept of the equal sign adequately prepares students for moving terms when solving for equations for the value of the variables in algebra. For the student who views the equal sign as a command to do something, Falkner, et al. asks, "What reasoning will suffice to make sense of subtracting or adding a number to both sides of an equation in algebra?" The computational concept of the equal sign is debilitating to the student as algebra approaches. The only option is "to memorize a series of rules for solving equations". Gavin and Sheffield (2015) believe that understanding the equal sign as a balancing agent can help students avoid the rules memorization that leads to errors and poor strategies in later classes in middle and high school. Since students arrive in kindergarten possibly already possessing an incorrect concept of the equal sign, it is incumbent upon educators according to Carpenter (2003) to determine methods of instruction which aid students in constructing an adequate foundation for algebra in relationship to their existing understanding.

Methods

With a desire to test a local group of students, I wanted to generate data to compare to the study conducted in 1999. I was able to utilize the second, fourth, and sixth grades at a local school. Having all the students at one school would enable the data to show possible trends from a smaller number of instructors. Using students who as a core group had advanced together through school would enable the research to be static with respect to gender, socio-economic, life experiences, etc.

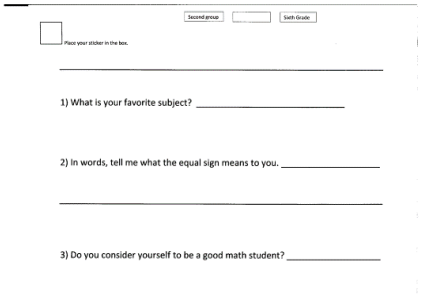
Unlike the original study, I included a survey which asked the students to name their favorite subject, use their own words to define the equal sign, and rate themselves as a math student. The students did not include their names on the two sheets they were given. I had assembled sets of matching stickers for the students to attach to the equation sheet and survey sheet. This enabled me to match the equation answer to the corresponding survey sheet from the same student. Here are the two sheets which the students were given. They were given the equation sheet first. After that sheet was collected, the survey sheet was handed out and completed.


Second group Sixth Grade

Place your sticker here.

Write your answer in the box.

8 + 4 = □ + 5


Second group Sixth Grade

Place your sticker in the box.

1) What is your favorite subject? _____

2) In words, tell me what the equal sign means to you. _____

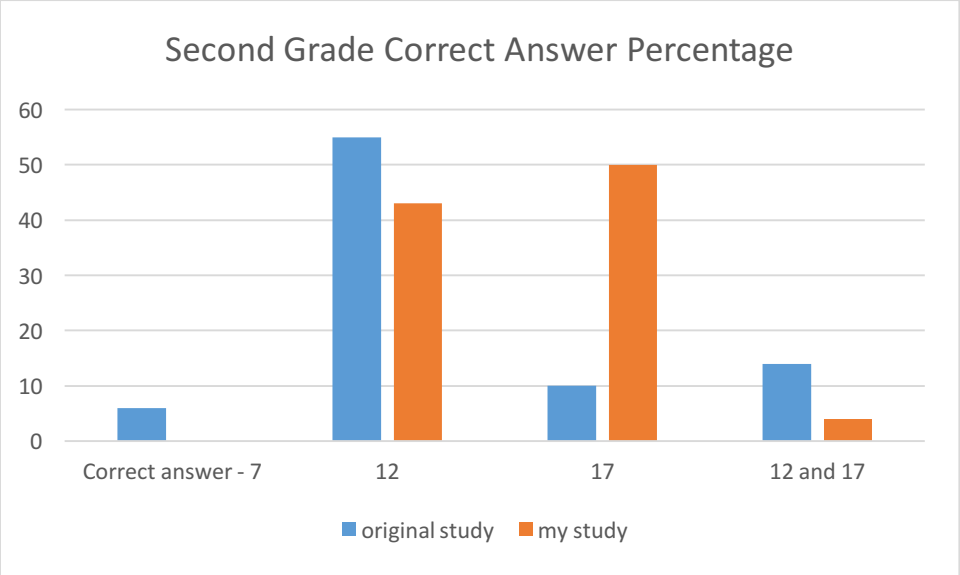
3) Do you consider yourself to be a good math student? _____

The use of the stickers also provided the opportunity to gather the data without the need for personal, identifying information.

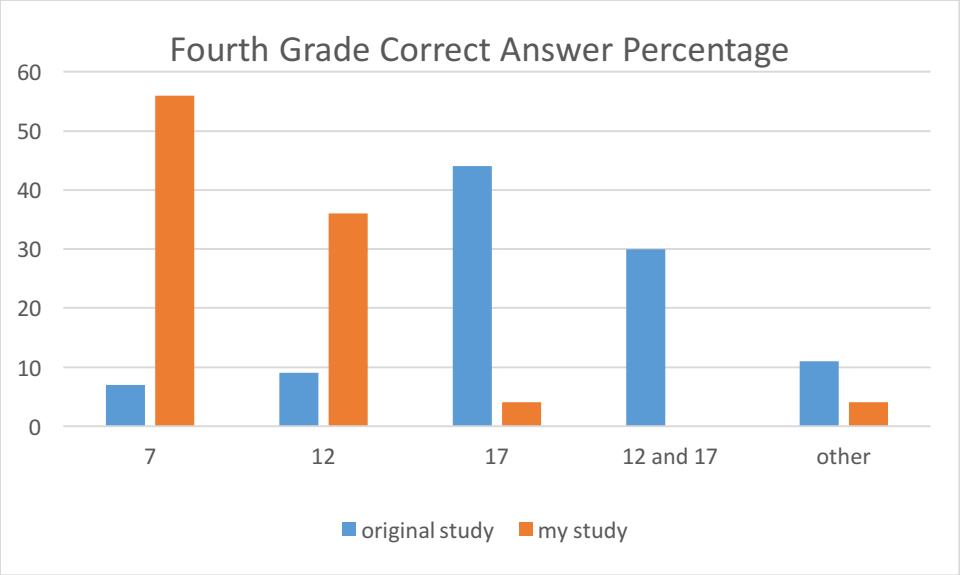
The data was categorized in several ways. The percentage of correct answers were tabulated by grade. The percentages were compared to the original study. The answers to the survey questions were grouped and compared to the correct answers to determine if any patterns existed to help explain the percentage of correct answers. The results for each grade were compared to the other grades.

Data Analysis

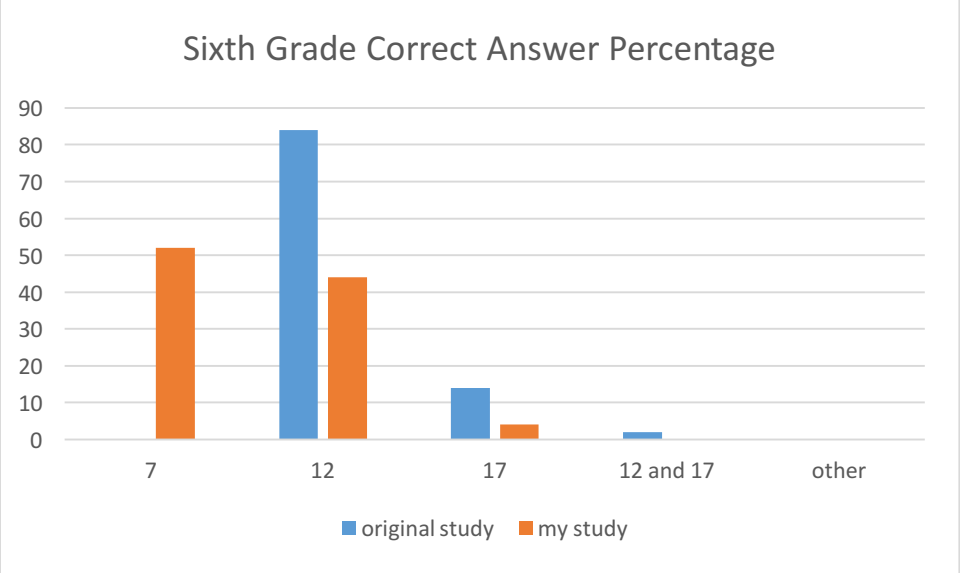
The following chart compares the second grade answers from both the original study and my study.



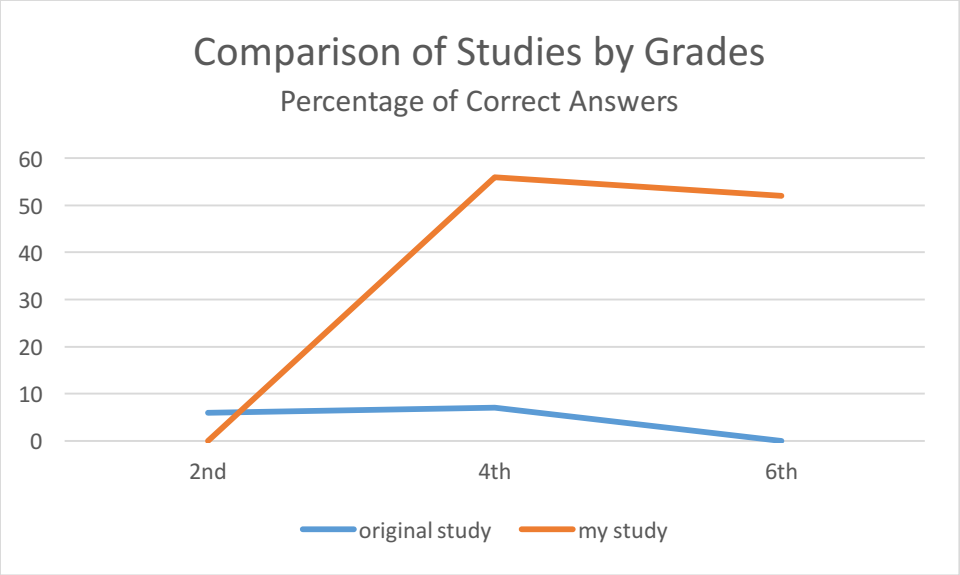
The second graders in my study performed more poorly than those in the original study. The following chart compares the fourth grade answers from both the original study and my study.



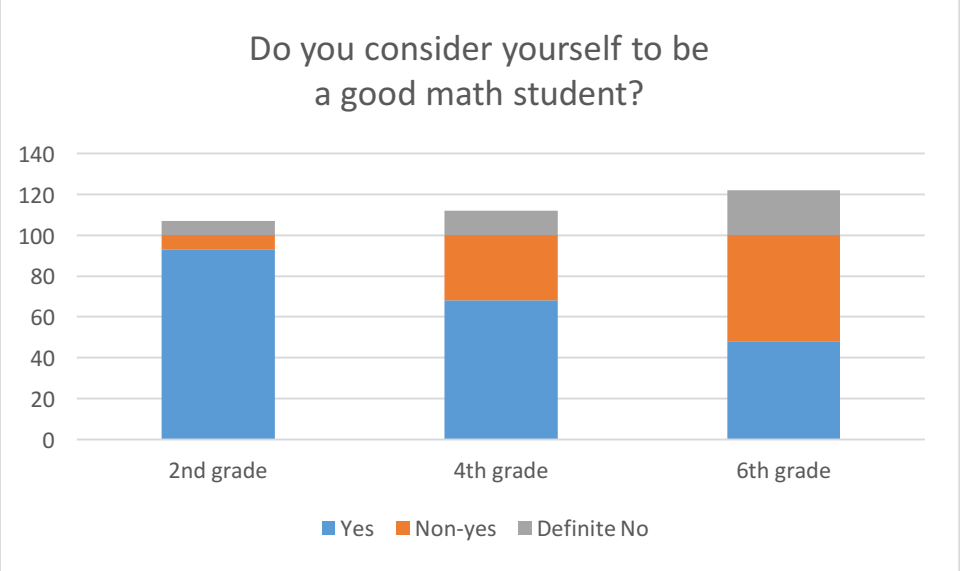
The fourth graders in my study vastly improved over those in the original study. The following chart compares the sixth grade answers from both the original study and my study.



While not matching the results of the fourth grade, the sixth graders from my study outperformed those from the original study. Interestingly, the fourth graders from both studies fared more favorably than the sixth graders. The following chart depicts that phenomenon.

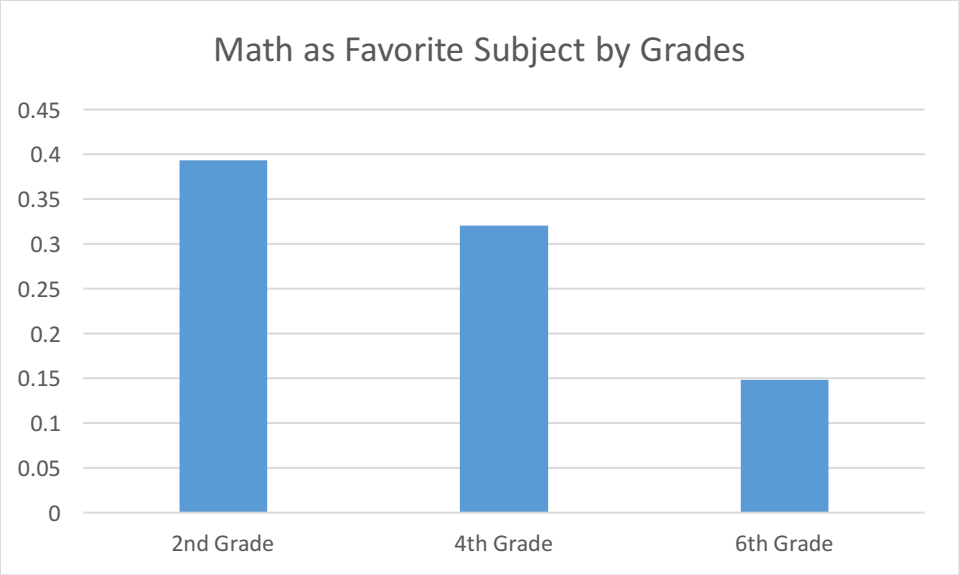


The survey questions were compiled to be compared to the answers in each grade. The following chart shows the decline of self-efficacy by grade.



Note, the portion that represents the 'Definite No' answers is also included in the 'Non-yes' section. It is a subset of that section and deserves extra scrutiny.

The answers in the survey concerning the relationship between the self-efficacy and correct answers are compared by grades in this chart.



Conclusions

Significant improvements were shown with the fourth and sixth grades, while the second grade performed more poorly. In my working with the second grade, I noted the teachers are responsible for all subject areas and teach the same group all day. Unless math is a priority, new techniques and insights won't easily be translated into improved learning for the students. In the original study, one of the researchers was the teacher for at least some of the second graders. I believe this created an environment where math concepts were more of a priority than the classes I studied.

In my research and as in 1999, the fourth grade scored the best. As far as my research, this is not surprising. The fourth grade teacher I dealt with was heavily involved and committed to staying current with education trends. We discussed the research, and she described her efforts to overcome some of the obstacles she had seen students have in regards to equation structure and students' reluctance to accept differing equation formats. Aside, she was the only teacher to request the results of my research.

Also parallel to the original study, there was an unexplained drop between the fourth and sixth grades. It is not clear what could cause this drop in both studies. It is possible that as the sixth graders are introduced to more topics, the correct concept of the equal sign fails to connect to new material. It may be that middle school teachers may not understand the need to continue to strengthen the correct concept of the equal sign. It is even possible that the onset of adolescence may have the potential to temporarily interrupt consistent mathematical capabilities.

Over 92% of my research students were unable to adequately define the equal symbol, which has the potential to negatively impact their future mathematical endeavors. Those students define the equal sign as a signal to compute an answer or an indicator that the answer is next.

Sadly, as students progress through school, their self-efficacy plummets. When thinking of school, most would believe that second grade has a safer, more comfortable perspective for students when compared to upper grades. Fast, et al. (2010) found "that students who perceived their classroom environment as more caring, challenging, and mastery oriented had significantly higher levels of math self-efficacy". This could explain why the self-efficacy is higher for the second graders and decreases as the environment becomes more competitive.

The rate of students describing math as their favorite subject is not as high as the self-efficacy rates, but it declines in a similar pattern. Both decline consistently from the second to sixth grades and do not have the trend the correct answer percentages have.

Implications

Given the drop between fourth and sixth grades, consistent, intentional efforts by the teachers and curriculum developers are necessary to continue to improve student understanding of the equal sign.

Educators should avoid the use singular equation formats (first set of equations), choosing instead lessons which utilize a variety of equation arrangements (second set of equations).

Set 1: 1 + 5 = __ 2 + 7 = __ 7 + 1 = __ 5 + 3 = __ 3 + 4 = __ 9 + 0 = __

Set 2: True or False: 6 = 2 + 4 12 = 12;

Open number sentences with mixed components and mixed formats:

__ + 2 = 5 + 3 3 + 4 = 7 + __ 9 = 2 + __ 2 + __ = 9

Given the students' regression to a possible default perspective of equations, it is imperative to reinforce the accurate perspective of the balancing concept of the equal sign through periodic review and practice. Manipulatives are beneficial for this work.

The recent research did show an improvement had been realized at least at the fourth and sixth grade levels. One could hope that the earlier research and related journal articles have had an impact on educators which has benefitted students. However, the worsening with the second graders and persistent unexplained drop for the sixth graders is a nagging reminder that enough has not been done to implement better teaching techniques that can help students understand the equal sign and make that successful transition to algebra.

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