Algebraic Understanding of College Algebra Students through Story Problems

Tricia Swift

Georgia College

Senior Capstone Fall 2013

Introduction

Teachers have been using story problems in mathematics education for a long period of time. They are a great way to engage and facilitate learning within a classroom. Story problems allow students to pull from their previous knowledge to formulate new mathematical ideas. This article will test college level students to try and discover where the difficulty lies. Past research shows “that relatively advanced students can experience serious difficulties in symbolizing certain meaningful relationships with algebraic equations” (Clement 16). We want to figure out why this is the case and how as teachers we can help students better understand mathematics through story problems. Other researchers have found that there are many misconceptions that students encounter when working with story problems or any critical thinking problem in math. One researcher assumed that “college students could translate between English and algebra, at least in simple situations” (Clement, Lochhead, & Monk 228). However, after discovering the difficulties, they found “students are rarely asked to construct a formula or to interpret one in a significant way” (Clement, Lochhead, & Monk 288). The researchers found that many students struggled with the reversal error where students struggle to translate a problem usually expressed in words into algebraic notation and retranslate a solution back into words.

What makes teaching (and learning) of these translation skills so difficult is that behind them there are many unarticulated mental processes that guide one in constructing a new equation on paper ((J. Clement)289). Research has noted a tendency that some students treat numerical variables as objects instead of numbers. The fact that students “cannot formulate or read such simple equations leads to concern about the extent to which students understand how equations are used to symbolize meanings” (Clement 29). These are a few misconceptions previously found and we hope to see if this is still the case or if new difficulties arise.

Through the math education program at Georgia College, we have been given many opportunities to work with high school students. We have noticed the many challenges students encounter when faced with the intricacies of mathematics. However, the one thing that many students seem to struggle with is word problems. In this study, we surveyed college algebra students giving them two different problems. In one problem they are asked to create their own story problem using an algebraic equation. For the second problem they are asked to analyze a story problem through a series of questions. Through such survey questions, we aim to better understand where students are struggling with algebraic reasoning. Prior to the study, we predicted the students would begin with a basic knowledge of what each variable represents. Their struggle would lie within their ability to create a story problem independently. Specifically, we foresaw students encountering great difficulty over how to understand and write a rule of a story problem.

Survey

The intent of this paper is to determine how well college algebra students at Georgia College understand how to analyze and work with story problems. We went to the Math Emporium where the college algebra students work and asked for volunteers who were willing to take the survey we had prepared. There were 28 students who participated in this survey. On average each student spent 5 minutes working on the first problem. On the first problem, students were given an equation with one variable where they were asked to write their own story problem using the given equation. They were then asked a few questions where they had to analyze what each variable or number represents in their story problem. In the second problem, students spent an average of 10 minutes. Students were given a story problem where they were asked to find a rule or expression that represented the sequence that they interpreted from the story problem. They were then asked a few questions where they had to rewrite the rule or expression they found when given different numbers for the original story problem. To view the full survey, go to Appendix A in the back.

We scored both pages of the survey using two different rubrics. The first page of the survey where the students were asked to write their own story problem, students were given a score of 0 to 4 where 0 was the lowest score and 4 was the highest. We took off points based on the common mistakes we saw throughout the first page.

No Story Action	“No story action errors are errors in which the student did not provide a story context for some component of the given equation” (Alibali et al, 2009).	-2 points
Added/Missing Mathematical Content	“Added/Missing mathematical content errors are errors in which students included/failed to include some of the mathematical content from the given equation in his or her story” (Alibali et al, 2009).	-1 point
Variable Errors	Variable errors are errors that involve responses indicating a lack of complete understanding for how variable changes affect the story problem context.	-1 point

[Figure 1]

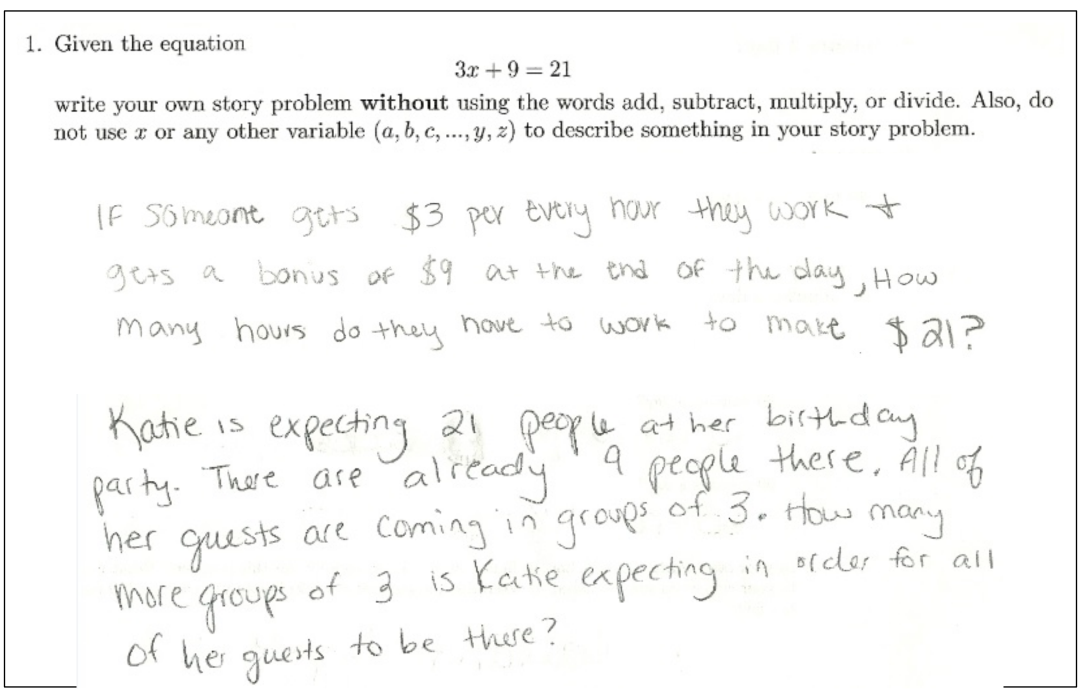
The second page where students were given the story problem and asked to describe a rule or expression, students were given a score of PA to D where PA was the lowest score and D was the highest score. We scored the second page based on their levels of understanding on how to conjecture a rule.

PA	No significant demonstration of algebra
A	“Systematically searches for a rule” (Driscoll, 2001).
B	“Attempts to describe a rule” (Driscoll, 2001).
C	“Conjectures a generalized rule” (Driscoll, 2001).
D	Conjectures correct rule

[Figure 2]

The two pages of the survey are in a sense complementary and attempt to describe a full circle of understanding. The first page is more conceptual: “conceptual knowledge of mathematics consists of logical relationships constructed internally and existing in the mind as a part of a network of ideas” (Van De Walle, 2004). The second page is more procedural: “procedural knowledge of mathematics is knowledge of the rules and the procedures that one uses in carrying out routine mathematical tasks and also the symbolism used to represent mathematics” (Van De Walle, 2004). However, neither page is exclusively conceptual or procedural.

Results



1. Given the equation

$$3x + 9 = 21$$

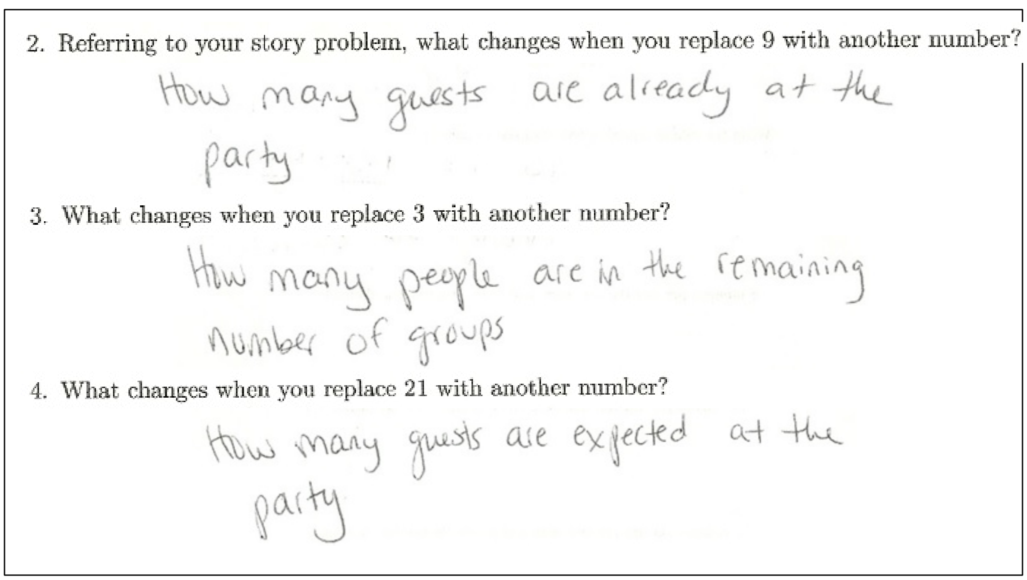
write your own story problem **without** using the words add, subtract, multiply, or divide. Also, do not use x or any other variable $(a, b, c, ..., y, z)$ to describe something in your story problem.

IF someone gets $3 per every hour they work + gets a bonus of $9 at the end of the day, How many hours do they have to work to make $21?

Katie is expecting 21 people at her birthday party. There are already 9 people there. All of her guests are coming in groups of 3. How many more groups of 3 is Katie expecting in order for all of her guests to be there?

[Figure 3]

After surveying the students, we read through each survey and gave each student a score according to the rubrics stated above in figures 1 & 2. We saw a range of students that scored between 0 and 4 on the first page. Figure 3 shows two students' story problems from the first page that clearly represented the equation given to them in number 1. Both students were able to assign each number to a variable and put the equation into a story context with a question that asks for the x-variable. Figure 4 shows a student's work on the three follow-up questions about what happens to their variables in their story problem when each individual number in the equation changes. Students who were able to give a story problem that correctly represented the equation and were able to answer how their story problem would change when given different numbers in the equation, were given a score of 4 on the first page of the survey.



2. Referring to your story problem, what changes when you replace 9 with another number?

How many guests are already at the party

3. What changes when you replace 3 with another number?

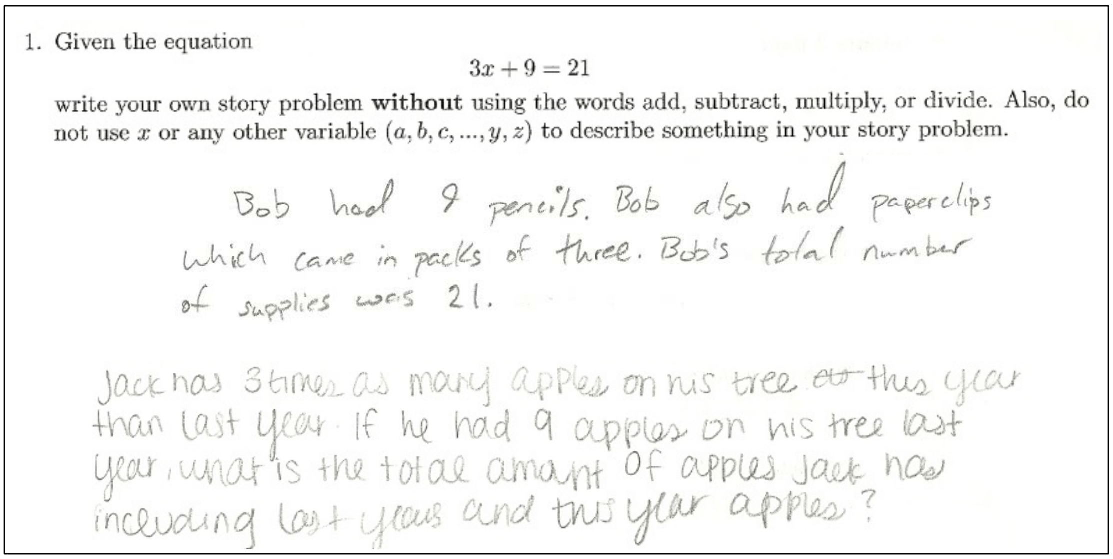
How many people are in the remaining number of groups

4. What changes when you replace 21 with another number?

How many guests are expected at the party

[Figure 4]

We took one point off on the first page for students who had any added or missing mathematical content. Figure 5 is an example of two students' who were both missing mathematical content. The first example is a student who was able to write a story problem but this student did not mention the variable x in their problem. The only thing the student needed to add was the question: “How many packs of paperclips does Bob have?” Had the student included that question, they would have received full credit for the problem. Likewise, the second student had no mention of the total 21 anywhere in their story problem. If the second student had mentioned 21, they also would have received full credit for the problem. Students that made these similar mistakes also had one point taken off.


1. Given the equation

$$3x + 9 = 21$$

write your own story problem **without** using the words add, subtract, multiply, or divide. Also, do not use x or any other variable $(a, b, c, ..., y, z)$ to describe something in your story problem.

Bob had 9 pencils. Bob also had paperclips which came in packs of three. Bob's total number of supplies was 21.

Jack has 3 times as many apples on his tree ~~at~~ this year than last year. If he had 9 apples on his tree last year, what is the total amount of apples Jack has including last years and this year apples?

[Figure 5]

We took two points off when a student had no story action to represent the given equation on the first page. Figure 6 is an example of two students who were unable to provide any story action. Both students ended up rewriting the given equation in words when the directions clearly stated to not use the words add, subtract, multiply, or divide. They both dodged the directions and found a way to write the equation using different words than the ones stated. The students in figure 5 were at least able to write a story problem whether they had all the correct information or not, but the students in figure 6 show no attempt of any kind of story problem. Any students that had work similar to the students in figure 6 also had two points taken off for no story action.

1. Given the equation

$$3x + 9 = 21$$

write your own story problem **without** using the words add, subtract, multiply, or divide. Also, do not use x or any other variable $(a, b, c, ..., y, z)$ to describe something in your story problem.

A number times 3, plus 9
equals 21.
some number of people, tripled in amount,
plus 9 more people gives you
21 people altogether

[Figure 6]

We also took one point off on the first page for any variable errors. Figure 7 is an example of a student who lacked understanding of how each variable was changing and how it affected their story problem. Instead of the student changing the specified variable, they changed the total each time. Our hope was that students would not only be able to write a story problem for the given equation but that they would be able to rewrite their story problem when given different variables. Questions 2, 3 and 4 were meant for us to be able to see if students could take their story problem to the next level and understand how each of the variables they created would change when given a different equation. Figure 8 shows all the different scores that the students received on the first page.

2. Referring to your story problem, what changes when you replace 9 with another number?

$3x + 9 = 9$

It changes the outcome answer

3. What changes when you replace 3 with another number?

$3x + 9 = 3$

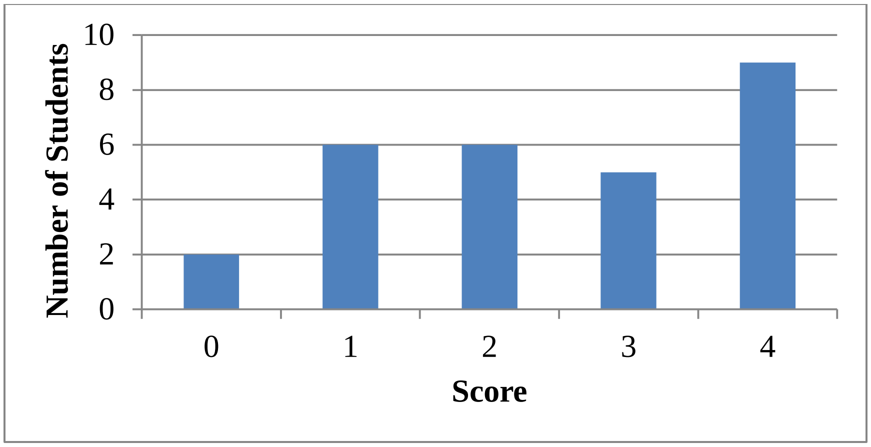
it changes the outcome answer

4. What changes when you replace 21 with another number?

$3x + 9 = 21$

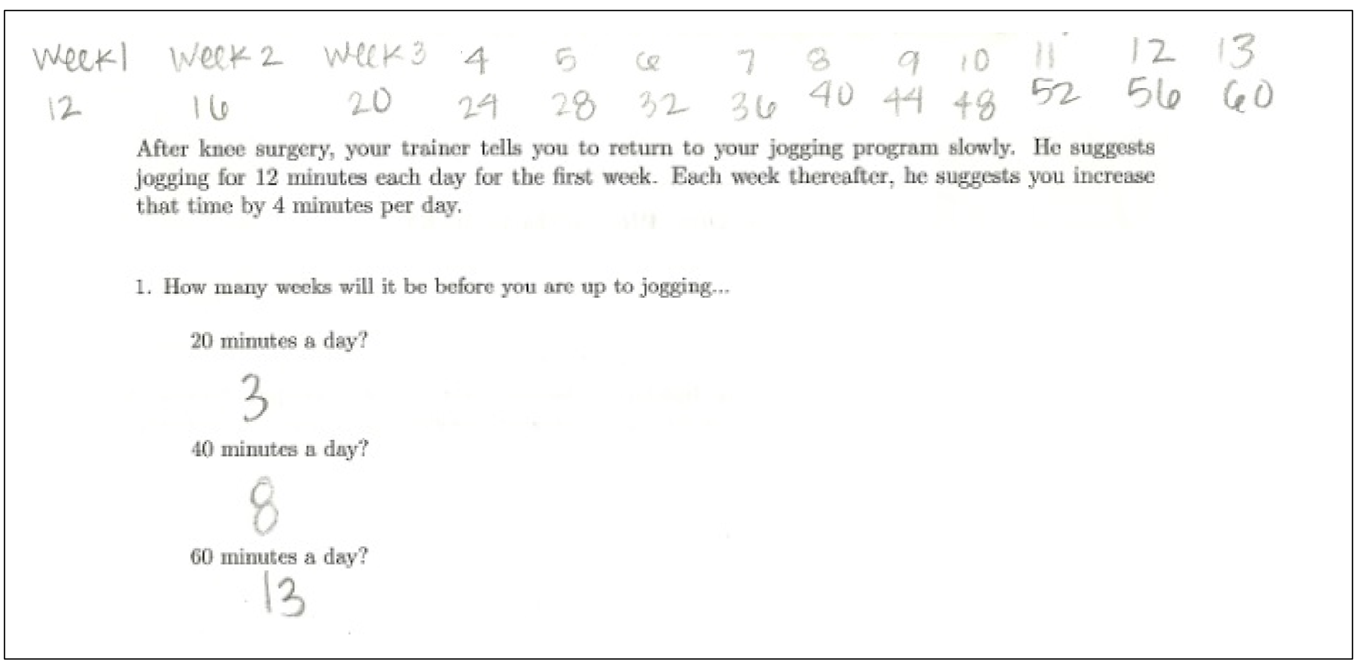
It changes the outcome answer

[Figure 7]



[Figure 8]

On the second page, we gave students a score that ranged from PA to D, where PA was the lowest score and D was the highest. Figures 9 & 10 show a student who scored a D for being able to conjecture a correct rule. The way we determined this was by looking at questions 3 and 4. This student was able to find a correct rule or expression in number 3 that represented the sequence that they found in the given story problem. In number 4, we asked the students how their rule or expression would change if the variables in the story problem were to change. If a student was able to answer both 3 and 4 correctly, then we assumed that they had a higher level of understanding and therefore received a D.



Week1	Week 2	Week 3	4	5	6	7	8	9	10	11	12	13
12	16	20	24	28	32	36	40	44	48	52	56	60

After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he suggests you increase that time by 4 minutes per day.

1. How many weeks will it be before you are up to jogging...

20 minutes a day?

3

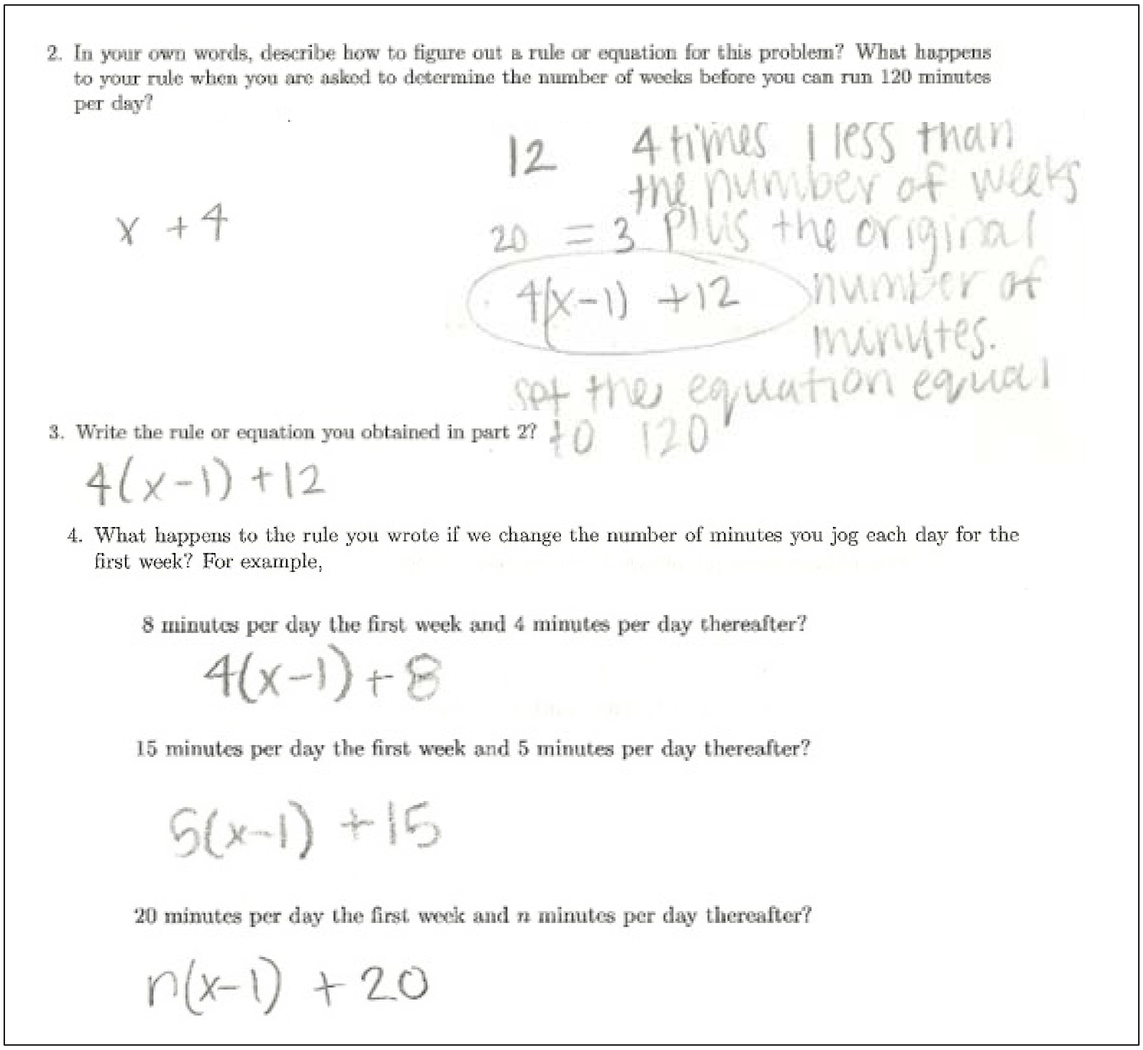
40 minutes a day?

8

60 minutes a day?

13

[Figure 9]



2. In your own words, describe how to figure out a rule or equation for this problem? What happens to your rule when you are asked to determine the number of weeks before you can run 120 minutes per day?

$x + 4$

12

20 = 3

$4(x-1) + 12$

4 times 1 less than the number of weeks plus the original number of minutes.

set the equation equal to 120

3. Write the rule or equation you obtained in part 2?

$4(x-1) + 12$

4. What happens to the rule you wrote if we change the number of minutes you jog each day for the first week? For example,

8 minutes per day the first week and 4 minutes per day thereafter?

$4(x-1) + 8$

15 minutes per day the first week and 5 minutes per day thereafter?

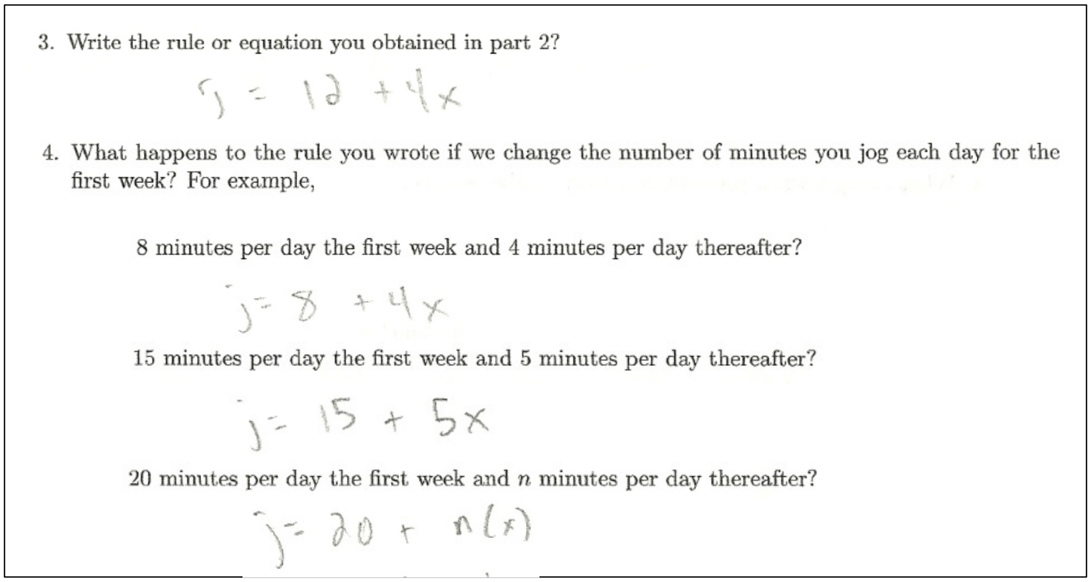
$5(x-1) + 15$

20 minutes per day the first week and n minutes per day thereafter?

$n(x-1) + 20$

[Figure 10]

A student scored a C if they were able to conjecture a generalized rule. If a student wrote a rule or expression that did not represent the sequence they found in the story problem but they were able to understand how their expression changed in number 4, then they were given a C. For example in figure 11, the student understood and was able to write a rule or expression; however the rule did not have to be correct. The fact that a student was able to still understand how the rule they found changed when given different variables is what allowed a student to score a C.



3. Write the rule or equation you obtained in part 2?

$y = 12 + 4x$

4. What happens to the rule you wrote if we change the number of minutes you jog each day for the first week? For example,

8 minutes per day the first week and 4 minutes per day thereafter?

$y = 8 + 4x$

15 minutes per day the first week and 5 minutes per day thereafter?

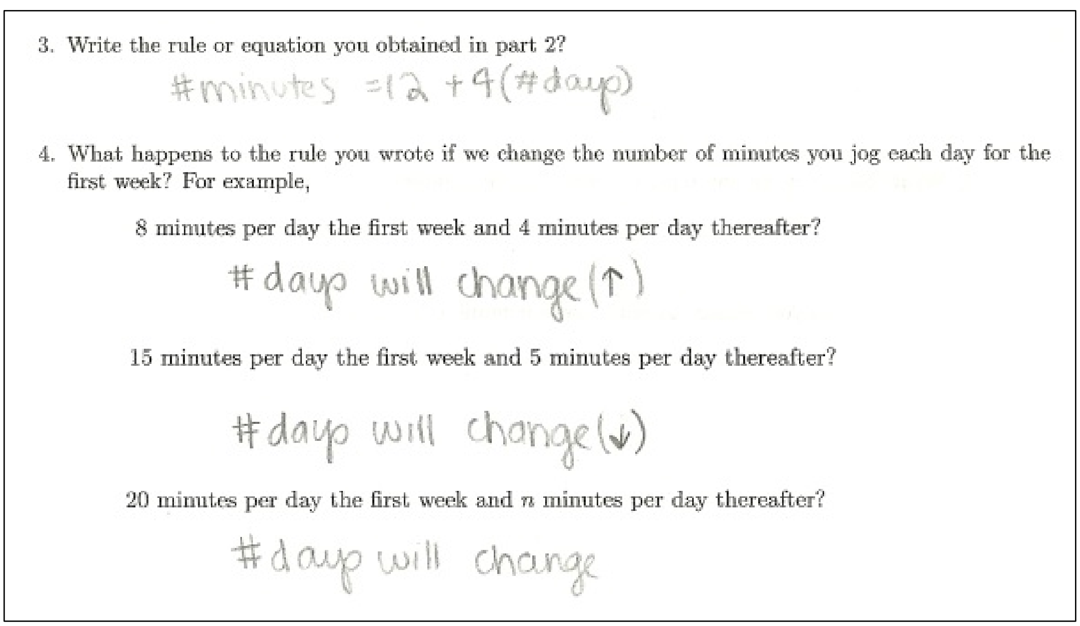
$y = 15 + 5x$

20 minutes per day the first week and n minutes per day thereafter?

$y = 20 + n(x)$

[Figure 11]

Students received a B for attempting to describe a rule. Figure 12 shows a student who was able to write a rule that did not correctly represent the sequence they found. The difference between this student and students that scored a C lies in their number 4 response. They were able to see something is changing in the expression they found, but they lacked some understanding as to what exactly was changing. Students who displayed a lack of understanding on number 4 but were able to write an expression on 3, received a B.



3. Write the rule or equation you obtained in part 2?

#minutes = 12 + 4(#days)

4. What happens to the rule you wrote if we change the number of minutes you jog each day for the first week? For example,

8 minutes per day the first week and 4 minutes per day thereafter?

#days will change (↑)

15 minutes per day the first week and 5 minutes per day thereafter?

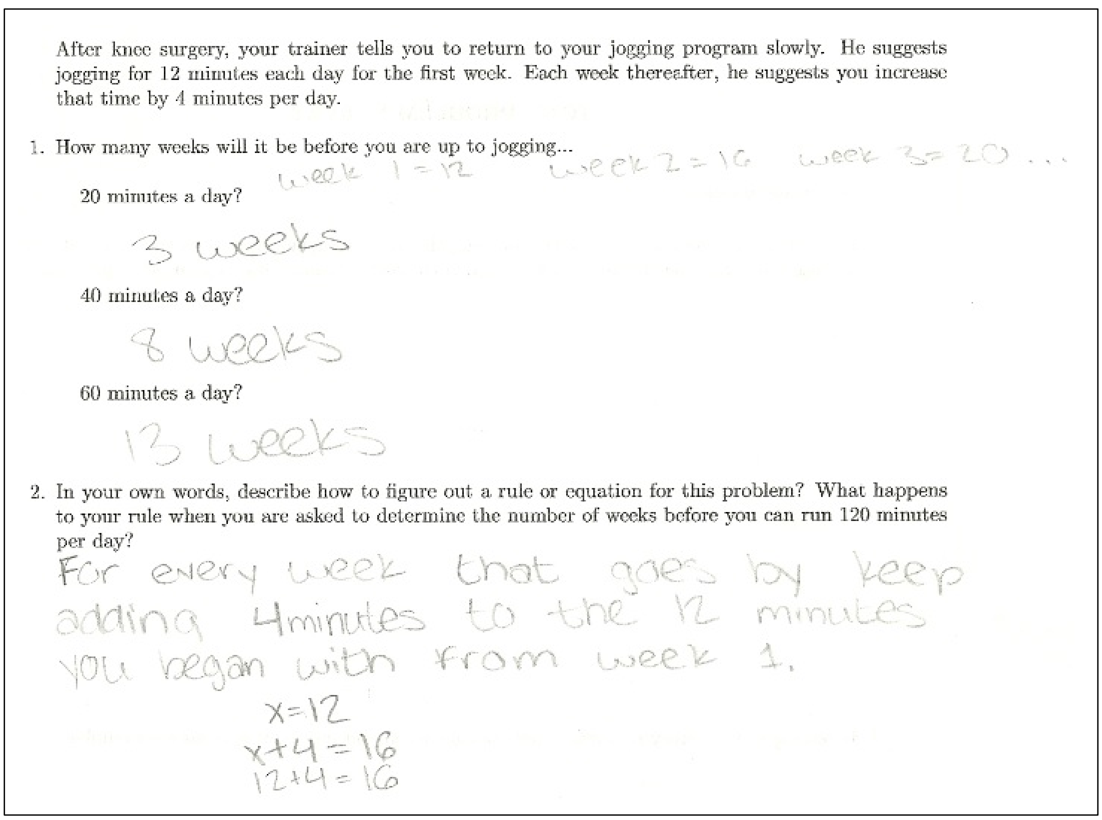
#days will change (↓)

20 minutes per day the first week and n minutes per day thereafter?

#days will change

[Figure 12]

Students scored an A if they systematically searched for a rule. As seen in figure 13, these were students who were able to find the pattern that represented the sequence, but they were unable to write down any form of a rule or expression. Figure 13 shows a student who was able to answer question 1 and was able to write out what is happening in their sequence in words. The lack of understanding lies in their ability to take one further step in order to write a rule. We know that this student and similar other students understand what is happening but lack the knowledge to put it into the form of an expression. We also gave a few students the score of PA, which was only given if we felt that students were at a pre-algebraic knowledge. This means they lacked the overall ability to do any of the second page, or their responses were too far off to see any level of understanding. Figure 14 shows all the scores given on the second page.


After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he suggests you increase that time by 4 minutes per day.

1. How many weeks will it be before you are up to jogging...

   week 1 = 12   week 2 = 16   week 3 = 20 ...

   20 minutes a day?

   3 weeks

   40 minutes a day?

   8 weeks

   60 minutes a day?

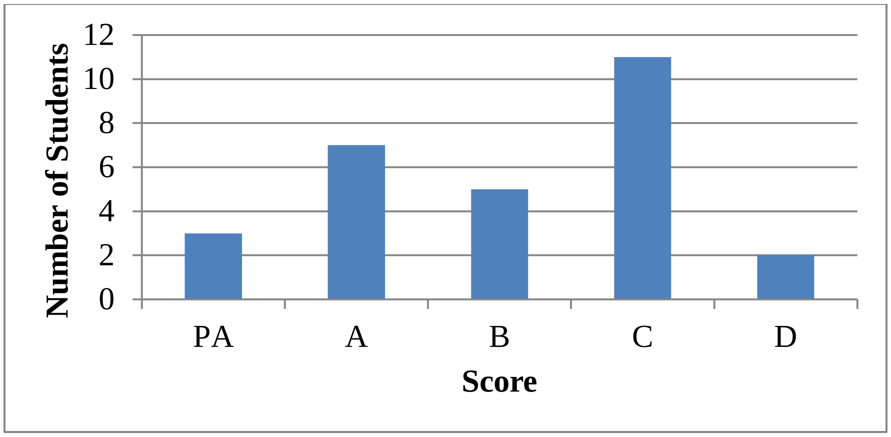
   13 weeks

2. In your own words, describe how to figure out a rule or equation for this problem? What happens to your rule when you are asked to determine the number of weeks before you can run 120 minutes per day?

   For every week that goes by keep adding 4minutes to the 12 minutes you began with from week 1.

   x=12
   x+4=16
   12+4=16

[Figure 13]



[Figure 14]

After sorting through all the surveys and assigning scores to each student we wanted to find a way to connect all the scores. Figure 15 shows a contingency table that we created to connect the scores using procedural and conceptual understanding. As stated earlier, we saw that the first page used more of a conceptual knowledge, whereas the second page used more of a procedural knowledge.

[Figure 15]

MORE PROCEDURAL

D	0	0	0	0	II
C	0	III	I	IV	III
B	I	I	II	0	I
A	0	I	III	0	III
PA	I	I	0	I	0
	0	1	2	3	4

MORE CONCEPTUAL

Figure 15 alone was not enough to show us how the first and second page tied together so we created another contingency table shown in figure 16. This table is broken into four quadrants. The top right quadrant shows students who had a higher level of conceptual and procedural knowledge. The top left quadrant shows students who had a lower level of conceptual knowledge but a higher level of procedural knowledge. The bottom right quadrant shows students who had a higher level of conceptual knowledge and a lower level of procedural knowledge. Lastly, the bottom left quadrant shows students who scored a lower level of both conceptual and procedural knowledge.

[Figure 16]

MORE PROCEDURAL

C-D	4	9
PA-B	10	5
	0-2	3-4

MORE CONCEPTUAL

Conclusion

After surveying and scoring all the students we saw that 12 of the 28 students were able to correctly write a story problem. This was surprising as students only had about 15 minutes to do the survey and were able to write a meaningful story problem. Of all the problems given, we assumed that asking students to write a story problem would have been the most difficult task, yet almost half of the students were able to do this. We saw that 16 of the 28 students were able to provide an algebraic rule on the second page, but they overlooked some details of the problem in order to write a correct rule. We know these students have the ability to write a rule however something is missing for them to write the correct rule.

Many of the students wrote their rule assuming that the first week of the sequence had already happened. This poses some questions that we wondered about. Do the students know they overlooked the first week? Do the students know how to check when their rule is right? We are not sure where the difficulty was for students that were not able to correctly write a rule, but half of the students struggled with this. We also saw that 5 of the 28 students demonstrated a higher level of conceptual understanding but surprisingly faltered with the more common algebraic tasks. Most students going through school are taught the step-by-step process of how to solve for a problem, whereas conceptual knowledge is harder for them to obtain. Most often students are not given the time or the resources for this to take place. We expect students that have solid conceptual understanding will handle more basic mathematical tasks with greater ease, which makes the 5 out of the 28 all the more interesting. Therefore we can conclude that conceptual and procedural knowledge are both important in the classroom.

Appendix A

STORY PROBLEM SURVEY

1. Given the equation

$$3x + 9 = 21$$

write your own story problem without using the words add, subtract, multiply, or divide. Also, do not use x or any other variable $(a, b, c, \ldots, y, z)$ to describe something in your story problem.

2. Referring to your story problem, what changes when you replace 9 with another number?

3. What changes when you replace 3 with another number?

4. What changes when you replace 21 with another number?

After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he suggests you increase that time by 4 minutes per day.

1. How many weeks will it be before you are up to jogging...

   20 minutes a day?

   40 minutes a day?

   60 minutes a day?

2. In your own words, describe how to figure out a rule or equation for this problem? What happens to your rule when you are asked to determine the number of weeks before you can run 120 minutes per day?

3. Write the rule or equation you obtained in part 2?

4. What happens to the rule you wrote if we change the number of minutes you jog each day for the first week? For example,

   8 minutes per day the first week and 4 minutes per day thereafter?

   15 minutes per day the first week and 5 minutes per day thereafter?

   20 minutes per day the first week and n minutes per day thereafter?

References

Alibali, M. W., et al. “Middle school students’ conceptual understanding of equations: Evidence from writing story problems. WCER Working Paper No. 2009-3.” *Wisconsin Center for Education* Research (2009): ERIC.

Clement, John. "Algebra Word Problem Solutions: Thought Processes Underlying a Common Misconception." *Journal for Research in Mathematics Education* (Jan., 1982): pp. 16-30. Vol. 13, No.1.

Clement, J. Lochhead, J. & Monk G. S. "Translation Difficulties in Learning Mathematics." *The American Mathematical Monthly* (1981): pp. 286-290. Vol. 88.

Driscoll, M.& Moyer, J. "Work As A Lens on Algebraic Thinking." *Mathematics Teaching In The Middle School* ((2001)): 282.

Walle, John A. Van De. *Elementary and Middle School Mathematics: Teaching Developmentally*. Boston: Pearson Education, Inc., 2004. Fifth Edition.