Student's Perception of Interactive Teaching Aids

In The Core Mathematics Classroom

Taylor Smoak

Georgia College

Senior Capstone Fall 2014

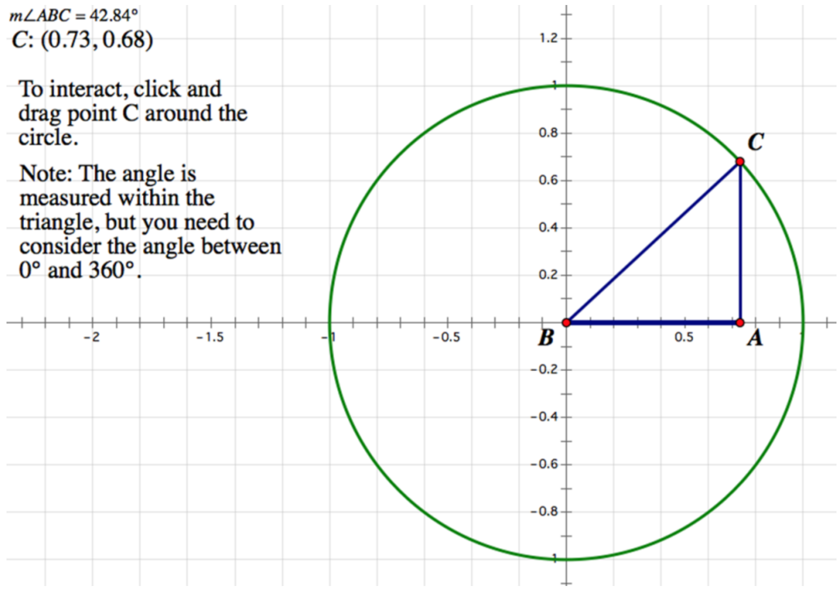
Introduction

Over the centuries, teaching techniques have evolved from one thing to another. From lectures and pictures, to technology based lessons, all school subjects have begun trying to keep up with the new way to best teach its topics while also trying to increase student achievement. Mathematics can be a struggle for many students because of the rigor and the fact that its' topics build on one another. It has been said that "manipulatives allow students to construct deeper meanings of math concepts and learn that math is more than memorizing a process" (Couture 27).While teaching techniques have evolved, the modern day student has as well. Not only do students need to listen, but many also seem to need an extra tool to help them learn; whether it is with a visual or a hands-on activity. Each students' learning preference is different because every student is different, but even if one student is struggling and another student is excelling, manipulatives can "help struggling students visually see a math process while allowing advanced learners to question themselves and dig deeper into a concept they have mastered" (Couture 27). Many studies have been done regarding this topic as well, and many have a positive outcome when using manipulatives. One study said students "increased their skills and showed more interest and enjoyment when learning was done through the use of manipulatives" and "were visibly more active in class and develop more self confidence in their math skills" (Allen 14). These studies have catered more to how a teacher feels about manipulatives in the classroom, but we need to understand how students feel towards this topic. Do students feel that they learn better by listening, seeing, or interacting with a lesson? Does this tactic that they feel is the strongest also yield a deeper understanding of that topic?

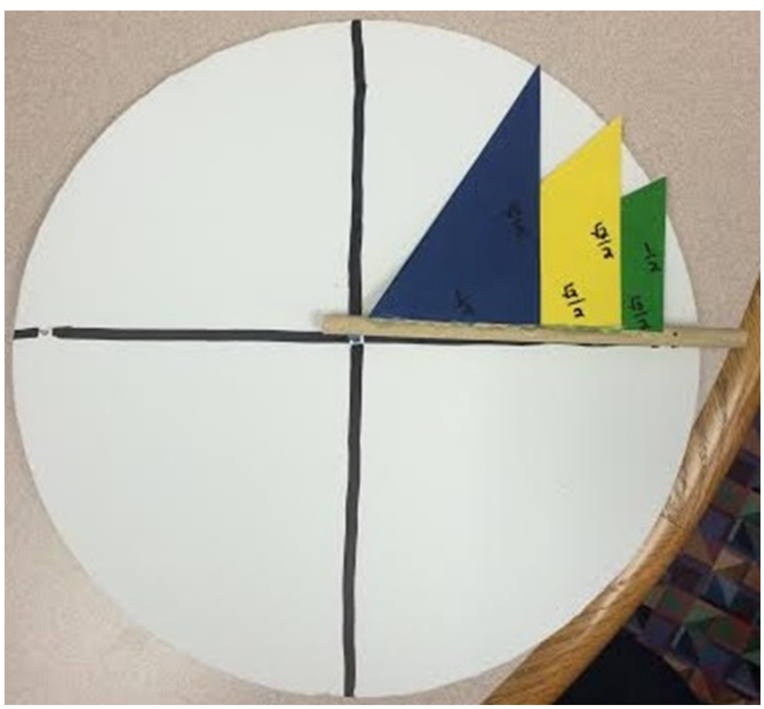
While taking one of the math education classes at Georgia College, many of the students have noticed how effective using manipulative based lessons have become to gather a greater understanding of different mathematical subjects. For this study, we surveyed probability and statistics students and calculus II students on their use of the teaching aid they are given and how it may or may not help them answer difficult questions pertaining to the unit circle. We split the students into three different groups. All of the groups were given the same worksheet and survey. One group was only allowed to see just a blank unit circle, the second was able to look and move a GSP version of the unit circle, and the third was given a hands on tool which represented the unit circle. After completing the worksheet, the students were then asked to complete a survey to see if they felt that their specific teaching aid gave them a better understanding of the unit circle. Prior to the study, we predicted that the hands on teaching aid group would gain a deeper understanding of the unit circle and feel that their teaching aid would be the best used aid when teaching the unit circle.

Worksheet, Survey, and Process

The intent for this study is to discover if specific teaching aids help students gain a better understanding of the unit circle. We used a Probability & Statistics class and a Calculus 2 class for volunteers to complete the worksheet and survey. We had a total of 47 students participate. Prior to giving out the worksheet and survey, we explained the few rules we had for the study such as no talking and no calculators. Then we randomly grouped the students with a computer program, handed out the worksheets and sent the groups to their respective areas. The technology group was sent to the computer lab with a chaperone, and the control group and manipulative group sat on opposite sides of the same classroom. The technology groups' aid can be seen in figure 1. They were given a unit circle in GSP with point C being able to move around the perimeter of the circle. Then we gave the manipulative group their hands on aid and explained how it moved. This aid can be seen in figure 2. The dowel rod with the triangles could be moved around the circle by flipping over the x-axis and the y-axis. The students in this group had to pass the manipulatives around because there were more students than tools. Little instruction was verbally given for this study since it is focused more on how the students used the aids.



[Figure 1]



[Figure 2]

The worksheet we created had questions that would assess a student's level of understanding when recalling or learning a specific topic. The first page of the worksheet had a blank unit circle that was to be filled out by each group. Each group then had the same visual on the second page to help them complete the unit circle. Following this was five questions related to trigonometry functions and the measurements on the circle. The third page asked deeper questions on the connections between the unit circle and geometry. This was the page that we felt would show the difference between the groups since each group had different aids which may lead them to understanding different concepts deeper than others. Once a student was done, the survey was then handed out separately and asked different questions about their teaching aid and depth of understanding. Many of the students in the control group finished the worksheet within 20-25 minutes and had plenty of time to do the survey. Some did use the whole class period of 50 minutes. The manipulative group took about 30-40 minutes to finish the worksheet and then used the rest of class to complete the survey. The technology group took about 25-40 minutes to finish the worksheet and then used the rest of the time to finish the survey and rejoin the class after the whole group was done with both the worksheet and survey. The complete worksheet and survey is in Appendix A in the back.

Levels and Grading

After collecting the data, we read through each worksheet and survey. The worksheet was graded as if it was a test. The first page of the worksheet was graded on correctness. If the student got a question wrong it was minus a point. The front page was graded out of 48 since each place on the unit circle had a degree, radian, and coordinate pair value. The second page also was graded on correctness and was graded out of 5 since there were 5 questions. The third page had more thought provoking questions so we used these questions to place students into different levels. To construct these levels, we began looking into the Van Hiele Levels of Geometric Thinking. These levels can be found in figure 3 (Mason 1).

Level 1: Visualization	Students recognize figures by appearance alone, often by comparing them to a known prototype. The properties of a figure are not perceived. At this level, students make decisions based on perception, not reasoning.
Level 2: Analysis	Students see figures as collections of properties. They can recognize and name properties of geometric figures, but they do not see relationships between these properties. When describing an object, a student operating at this level might list all the properties the student knows, but not discern which properties are necessary and which are sufficient to describe the object.
Level 3: Abstraction	Students perceive relationships between properties and between figures. At this level, students can create meaningful definitions and give informal arguments to justify their reasoning. Logical implications and class inclusions, such as squares being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood.
Level 4: Deduction	Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. At this level, students should be able to construct proofs such as those typically found in a high school geometry class.
Level 5: Rigor	Students at this level understand the formal aspects of deduction, such as establishing and comparing mathematical systems. Students at this level can understand the use of indirect proof and proof by contrapositive, and can understand non-Euclidean systems.

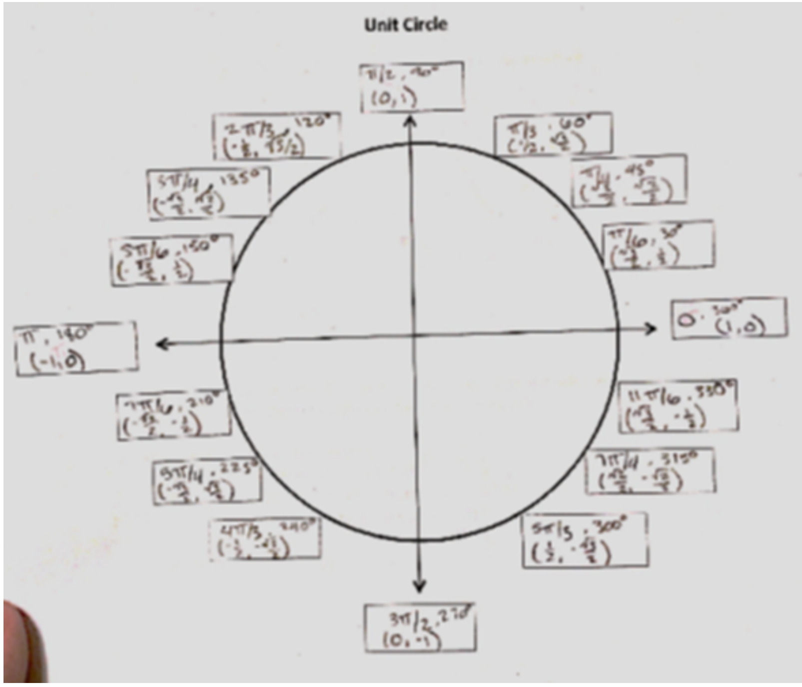
[Figure 3]

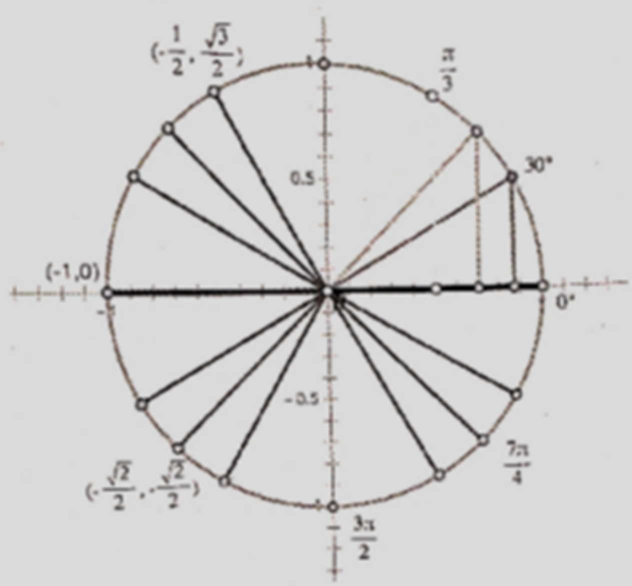
From these levels, we were able to construct our own for this specific study. We constructed 4 levels, with level 1 being split into two levels for algebraic and geometric thinking. We then used the three worksheet pages to get a sense of the students understanding. We looked at the amount of correctness on both page 1 and 2, and the short answers on page 3. From these answers, we then placed the student into a level. These levels can be seen in figure 4.

Level 0	Students observe the circle and the elements of the circle, but do not exhibit an understanding of the geometry of the circle as the basis for the elements (i.e. they are simply memorized at best)
Level 1a	Students demonstrate that there is a connection of cosine and sine to the coordinates of the circle
Level 1b	Students demonstrate that there is a connection of triangles within the circle to the coordinates
Level 2	Students make reference to the connection between the coordinates, the trig functions cosine and sine, and the triangle geometry of the circle, but cannot precisely explain the connection
Level 3	Students make reference to the connection between the coordinates, the trig functions cosine and sine, and the triangle geometry of the circle and can precisely explain or justify the connection

[Figure 4]

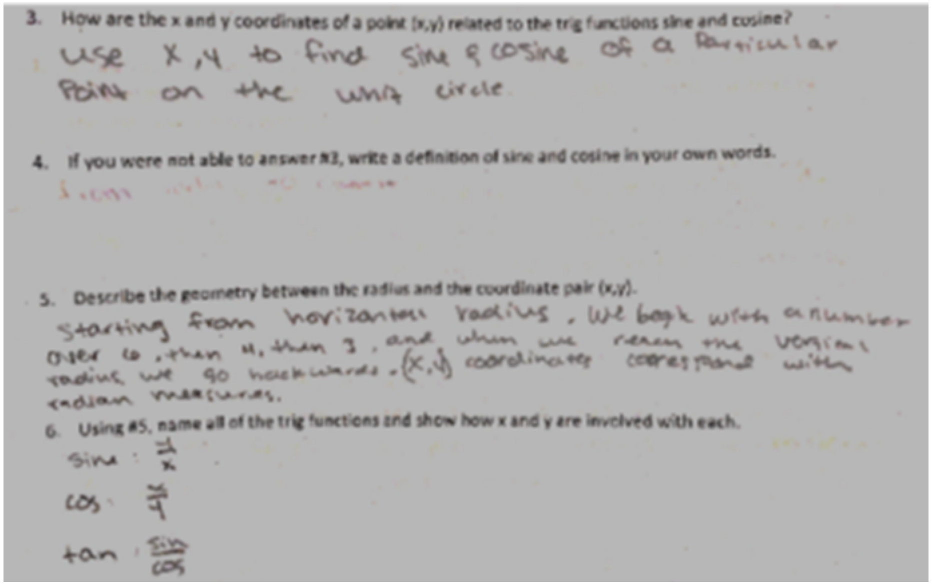
The first level that we saw students performing at was level 0. This level was where students either was not able to fill out the unit circle or they showed that they could only fill out the unit circle. Figure 5 shows a student who filled out the unit circle completely and was able to even answer the trigonometry questions on page 2, but when the short answer questions came on page 3 this student was not able to show any deeper connections. This student tried to answer the questions, but seemed to not understand the definitions of sine and cosine which kept them from being able to answer the other questions. For these reasons, we placed this student in the level 0 category.





1. Fill in the each square in the above unit circle with the degree and radian angle measures, and the coordinate pair that corresponds to that angle on the circle.

2. Using the circle, answer the following:
   a. What is the coordinate pair at 45°? $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
   b. What is the angle measure (in degrees) at π? 180°
   c. What is the radian measure at the point (0,-1)? $\frac{3\pi}{2}$
   d. What is sin(30°)?
   e. What is cos(225°)? -1



3. How are the x and y coordinates of a point (x,y) related to the trig functions sine and cosine?

use x, y to find sine & cosine of a particular point on the unit circle.

4. If you were not able to answer #3, write a definition of sine and cosine in your own words.

5. Describe the geometry between the radius and the coordinate pair (x,y).

starting from horizontal radius, we begin with a number over 6, then 4, then 3, and when we reach the vertical radius, we go backwards. (x,y) coordinates correspond with radian measures.

6. Using #5, name all of the trig functions and show how x and y are involved with each.

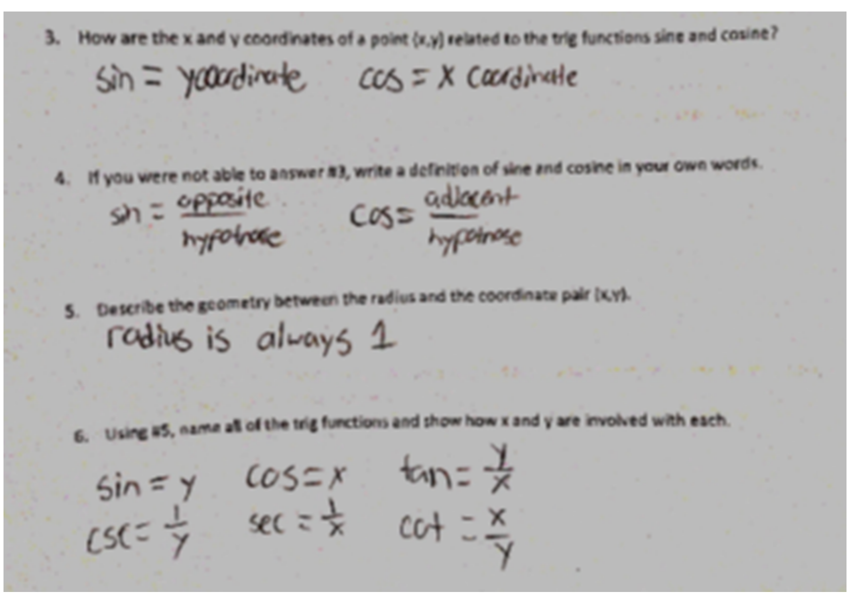
sine: $\frac{y}{x}$

cos: $\frac{x}{y}$

tan: $\frac{\sin}{\cos}$

[Figure 5]

In figure 6 we see an example of a student who received a placement of level 1a. This student was able to correctly fill out the unit circle and answer the trigonometry questions, but was also able to define sine and cosine. This student focused on sine and cosine as more of an algebraic concept. They did not seem to connect to any geometric concepts with the unit circle. Since they focused more on algebraic concepts we gave them a placement of level 1a.



3. How are the x and y coordinates of a point (x,y) related to the trig functions sine and cosine?

sin = y coordinate cos = x coordinate

4. If you were not able to answer #3, write a definition of sine and cosine in your own words.

sin = $\frac{\text{opposite}}{\text{hypotnuse}}$ cos = $\frac{\text{adjacent}}{\text{hypotnuse}}$

5. Describe the geometry between the radius and the coordinate pair (x,y).

radius is always 1

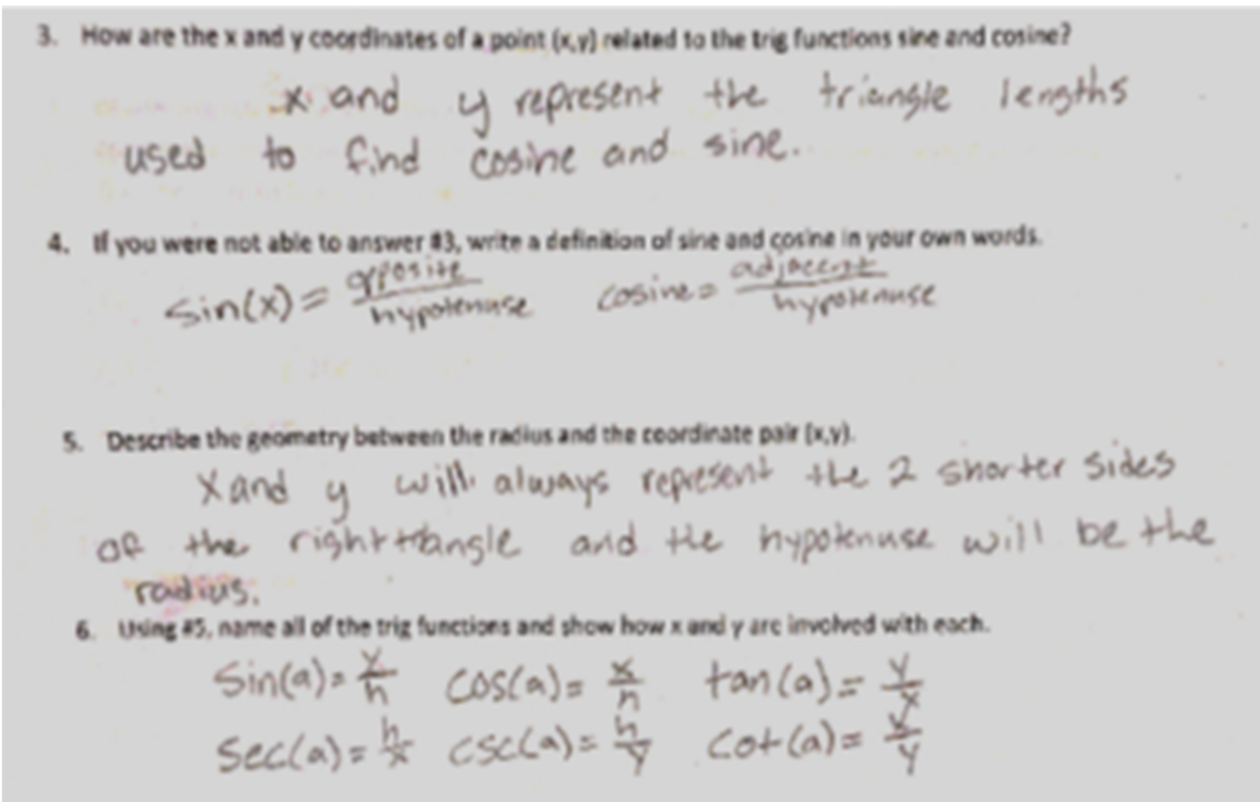
6. Using #5, name all of the trig functions and show how x and y are involved with each.

$\sin = y$ $\cos = x$ $\tan = \frac{y}{x}$

$\csc = \frac{1}{y}$ $\sec = \frac{1}{x}$ $\cot = \frac{x}{y}$

[Figure 6]

The next level we placed students in was level 1b. This level had more of a focus on the geometric connections within the unit circle. This student discussed more about the importance of the triangles that are used to make up the unit circle. They used vocabulary such as “lengths”, “sides”, and “right triangles” in their explanations and focused more on geometry. This is why we placed this student in level 1b. This is shown in figure 7.



3. How are the x and y coordinates of a point (x,y) related to the trig functions sine and cosine?

x and y represent the triangle lengths used to find cosine and sine.

4. If you were not able to answer #3, write a definition of sine and cosine in your own words.

$\sin(x) = \frac{\text{opposite}}{\text{hypotenuse}}$ $\quad \text{cosine} = \frac{\text{adjacent}}{\text{hypotenuse}}$

5. Describe the geometry between the radius and the coordinate pair (x,y).

X and y will always represent the 2 shorter sides of the right triangle and the hypotenuse will be the radius.

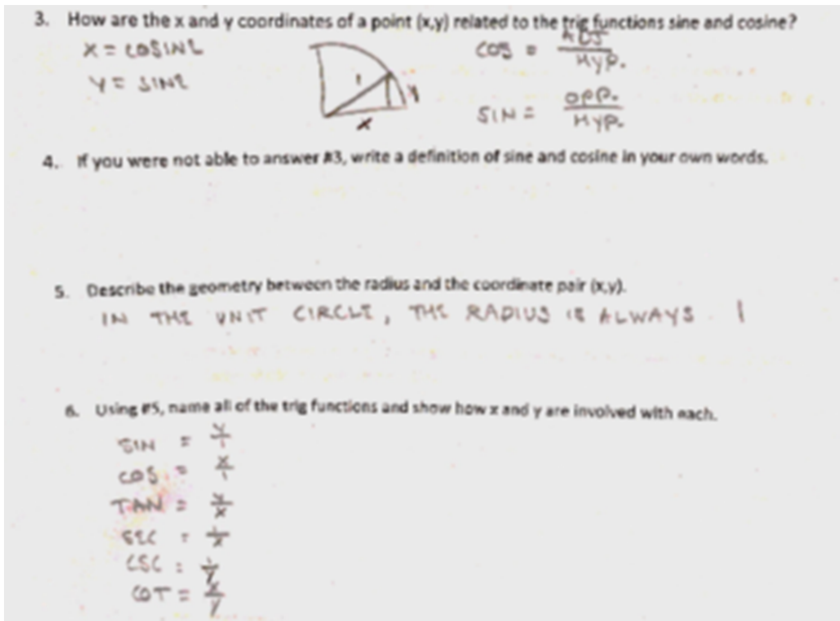
6. Using #5, name all of the trig functions and show how x and y are involved with each.

$\sin(a) = \frac{y}{h}$ $\quad \cos(a) = \frac{x}{h}$ $\quad \tan(a) = \frac{y}{x}$

$\sec(a) = \frac{h}{x}$ $\quad \csc(a) = \frac{h}{y}$ $\quad \cot(a) = \frac{x}{y}$

[Figure 7]

The last level that we were able to place students in was level 2. This level showed that students were able to make a connection between the algebraic and geometric concepts in the unit circle, but they were not able to explain the reasons why the connection was there. This is where we began seeing more pictures being drawn on the worksheets, and where the definitions of sine and cosine had both an algebraic and a geometric connection. The connection discussed how sine and cosine not only made up a coordinate pair, but also made up the right triangles that are seen in the circle. This is seen in figure 8. We also began to see discussion about the Pythagorean Theorem and its connection to the unit circle. This was something that we thought we would not see, but it was a pleasant surprise that multiple students were making the connection between the unit circle and the Pythagorean Theorem. An example of this is shown in figure 9.



3. How are the x and y coordinates of a point (x,y) related to the trig functions sine and cosine?

X = COSINE
Y = SINE

$\cos = \frac{ADJ}{HYP.}$

$\sin = \frac{OPP.}{HYP.}$

4. If you were not able to answer #3, write a definition of sine and cosine in your own words.

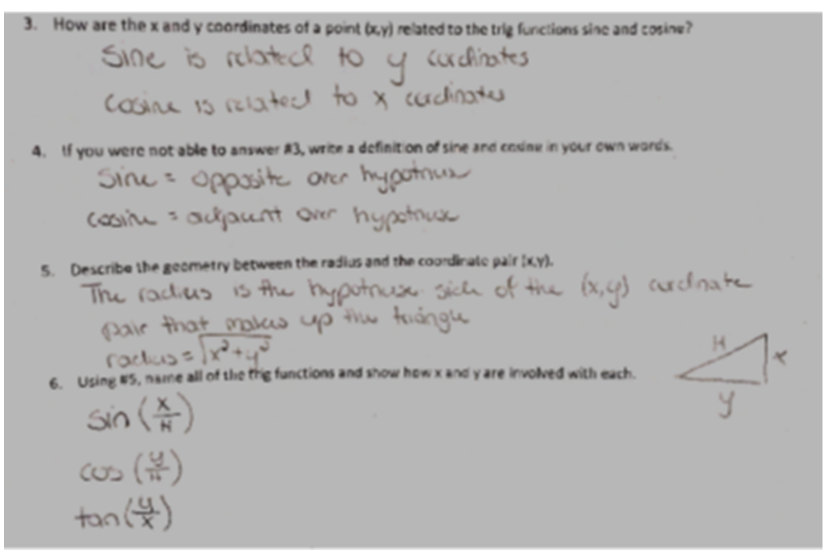
5. Describe the geometry between the radius and the coordinate pair (x,y).

IN THE UNIT CIRCLE, THE RADIUS IS ALWAYS 1

6. Using #5, name all of the trig functions and show how x and y are involved with each.

SIN = $\frac{y}{1}$
COS = $\frac{x}{1}$
TAN = $\frac{y}{x}$
SEC = $\frac{1}{x}$
CSC = $\frac{1}{y}$
COT = $\frac{x}{y}$

[Figure 8]



3. How are the x and y coordinates of a point (x,y) related to the trig functions sine and cosine?

Sine is related to y cordinates
Cosine is related to x cordinates

4. If you were not able to answer #3, write a definition of sine and cosine in your own words.

Sine = opposite over hypotnuse
cosine = adjacent over hypotnuse

5. Describe the geometry between the radius and the coordinate pair (x,y).

The radius is the hypotnuse side of the (x,y) cordinate pair that makes up the triangle
radius = $\sqrt{x^2+y^2}$

6. Using #5, name all of the trig functions and show how x and y are involved with each.

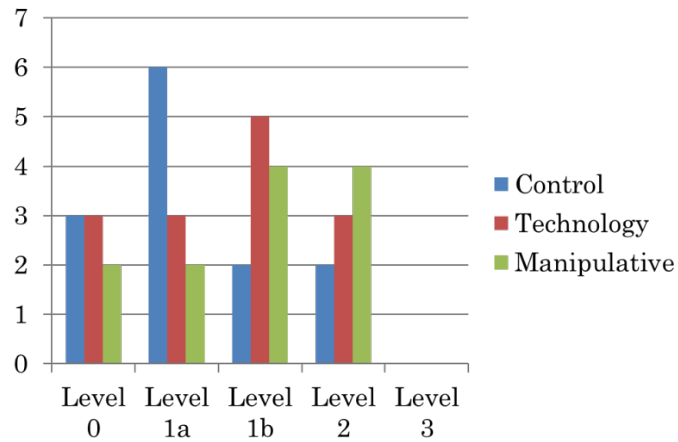
$\sin\left(\frac{x}{H}\right)$
$\cos\left(\frac{y}{H}\right)$
$\tan\left(\frac{y}{x}\right)$

[Figure 9]

The only level that was not seen in this study was level 3. This level would have been reached by students who fulfilled all of the previous levels and could explain their reasoning. There were a few variables that may have kept students from reaching this level. There was a time constraint from this study since we were only able to meet the students one time for an hour. If we had more time to conduct this study we might have been able to see more level 3 students. Some of the questions may have also kept students from reaching level 3. The questions may not have allowed the students to reach the level of thinking that we wanted to see for level 3, and only allowed them to reach level 2. These two variables could play a significant role in the fact that we had no students at level 3, which if we could conduct the study again we could try to keep these variables from being a factor in our results.

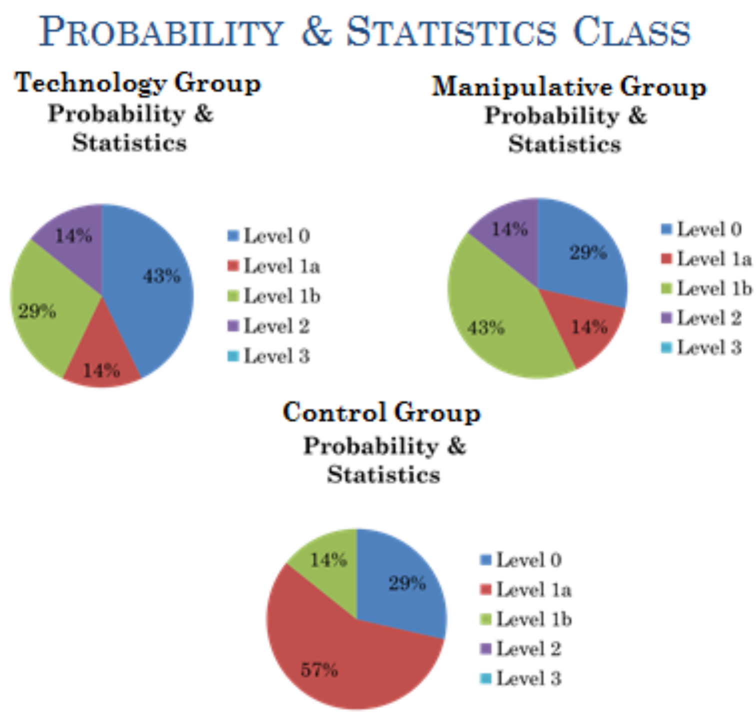
Results

From the results, we were able to place 39 students into a specific level based on their answers to our questions. Some students' worksheets were not able to be placed into a level, because they did not try to answer our questions or their answers were not clear enough for us to place them into a specific level. From these 39 students there were no students at a level 3 understanding in both the Probability & Statistics class and the Calculus II class. As stated above this could be from multiple variables that affected the study. Figure 10 shows the amount of students in each level based on the group they were placed in. This figure is of the 39 students whose worksheet and surveys we were able to use.



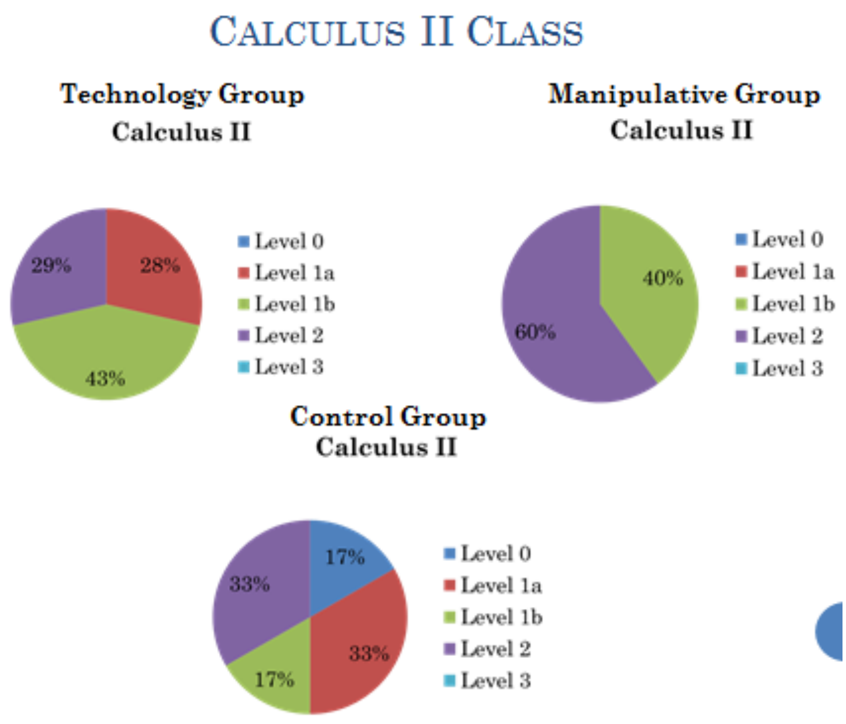
[Figure 10]

From this chart we were able to see how many students from each group were functioning at each level. This is where we began to notice that the control group students were functioning at mostly a level 0 and level 1 (a and b) understanding and less at a level where connections were beginning to be made. The technology group seemed to stay even at each level with the exception of level 1b, and the manipulative group showed an increase in the students understanding. We then decided to look at the classes separately and see how the groups compared to each other within the class. We looked at the Probability & Statistics class first. We first noticed that the control group was all at either a level 0 or a level 1 (a and b) understanding. We saw no level 2 understanding in this group. In the groups with the teaching aids though, we saw an increase in students at a level 2 understanding in both the technology and the manipulative groups. So not only did the level 0 decrease in one case, but the level 1 sections as a whole decreased as well. We did notice however that the amount of students at level 0 increased in the technology group. This was interesting to us, but after we read the surveys that the students filled out at the end we were able to see that many students struggled with the use of technology. They did not understand how to use the technology in a way that would increase their understanding of the unit circle. This showed us that just because a teaching aid is used in the classroom does not necessarily mean that it will benefit a student immediately. It is important for students to understand how to use the aid that they are given so that they can benefit from the aid as much as possible. The important thing for us though was that the use of aids increased the amount of students who were understanding at a higher level. The percentages can be seen in figure 11.



[Figure 11]

Following the Probability & Statistics class, we then looked at the Calculus II class. The control group in this class had all levels of students except level 3. Most of the students in this group were at a level 0 or level 1 (a and b), with some students at a level 2. Then when we looked at the groups with the teaching aids we were able to see more students understanding at higher levels. The technology group had no level 0 students. These students were either at a level 1 or 2. This technology group also seemed to understand how to use the technology aid to their benefit more than the other technology group, which may have led to no level 0 students. When looking at the manipulative group, we were able to see that the majority of students in this group were at a level 2 understanding. All of the students were also at either a level 0 or level 1just like the technology group, but the manipulative group had more level 2 students than the technology group. This progression between the groups and levels was what we were hoping to see and supported our hypothesis. This can be seen in figure 12.



[Figure 12]

Reflection and Conclusion

As a future teacher I feel that reflection is something that should always be done no matter if a lesson goes well or not. A teacher that takes the time to reflect is willing to make changes for their students' benefits and is not afraid to take note if something did not go as planned. With this in mind, I decided to take some time and reflect on this study. I mostly wanted to think about what I could have done better or differently if I conducted this study again. I decided that if I ever was to conduct this study again I would use more time. I would conduct this study over many days. I would then also have every individual student be in each group once. This is because while reading the surveys I noticed that many students did not like the group that they were placed in. Many of the students in the control group said that they would prefer to be placed in a hands-on group because they learn better that way. Other students said that they would have rather been lectured. Other students said that they prefer constant repetition of problems. This was something we expected to happen, but with more time we may have been able to keep this dislike to a minimum. I would have also liked to have made more and sturdier manipulatives for the manipulative group because there were not enough for everyone to have their own manipulative. This was due to lack of materials as well as time. The last thing I would change is some of the questions. As previously talked about, I felt that maybe some of the questions had a ceiling of a level 2 understanding. I might not have worded them well enough for students to understand that I wanted an explanation. Some students might have been at a level 3, but since I did not ask for an explanation we placed them at a level 2. These were the main things that I felt could be changed for further studies, but there could be other changes as well.

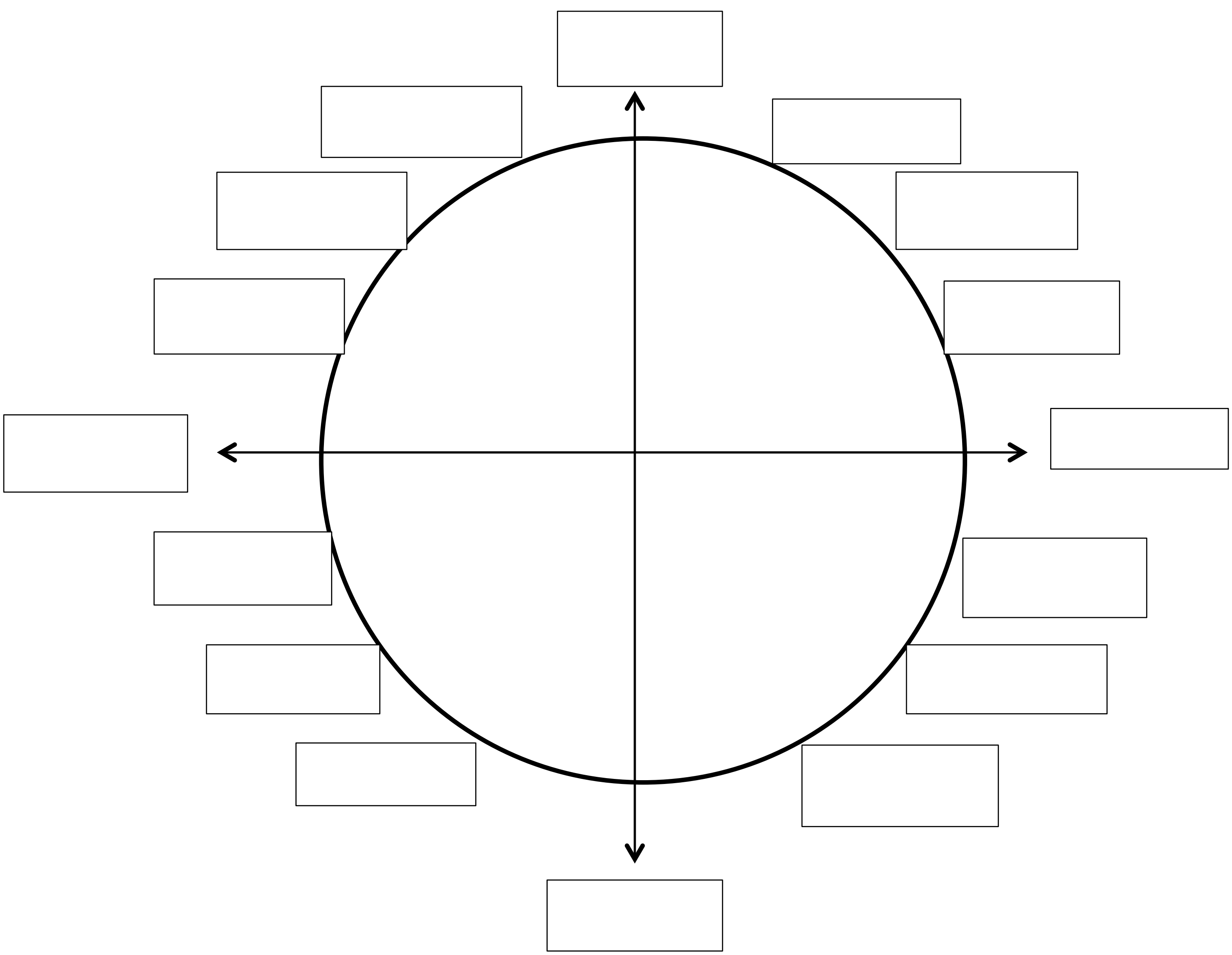
In conclusion we saw that the use of teaching aids increased the level of students' understanding. This is particularly true for the manipulative groups. The aids led to a better understanding of both the algebraic and geometric concepts as well as the connection between the two. With more time, we may have been able to see even more benefits that the aids bring to the classroom. This study has also shown me how important the use of teaching aids can be in the classroom, and because of this, I plan on using teaching aids in my future classroom in a way that will benefit me and my students.

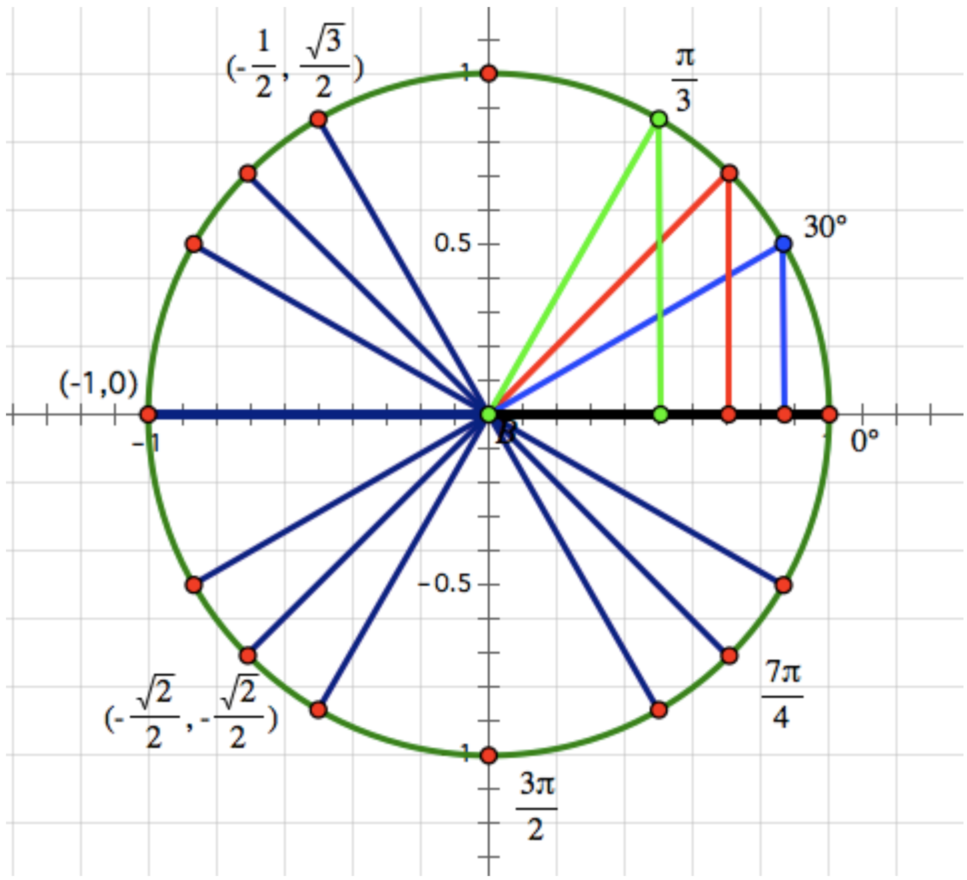
Appendix A

Unit Circle Worksheet and Survey

Directions: On this worksheet, you will recall your knowledge on the Unit Circle. Answer the following questions to your best ability. You can only use what is given on the sheet and what you remember about the unit circle. After completing the worksheet, please answer the follow up questions.

Unit Circle





1. Fill in the each square in the above unit circle with the degree and radian angle measures, and the coordinate pair that corresponds to that angle on the circle.

2. Using the circle, answer the following:
   a. What is the coordinate pair at 45°?
   b. What is the angle measure (in degrees) at π?
   c. What is the radian measure at the point (0,-1)?
   d. What is sin(30°)?
   e. What is cos(225°)?

3. How are the x and y coordinates of a point (x,y) related to the trig functions sine and cosine?

4. If you were not able to answer #3, write a definition of sine and cosine in your own words.

5. Describe the geometry between the radius and the coordinate pair (x,y).

6. Using #5, name all of the trig functions and show how x and y are involved with each.

Survey Questions

1. Would you rather use a hands-on learning activity, a visual learning activity, or a lecture comprehension activity? Which of the above do you feel would be the best method of teaching the unit circle to first time learners? Why?

2. Do you believe that the activity given to your group will allow you to remember the construction of the various elements of the unit circle? Why?

3. Do you believe that the activity given to your group has helped you understand the connections between the algebraic and geometric elements of the unit circle? Why?

4. Do you believe that the activity given to your group is the best way to assess your knowledge of the unit circle? Why?

References

Allen, Crystal. "An Action Based Research Study on How Using Manipulatives Will Increase Students' Achievement in Mathematics." (2007). Print.

Couture, Katie. "Math Manipulatives to Increase 4th Grade Student Achievement." (2012). Print.

Mason, Marguerite. "The Van Hiele Levels of Geometric Understanding." *Professional Handbook for Teachers, GEOMETRY: EXPLORATIONS AND APPLICATIONS*. McDougal Littell. 4. Print.

Yildirim, I. (2013). Use of Technology Assisted Mathematics Education and Alternative Measurement Together. Cukurova University Faculty of Education Journal, 42 (1), 65-73.