Effectiveness of CRA Method Implementation in Secondary and Post-Secondary Mathematics Instruction

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ABSTRACT

The concrete-representational-abstract (CRA) method of teaching mathematics content has shown great success in primary education. The specific sequence of tasks required in this method make for a unique learning experience that has shown many benefits for both students and teachers at lower levels, particularly in special education. This research was designed to evaluate the success of the CRA method in secondary and post-secondary mathematics, specifically Calculus I and Calculus II courses. During a pre-survey, the students' work was evaluated to gauge their content understanding and personal feelings about their own ability to follow material. After a CRA activity and post-survey, the students' responses were then compared to their pre-survey to observe potential improvements in their knowledge and notion of tangency along with their opinion on how well they were able to follow the activity. The goal of this research was to determine the effectiveness and feasibility of implementing the CRA method in upper division mathematics courses.

INTRODUCTION

Throughout their elementary years, many students experience various methods of teaching that aid in their understanding of mathematical concepts. Oftentimes, manipulatives are present, and pictures and graphs appear to help students grasp new ideas. Somewhere along the line these methods have gotten lost. This research is not designed to examine why the use of these methods has dwindled but rather to observe whether their use in upper division mathematics would be productive. Specifically, the data collected will study the effectiveness of a model of teaching known as the concrete-representational-abstract (CRA) method. This method uses manipulatives along with pictorial representations and words and numbers in a specific sequence to introduce and improve understanding of mathematical concepts. This method has shown great productivity in lower level mathematics courses, but our interest falls in the following questions:

- Can the CRA method of teaching content be applicable in secondary and post-secondary mathematics courses?
- What problems could potentially arise while trying to translate the CRA method into upper-level courses?
- During which part of the CRA method will the students be most engaged?
- Will the effects of using the CRA method in upper-level mathematics be as obvious as the effects of its use in lower level courses?
- Will the concrete aspect of the method be necessary at higher levels?
- Is it possible to get students to automatically process new information through the CRA method in order to have better content understanding?

LITERATURE REVIEW

There are criteria for the CRA method to be implemented correctly so that it can be successful. The sequence in which the aspects are applied is very important with respect to the effectiveness of the method. “Teachers ought to start at the concrete level before moving to the representational level and, finally, the abstract level” (Mudaly, 2015).

In an effort to nail down the uncertainties of the CRA method, previous research was examined. The soundest definitions of concrete, representational, and abstract are as follows:

Concrete- *learning through hands-on instruction using actual manipulative objects.*

Representational- *learning through pictorial [type] representations of the previously used manipulative objects during concrete instruction*

Abstract- *learning through abstract notation such as Arabic numbers and operational symbols*

(Witzel, 2008)

Each step of CRA teaching is irreplaceable if the method is to accomplish its goal of ensuring content understanding for the majority of students. The idea behind starting with the concrete level and using manipulatives is that “interactions with concrete objects increase the likelihood that learners would remember stepwise procedures in mathematics, because this allows learners to retrieve information in a variety of ways including visual, auditory, tactile and kinesthetic” (Mudaly, 2015). Differences in learning styles can have massive effects on an individual’s ability to truly know content. The ability to use given tools to visualize a process or occurrence is invaluable in the learning process but is oftentimes left out due to time constrictions or differences in teaching styles.

The next part of CRA, representational style teaching, is equally as important as concrete style teaching. Without having done the concrete aspect first, the student would be missing out on the majority of the tactile learning and real-life applications. Representational learning is used to bridge the gap between the concrete and the abstract. More specifically, “representational understanding is achieved by using an appropriate drawing technique and, finally, appropriate strategies are used to assist learners in moving towards the abstract level of understanding of the concepts and symbols for a particular mathematical idea” (Mudaly, 2015). At the representational level, students will observe a picture, graph, table, etc. that represents the manipulatives used in concrete learning. This will help to show the students another view of the same content.

“Once pictures are drawn without hesitation and with clear explanations of the procedures, then the students can move to abstract notation” (Witzel, 2008). It is important that the abstract learning come last because without the concrete or representational parts the student will not be able to fully grasp the applications and meaning of the content. The abstract portion of the methodology includes words and numbers which are meaningless if not given context that is provided by the concrete and representational portions.

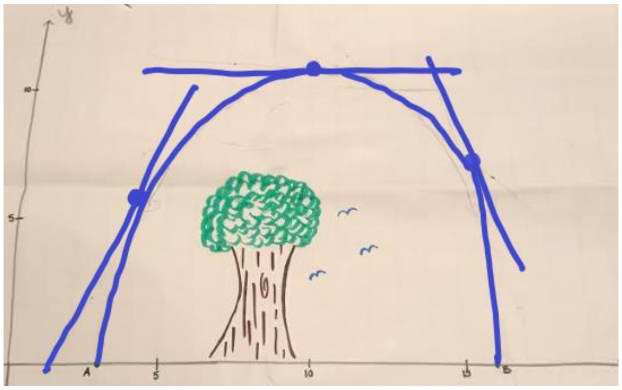
Many students claim that math is too hard or that they just do not understand the subject matter. “Researchers have shown that the use of the CRA sequence of instruction has been very effective and beneficial to learners who struggle with understanding mathematical concepts and procedures” (Mudaly, 2015). This is most likely because of the division of the work into its concrete, representational, and abstract aspects. By splitting the learning up into different styles, the teacher attacks several areas of questioning for the students relating to real life applications, relations to graphs/pictures, and memorized and procedural knowledge. “The use of manipulative objects is not unique to this method. The combination of teacher demonstration, guidance, and student demonstration of mastery over three lessons differentiates this from other methods” (Mudaly, 2015).

There are many potential results that could be yielded should the method be carried out successfully. In one case, researchers found that their “students’ confidence in their mathematics ability appeared to increase after the CRA interventions as demonstrated by (a) teachers’ reports of increased volunteering during math class, (b) an increase in positive comments made during intervention sessions with the researcher, (c) an increase in their willingness to actively participate in sessions with the researcher” (Flores, 2010). The results in this case help to show that the students’ content knowledge drastically improved while using the CRA method. On another note, the CRA method has shown great progress in getting students to take ownership of their learning. The students were able to gain majority understanding of the concept on their own through the activity. “It was apparent in all the classrooms that the master teacher was not the distributor of knowledge, but rather acted as a guide for the educational experience of his/her learners” (Mudaly, 2015). By using the CRA method, “Learners played a more active role in their own learning, and this led to an intrinsic motivation. The learners’ enjoyment, fulfilment and interests were emphasized (Mudaly, 2015). Encouraging students to have inner motivation to learn will help ensure that they understand the content that they are presented. Although it is not possible to guarantee complete understanding of mathematical concepts, “the explicit sequence and multisensory approach of the CRA instructional sequence provides flexibility for implementation across various mathematical concepts” (Witzel, 2008).

METHODOLOGY

The goal of the study is to gauge the success of the CRA method in post-secondary calculus courses. In an effort to gather data about the CRA method's effectiveness, thirty-five Georgia College Calculus I and Calculus II students participated in the study. The students were from various backgrounds, but the majority of them were choosing to pursue a STEM (Science, Technology, Engineering, Mathematics) major throughout their college career. The research began with an instructor handing out the necessary information for the study. Each student filled out their consent form and turned it in. Next, they were asked to complete the pre-survey. In this, they were asked general questions about basic calculus knowledge, conceptualization of tangency and derivatives, their previous mathematics courses, and their previous instructors.

After the pre-survey was completed, the students then moved on to the activity. The methodology behind planning the activity was to give the students experience using the CRA method to see if it helped with their understanding and conceptualization of tangency. Implemented within the activity were the three main criteria for teaching using the CRA method. The first part had students using manipulatives (physical objects) to model the tangent lines of a parabola. The next part, meant to be the representational part of the activity, was for the students to analyze their created graph and pictures. Finally, the abstract aspect of the method was demonstrated when the students were asked to take the derivative of quadratic equations. They were then asked to describe their findings in words. More specifically, the activity contained a large piece of graph paper with either the first quadrant or first and fourth quadrant drawn on it. There were pictures drawn in either quadrant. The students' goal was to construct a parabola using the created manipulative around the pictures from point A to point B on the x axis avoiding the pictures. This would be considered the concrete part of the CRA method. In this case, the students used a small pencil placed inside a chalkboard eraser. At seven points along the parabola the students were instructed to stop at a point and create a tangent line corresponding to that specific point. To do this they used the ends of cardstock that were placed directly in the middle of the eraser.



Once the tangent line was created, they were to record the *x* coordinate and *y* coordinate of the point on the parabola and the slope of the tangent line in a given table. This is considered the Representational part of the CRA method. After the entire parabola was created and all the values were recorded, the students moved on to the next step which is considered the abstract portion of the CRA method. They used a quadratic regression with the *x* coordinates and *y* coordinates for their parabola points to find an approximate equation for the parabola that they drew. Then, they were instructed to take the derivative of that function. In the next step, they completed a linear regression using the x coordinates of the parabola points and the recorded slopes of the tangent lines. They were then asked to compare their linear regression finding with the derivative of their quadratic regression finding. To do this they looked at the equations and at the graphs of both functions. In a perfect scenario these values should have been equal, but the construction of the parabola and the recording of points was an approximation. Once a conclusion had been drawn about the activity the students were instructed to complete a post-survey. The post-survey again questioned their ideas on the concept of tangency and derivatives to see if the activity changed their mindsets. The post-survey was also used to see if the CRA method used in the activity had any effect on the perception of the content for the students. When the students had finished all of their work, their packet was collected, and each student's response was evaluated. Based on their answers, they received a score of zero through four from the rubric shown below. The students' work was then classified based on their deficiencies using the criteria shown below. If a student received a two or lower on a specific question from the surveys, they are considered to be deficient in that specific category. Their pre-survey answers and post-survey answers were compared to gather information on themes seen throughout the students' work to see if the CRA method made a difference on their content knowledge and general understanding of the material.

Rubric for Grading Pre-Survey (Figure 1)

0	1	2	3	4
The student showed no knowledge of content. No work was shown.	The student showed almost no knowledge of the content and no correct work was shown.	The student showed some understanding of the content with little correct work.	The student showed very good understanding of the content. Minimal incorrect work.	The student showed full knowledge of content. All work was correct.

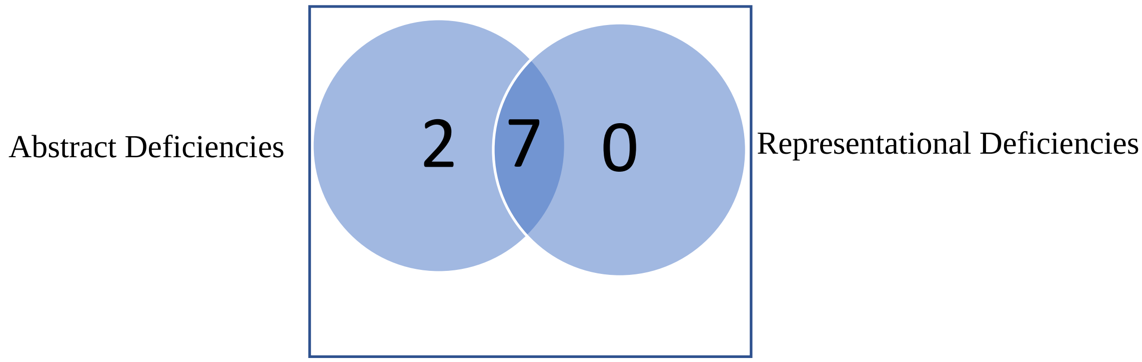
Criteria for Deficiency (Figure 2)

Deficient Using Representational Methods	Deficient Using Abstract Methods
The students were unable to pictorially represent information given to them. They guessed when asked questions about graphs and did not completely know how to find and draw the tangent line to a curve on a graph.	The students were unable to find solutions to problems through abstract methods. The students either guessed or got the majority of the problem incorrect.

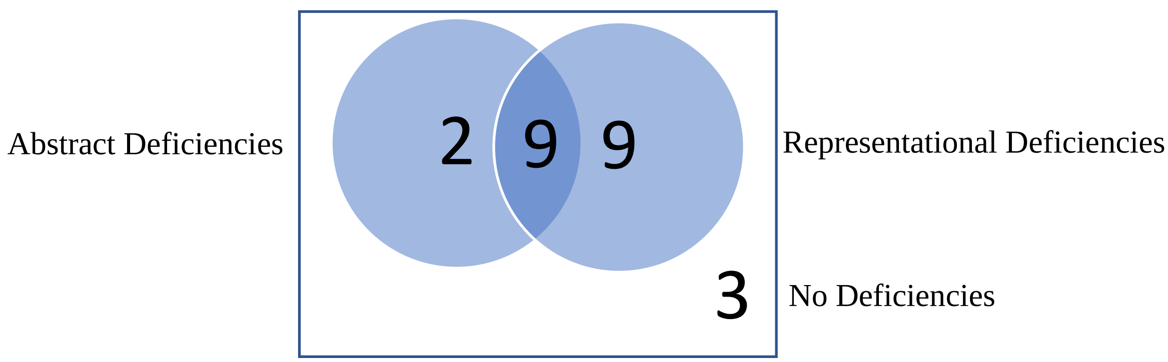
FINDINGS

When evaluating the students' pre-surveys, their work was split up into the different classes they were taking. In the Calculus I class (Figure 1), it was found that seven out of the nine total students were deficient when using both representational and abstract methods. The other two students only showed deficiencies when using abstract methods of learning.

Deficiencies in Calculus I Students (Figure 3)



Deficiencies in Calculus II Students (Figure 4)



In the Calculus II courses (Figure 4), 9 out of the 23 students were classified as deficient when using both representational and abstract means of learning. The same number of students (9 of 23) were classified as solely representationally deficient while only two out of the 23 were found to be deficient in using abstract methods. Three students showed no deficiencies in their pre-surveys. It is important to note that although a student may be considered to be deficient in one or more of these categories, they might not fall under these classifications for different content. In this case, the students were asked to find derivatives given a function and to graph a curve and its tangent line at a specific point. For example, should the same students be asked to solve an equation or find the area of a circle, they may have different answers, and thus, they would fall under different categories.

Many of the students felt that they had experienced learning in a similar sequence but at much younger ages. This would be plausible considering there is much evidence about the effectiveness of the CRA method in lower-level mathematics courses. One student said that the activity was “a hands-on example problem” and that they had experienced similar methods in “elementary school but not upper level classes”. When asked if the students had seen this type of teaching, another student answered, “Yes, mostly in lower level schooling, only a few times in high school”. Many pieces of evidence suggest great success of the CRA method in lower-level courses; however, our findings suggest the potential for success in upper-level courses should the method be more regularly implemented.

A few discrepancies came about when working through the concrete and representational aspects of the activity. One student said that the activity was “more hands-on”, but that “there was more room for error”. This same student also suggested that the potential for errors could make the activity and its applications “more interesting”. Similarly, another student said that the activity was “subject to error” because of its hands-on nature. This aspect of the method can be appreciated because although there can be small variations in data collection, the data is more lifelike and relates back to real-world applications better than strictly using abstract, theoretical methods.

Some students commented on the specific concrete method used. In this activity, students were asked to construct a parabola given a chalkboard eraser with a pencil through the middle and cardstock extensions to draw the tangent lines (refer to picture). Regarding this subject, the feedback was mostly hit or miss. One student said that the activity was “hands on but made it more challenging to focus on what was actually going on”. This is a respectable answer as the concrete part of the activity could have been thought out more. With regards to improving the activity, instead of using an eraser with a pencil, students could use a device that moves smoother and is easier to see. The directions could have been clearer when directing students to use the manipulatives given. Another student claimed that it was a “bad activity” saying that the students’ success and accuracy was “based on perception of the problem and how one uses the tools”. Again, this relates back to the potential for error. If a student is unable to use the manipulatives correctly, their success can possibly be hindered or altogether prohibited. The tools used in this research could have been reevaluated to be more efficient although the majority of the students did not have major issues using the given manipulatives. Another small change that could have been made was giving the students more time to properly synthesize the whole activity.

When asked if the students’ definitions of tangent had changed, a few replied that their definitions had not but only because they were fully familiar with the concept. They did however provide relevant data when it came to the effectiveness of the activity. One student said that the activity was “an excellent refresher” even though they “already knew how [tangent lines] work”. Another student said they had “forgotten some of the steps to create the tangent line” and again that the activity was a “good refresher”. Even if the students are already completely familiar with the content or had been at one time, this method is efficient in ensuring that no aspect gets left out. This activity helped one student to “remember vocab words previously forgotten”. The concrete aspect is important in this case because it helps students apply mathematical ideas to a manipulative that will hopefully help them to remember the content for a longer period of time. Some students commented that the activity would be a “great activity to get a better understanding for those who are unaware of the meaning [of tangent line]” and that the activity provided a “swell example” of the use of tangent lines and derivatives. One student claimed that the concrete aspect “seemed unnecessary”. Should a student already have full comprehension of the subject at its abstract level, the concrete aspect could potentially seem unneeded, but to a student who is not yet fully versed on the subject, using the manipulatives can help them begin to see the mathematics on a grander scale.

Students who did not have a full grasp of the content before the activity had some useful feedback as well. One student said that the “tools made it easier to visualize the line”. That means that they found value in the concrete aspect of the activity and that it helped them to have a better understanding of tangency. This supports CRA because you must start with concrete learning to lay a foundation for further understanding. Another student said that their definition of tangent changed to say “[tangent lines] provide representation of the graph as it was drawn, not after” which shows that the student understands the instantaneous part of tangency. Their new definition is closely related to the concrete and representational parts of the activity. Many students claimed that by using tracing and the graph, they were able to better visualize tangent lines, and realize where they come from. All of this speaks to the productivity achieved through the CRA method.

The activity performed in the three classrooms was to help ensure the understanding and conceptualization of tangent lines as they relate to derivatives. The intent was not to teach a new idea or to convert students' thinking to become strictly abstract. In a few cases, there was success shown between the students' perceptions in the pre-survey and their thoughts in the post-survey. Before the activity, one student claimed that a tangent line was "a line that intersects a curve at one point". Afterwards this same student said their definition of tangent line changed to "slope of the graph at that instant". Although the derivative function is not the slope of the line but rather a way to find the slope of tangent lines at specific points, the student showed increased knowledge about the instantaneous nature of tangent lines and their connections to slope. Another student said in the pre-survey that a tangent line was "a line that passes through a single point of a function". While this is true, this is only one part of the criteria for tangent lines. After the activity the student said that their definition of tangency changed to "representation of the graph as it was drawn, not after". Although this definition does not make much sense, it also speaks to the rate of change of functions. A third student said before the activity that the definition of tangent line was "the line that sticks to one point of the function" while afterwards they claimed tangent lines were "closely related to the slope of the function" and that "the tangent line [for] (a, f(a)) is the slope of point a, instantaneous rate of change)". While it is evident that none of these three students grasp the idea completely, it is obvious that the activity helped them explore the relationship between tangent lines and derivatives more extensively.

Many students claimed in the pre-survey that they had struggled in general when it came to following material in their previous mathematics courses. For this specific activity, numerous students said that they felt they were able to follow the material better than they previously had in their lower level classes. One potential cause of this result would be the use of manipulatives and representations in the activity. Although this might not be the only contributing factor to their improvement (consider specific material and previous instructor methods), it would be wrong to completely omit the idea that the specific sequence of steps in the CRA method helped in their ability to follow material. A chart was created that shows students confidence in the CRA activity vs. their previous general confidence in following mathematical material (Figure 5).

Following the CRA Activity (Figure 5)

In General	Very Bad	Somewhat Bad	Neutral	Somewhat Good	Very Good
Very Good		|||	|		|
Somewhat Good	|	|	|	|||||||||||| (12)	||
Neutral			|	||	|
Somewhat Bad			||	|||	
Very Bad				|	

CONCLUSIONS AND APPLICATIONS

Although no extensive conclusions can be made about the CRA method in calculus classrooms, there are small bits of evidence that suggest modest enhancements of understanding due to this specific activity. Some of the students' ability to recognize the connection between derivatives and tangent lines improved as well as their ability to follow a sequence of steps (CRA). Some students felt that they were able to understand the process of finding tangent lines better and that the CRA method helped them understand the concept better. In moving forward, there are many potential positive outcomes that could result from more intentional use of the CRA method.

Should teachers choose to implement the CRA method in their classrooms, they may help their students in more ways than one. The students could benefit from seeing different types of activities and the relationships between them. The students would get hands on work with manipulatives, along with representations, and abstractions to benefit students that learn in different ways. It is also easy to implement collaborative group work into the CRA method. Teachers could spread out the method over different class periods or even different days to ensure that the students have full understanding of one part before moving on to another. If well planned and executed carefully, teachers will be able to use the CRA method of instruction at many different times throughout their courses. From this research it is quite possible that CRA can have some capacity of success at any level of mathematics should it be implemented correctly.

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APPENDIX A

CRA IN CALCULUS COURSES: PRE-SURVEY

In this survey, you will be asked questions pertaining to your general calculus knowledge and background, general mathematics classroom experience, and the method of teaching mathematics that is to be researched.

1. Which of the lines in the following picture are tangent to the given curve, f(x)? (Circle all that apply.)

a.

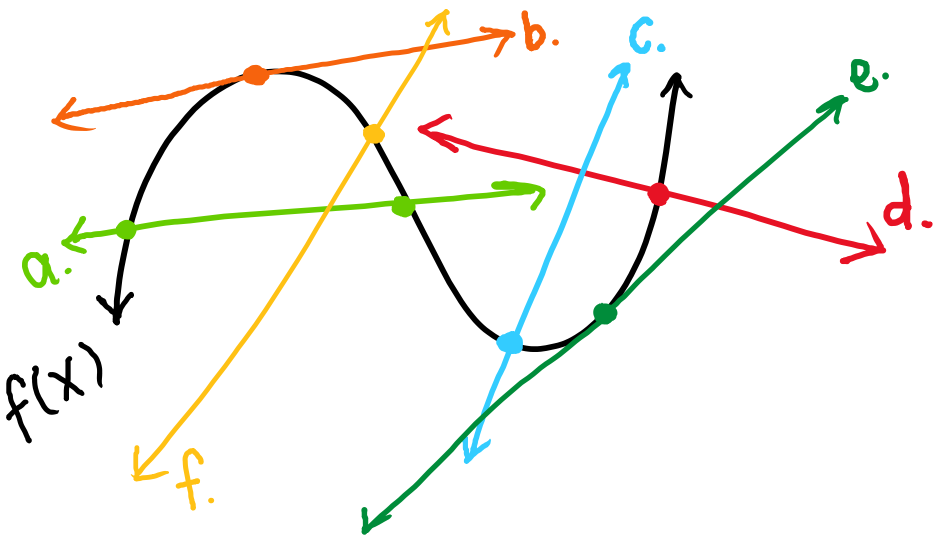
b.

c.

d.

e.

f.



2. After choosing your tangent line(s) from above, write in your own words a definition for tangent line.

3. Do you know how to find the derivative function, f '(x), for the function $f(x)=12x^4+6x^2-13$? If so, find it and show your work.

4. How about finding the derivative, g'(x), for the function g(x)=3cos(x)-4sin(2x)? If so, find it and show your work.

5. Throughout the majority of my mathematics courses I would rate my ability to follow material in class as:

a. Very Good
b. Somewhat Good
c. Neutral
d. Somewhat Bad
e. Very Bad

6. Throughout the majority of my mathematics courses I would rate my ability to do independent work at home as:

a. Very Good
b. Somewhat Good
c. Neutral
d. Somewhat Bad
e. Very Bad

7. Choose true or false for the following questions:

T/F My teachers have generally used more than one approach to teaching their class. (e.g. lecture, activity, worksheets, etc.)

T/F I don't need to apply math to real life to understand the concepts.

8. For the following questions, consider a previous mathematics course where your experience was the most positive and your learning was at its highest.

a. How often did you explore mathematics using manipulatives and mathematical tools (physical objects that can be used to illustrate and discover mathematical concepts, e.g. string, ramp, wheels, etc.)?

Always
Occasionally
Rarely
Never

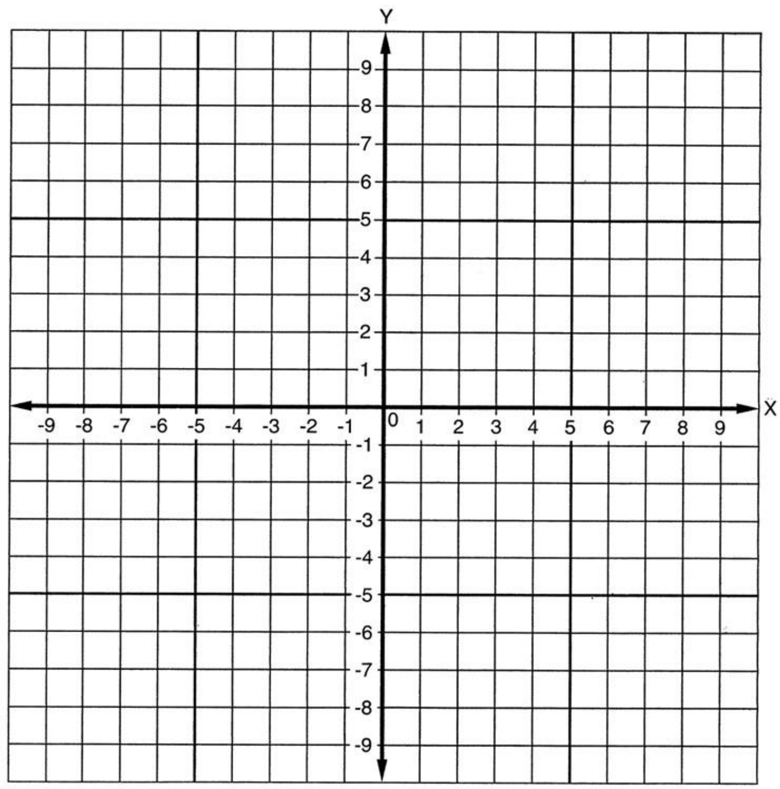
b. How often did you explore mathematics using visual representations such as graphs, pictures, animations, etc.?

Always
Occasionally
Rarely
Never

9. Which of these represents a quadratic relationship?

a. $y=x^2$
b. parabolic graph
c. sideways parabolic graph
d. side length of a square to the area of a square
e. triangle numbers: {0, 1, 3, 6, 10, 15…}
f. population growth

10. Construct the curve $f(x)=x^2+1$, and then construct the tangent line when x=1. Do NOT write any of your calculation work down on the paper.



11. What equation did you find for the tangent line? What steps did you take to find this?

APPENDIX B

CRA IN CALCULUS COURSES: ACTIVITY

Directions for Activity:

1) Take the eraser with the small pencil in it and begin to draw a parabolic path from A to B either above or below the obstacles (Read Step 2 before starting).

2) At seven points while tracing, stop the eraser and record the coordinate pair where the pencil tip lies. Use the paper pieces as endpoints for a tangent line, and write down the slope of the curve at the respective points. Record them in the data table below.

x-value	**y-value**	**slope of the tangent line**

3) Calculate the approximate equation of your parabola by inputting your x and y values into the calculator and calculating a quadratic regression (directions on next page). Record your equation below the graph.

4) Find the derivative equation of the function found in step 3 and write it below.

5) Calculate the approximate equation of the derivative by inputting the x coordinates and related tangent line slopes into the calculator and calculating a linear regression (directions on next page).

6) Graph both equations found in steps 4 and 5 at the same time in your calculator. What is similar about the lines? What is different?

7) Pick an x value that you recorded above and substitute it into your equations for steps 4 and 5. What is the difference? Were you close?

How to Use the Calculator in Step 3

1) Press STAT.
2) Underneath the EDIT column choose Edit (option 1)
3) Make sure your L_1, L_2, and L_3 columns are empty by going over the header and pressing CLEAR.
4) Put all your x values in the L_1 column and all of your y values in the L_2 column.
5) Once all of your values are in the lists, push STAT again.
6) Go to the CALC tab and choose QuadReg (option 5).
7) Put your L_1 for your Xlist and L_2 for your Ylist.
8) Press Calculate.
9) Write your equation based on the formula $y=ax^2+bx+c$ with the values found.

How to Use the Calculator in Step 5

1) Press STAT.
2) Underneath the EDIT column choose Edit (option 1)
3) Make sure all of your x values are still in the L_1 column, and all put all of your **SLOPES** in the L_3 column.
4) Once all of your values are in the lists, push STAT again.
5) Go to the CALC tab and choose **LinReg (ax+b) (option 4)**
6) Put your L_1 for your Xlist and L_3 for your Ylist.
7) Press Calculate.
8) Write your equation based on the formula $y'=ax+b$ with the values found.

If your calculator is NOT updated…

Once you press the specific regression button you will have to enter the lists manually. You must put the corresponding lists in parentheses separated by a comma. For instance, in step three you will have to input **(L_1,L_2)** and then press enter.

APPENDIX C

CRA IN CALCULUS COURSES: POST-SURVEY

1) In the task you were asked to construct the path of a moving object. In what ways did the activity help your connections to the idea of tangency? After completing the activity, has your definition of tangent line changed from the pre-survey? If so, how?

2) How do you feel that this activity differs from previous teaching methods you have experienced in prior classes? Have you experienced activities like this in your previous math classes? If so, which classes?

3) What value did you find in tracing the curve with the eraser? Why or why not might the tracing task be a good example of developing the idea of tangency? Had you not drawn the parabolic curve, would you be able to find the slope of the tangent lines using only coordinate pairs?

4) Why do you believe or not believe that it is imperative to understand where the information on a graph or in a table came from or how it can be inferred from something you construct or observe?

5) What is the purpose of putting an x-value into the derivative function? What does it mean when the value of the derivative function changes?

6) How does putting a specific x-value into the derivative function compare to putting that same x-value into the original function?

7) This activity helped me understand the material ____________ than strictly learning new material by lecture.

a. Much Better
b. Somewhat Better
c. The Same
d. Somewhat Worse
e. Much Worse

8) I would rate my ability to follow the steps in the completed activity as:

f. Very Good
g. Somewhat Good
h. Neutral
i. Somewhat Bad
j. Very Bad