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Making Our Partial Understanding of Fractions Whole

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**ABSTRACT**


> "Mathematics is a universal, utilitarian subject—so much a part of modern life that anyone who wishes to be a fully participating member of society must know basic mathematics. Mathematics also has a more specialized, esoteric, and esthetic side. It epitomizes the beauty and power of deductive reasoning. Mathematics embodies the efforts made over thousands of years by every civilization to comprehend nature and bring order to human affairs" (Findell, Kilpatrick, & Swafford, 2001, p. 15).

Fractions are a fundamental aspect of mathematics, and a student's understanding or lack of understanding tends to follow from elementary school through college and to the real world. Fractional understanding chases students in telling time, measuring ingredients for recipes, medication doses, and even splitting of groups of items. Students have always had difficulty understanding fractional concepts. The researcher contends that it is because of the procedural methods used to teach them. Research suggests that teachers who lack a conceptual understanding of fractions are unable to help students understand fractions in ways that allow them to use fractions beyond the classroom. At best, this has led students to be able to divide fractions by the "keep it, change it, flip it" rule, but not recognize when to use division of fractions when the division symbol is not used. This type of understanding is called an instrumental understanding. The researcher believes that teachers with a relational understanding, where they have an interconnected web of ideas for mathematical topics, would be more capable of helping students have a more conceptual understanding of fractions. When students are able to make sense rather than recall, this will likely alleviate the fear of fractions. The researcher will find out through data collection how little or how much a convenient sample

of students understands both conceptually and procedurally about fractions. The project aims to recognize any weaknesses or strengths of prospective teachers concerning their fractional reasoning. The researcher will also utilize available resources, including mathematics teacher educators and the literature on fractions to develop different strategies to teach teachers conceptually so that they can stop the cycle of fraction fear.

## BACKGROUND AND ASSUMPTIONS

Quantifying students' mathematical proficiency can be broken down into five main intertwining strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Conceptual understanding refers a students' knowledge of more than just isolated facts and methods, instead encompassing connections and relationships between concepts. The skill of flexibly carrying out procedures and doing so with accuracy refers to procedural fluency. Another strand of mathematical proficiency is strategic fluency and is defined as the ability to formulate, represent, and solve mathematical problems presented. Adaptive reasoning characterizes the capacity for logical thought, reflection, explanation, and justification of mathematics. The habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with the belief in diligence and one's own efficacy is how productive disposition is defined. Teaching for mathematical proficiency requires teachers to have all the strands of mathematical proficiency and the deep understanding of the relationship between all five. Large scale surveys about United States students' mathematical knowledge indicate students hold some proficiency with some individual strands, but only when disconnected from the rest of the strands. Teachers fail to elevate their students' mathematical proficiency when they concentrate on only one type of proficiency and exclude the others. Progress in mathematical proficiency can only be made if teachers make efforts to connect all strands (Findell, Kilpatrick, & Swafford, 2001). Teachers must also consider different interconnected aspects that may affect students' learning such as teaching-learning environments and methods of presentation of the material. All strands of mathematical proficiency and different learning situations are interconnected and changes in one factor affect the others.

The United States is in a war on mathematical teaching paradigms: teaching of more repetitive procedures versus teaching of more conceptual methods. When used independently, each threatens to undo each other in the classroom (Davis, Maher & Noddings, 1992). It then falls on the teacher to find an adequate balance of the two that fulfills students' needs and encourages learning. However, few teachers recognize the need for this balance, instead continuing the battle between drilling students with algorithms and procedural teaching versus teaching conceptually.

## LITERATURE REVIEW

Unfortunately, the hypothesis that computation teaching method still constitutes the majority of a students' learning during elementary mathematics is true. Later on we will discuss the TIMSS and how this study is evidence proves our hypothesis to be true. The majority of students' mathematics competency lies in doing simple computations or simply guessing for the answer since there is no fundamental basis after having been through elementary mathematics. Making students perform drills in mathematics simply makes them faster at basic procedures, regrettably without any basis. Drills completely lack the majority of mathematical proficiency and make zero connections between other concepts students may be learning. Repetition does not lead to understanding, but is regarded as a way to ensure skilled quantitative thinking through repeated practical reinforcement (Ford & Resnick, 1981).

Procedural teaching methods and drilling typically go hand in hand. Procedural teaching occurs when teachers begin teaching a topic by giving a formula or a shortcut quick procedure for students to complete a problem. There are two types of teachers, teachers with calculational orientation and teachers with conceptual orientation. Calculationally oriented teachers teach procedurally and are focused on getting the correct answer in a problem. Conceptually oriented teachers use story problems to reflect on reasoning behind the problem and focus on images of systems of ideas. Conceptual teachers want to develop these ideas through activities and explorations with constant student engagement (Philipp). This is the pit fall that many teachers have led their students into when the students are able to divide fractions by the "keep it, change it, flip it" rule, but not recognize when to use division of fractions when the division symbol is not used.

## DATA COLLECTION AND ANALYSIS

A group of pre-service early childhood teachers were given a fractional mathematics test in order to gauge how little or how much they understand conceptually and procedurally about fractions. When asked how to show conceptually how to change 2 $\frac{3}{4}$ to an improper fraction, 15 out of 25 students wrote  down the following shortcut:

$$2 \, {}^{\times}_{+} \, \frac{3}{4}$$

(MAED, 2012). These students, two thirds of the classroom population, wrote down a common procedure they learned in elementary school even though they were asked to show conceptually. These pre-service teachers are falling into the same cycle their mathematics teachers are in unless they learn how to conceptualize mixed numbers. The shortcut that these pre-service mathematics teachers used stems from their own lack of conceptual knowledge in what mixed fractions actually are, which is why when these pre-service teachers were drilled on converting mixed fractions into improper fractions they reminded themselves “all you have to do is multiply the denominator and the big number, then add the numerator”.

Written notations and spoken notations can also contribute to student errors. While mixed numbers use whole numbers and fractions, unfortunately there is nothing in either spoken or written notation to help convey the whole numbers’ meaning of fractional parts (Findell, Kilpatrick, & Swafford, 2001). Chinese mathematics teachers make distinctions in spoken notation that we as American mathematics teachers tend not to do. Where we as Americans use the same spoken notation when talking about fractions and the ordering in a line, fifth in line and three fifths, the Chinese use spoken notation specific to fractions, out of 5 parts take 3 (Findell, Kilpatrick, & Swafford, 2001). This is yet another instance where a mathematics teachers’ conceptual knowledge of fractions and the spoken notation associated with them need to be passed on to their students for a full understanding of mixed fractions.

Another classic example of a lack of conceptual fractional knowledge occurred on the pre-service teachers’ test. Again 15 out of 25 pre-service teachers were not able to draw a picture to solve the expression $\frac{1}{2} - \frac{1}{3}$; they were only able to write down the equation and solve for the correct answer. Virtually every one of the pre-service teachers that was unable to draw a picture

$$\frac{1}{2} - \frac{1}{3} = \frac{(1 \text{ x } 3)}{(2 \text{ x } 3)} - \frac{(1 \text{ x } 2)}{(3 \text{ x } 2)} = \frac{(3-2)}{(2 \text{ x } 3)} = \frac{1}{6}$$

to solve the problem wrote down instead some version of the following algorithm

(MAED, 2012). Here the same pre-service teachers as before were unable to conceptualize a basic fractional operation. Because all pre-service teachers knew the correct answer, the researcher knows they do not lack procedural knowledge on fractions, but that they lack any form of conceptual fractional knowledge. However, unlike these pre-service teachers, the majority of student errors occur because they are attempting to apply poorly understood calculation rules for fractions (Findell, Kilpatrick, & Swafford, 2001). Applications of fractional algorithms on a problem are only valid when the student is able to first answer the problem conceptually. Teachers must be able to conceptualize fractional operations to begin with in order for students to be able to develop algorithms and understand where these algorithms come from.

Most teaching methods, both procedural and conceptual alike, begin the topic of fractions with the idea of comparing sizes or quantities of fractions. However, a problem in procedural methods arises when putting a set of fractions on a number line because they have different denominators and students are not able to tell without a calculator which fractions are bigger than others (Findell, Kilpatrick, & Swafford, 2001). Case in point, on the pre-service teachers' test they were asked to put the following fractions in ascending order:

$$\frac{5}{7}, \frac{1}{12}, \frac{2}{3}, \frac{1}{4}, \frac{19}{22}, \frac{3}{5}$$

Only 11 out of 25 pre-service teachers were able to correctly put the fractions in ascending order. The rest of the pre-service teachers made errors ranging from putting the fractions in order based on numerator or denominator value, guessing, or miscalculations when they performed division. Startlingly only one pre-service teacher was able to put the fractions in order using conceptual methods by drawing each fraction out and ordering them based solely off the pictures, while the rest were able to do correct calculations in order to answer the problem (MAED, 2012). These statistics on the pre-service teachers test only solidify the need for conceptual teaching methods, especially when it comes to the foundations of fractions.

One country we typically compare ourselves to in order to gauge our mathematical standing in the mathematics world is the Chinese. We fall behind the Chinese almost constantly in every international mathematics knowledge test. One main reason we are continuously behind the Chinese is because of our core teaching methods. The key motto for Chinese teaching is "know how, and also know why", we tend to follow the ill-fated unofficial motto of "know how, and know how to quickly". The Chinese motto instills in students and teachers alike the demand for conceptual learning and connections between mathematical ideas (Ma, 1999). However, our typically adopted unofficial motto promotes the learning of algorithms and rote procedures through procedural teaching methods. While we coin the notorious phrase "keep it, change it, flip it" in regards to dividing fractions, the Chinese refer to the dividing of fractions as "equivalent to multiplying by its reciprocal," making use of the inverse relationship between multiplication and division. In addition, Chinese teachers would require students to know why the sequence of steps in the computation makes sense (Ma, 1999). Unfortunately for our teachers and students as a whole, we fundamentally teach mathematics differently than the Chinese.

The Trends in International Mathematics and Science Study, commonly referred to as the TIMSS, is a study of the knowledge of fourth and eighth grade students in over 60 countries and jurisdictions. A grand total of approximately 500,000 students participate in this study worldwide in 2007. While both U.S. fourth and eighth graders on average placed slightly above the 2007 TIMSS average scale of all countries, the percentages of students performing at or above the advanced international mathematics benchmark are more telling of our mathematical knowledge. Our fourth and eighth grades percentages of performing at or above the advanced benchmark were 10 percent and 6 percent respectively and blatantly point out our mathematical shortcomings compared to the Chinese with 41 percent and 45 percent respectively ("Trends in international," 2007). What is worse, our resulting percentages for twelfth graders were even worse. The staggering differences in percentages of students at or above the advanced benchmark are yet another indicator that the mathematic teachers of China are on to something we are not that is exponentially useful to their students, conceptual learning. Gone are excuses such as Chinese students spend significantly more time in instruction and practicing mathematics than our American students; Chinese students just learn more effectively by using conceptual teaching. What a teacher's definition of math is affects his/her approach to teaching it in the classroom (Davis, Maher, & Noddings, 1992). The U.S. teachers have the misconception that elementary mathematics is basic, clearly by our TIMSS scores and students' general lack of fractional knowledge, it is not. Therefore we need teachers to have a more comprehensive and conceptual understandings of fractions in order to be able to teach operations that have their foundation in elementary mathematics. As unfortunate as it is, data shows that a group of ninth grade Chinese students were more competent in elementary mathematics than a group of U.S. teachers. The knowledge gap between U.S. and Chinese teachers is a parallel to the learning gap between students in each country (Ma, 1999).

## FINDINGS AND DISCUSSSION

Consider the undergraduate degree that pre-service teachers of high school mathematics at Georgia College must accomplish. Since at Georgia College there does not exist a mathematics education major for pre-service high school teachers, those pre-service high school teachers must major in mathematics and obtain a concentration in teaching. This requires all pre-service high school mathematics teachers to complete the same mathematics degree that every mathematician seeking a mathematics degree is required to complete. A mathematician is being defined as an expert in or a student of mathematics (Webster, 2012). Hence Georgia College prepares pre-service high school mathematics teachers extremely well in mathematical proficiency and professional development. Georgia College does its part to stop the cycle of procedural learning and teaching by having all mathematics and education majors alike take math education classes that only teach pre-service teachers how to learn and teach conceptually.

Teacher mathematical proficiency and professional development help teachers understand the mathematics that they teach, how to facilitate their students' learning, and how their students actually learn. Proficiency in teaching mathematics to students parallels the teacher's proficiency in mathematics. Mathematics teachers should have a high level of professional mathematical development as well as continued education on how to teach most effectively (Findell, Kilpatrick, & Swafford, 2001). The low quality of studying mathematics in the education of teachers is recapitulated in students' instrumental understanding of mathematics taught in school by those teachers. This tends to happen because the majority of U.S. teaching programs for math education focus on the teaching of mathematics, and not actual mathematics. Also in the majority of the math education programs there is a fundamental lack of connection between the study of mathematics and the teaching of mathematics; both of which are completely necessary for conceptual teaching methods. Even seasoned mathematics teachers who were mathematically confident and energetically participate in mathematics teaching reform did not seem to have a thorough knowledge of mathematics taught in elementary school (Ma, 1999). Virtually none of Randolph Philipp's, another researcher in the mathematical education field, research of elementary and secondary mathematics teachers could provide a conceptually orientated explanation for how we divide fractions. After following a conceptual lesson plan showing the elementary and secondary teachers, many questioned why they never had the opportunity to understand factions when they were themselves students (Philipp). Most teachers and schools are not even able to achieve new educational standard goals set forth, not because they are unwilling to achieve the goals, but because they lack the mathematical knowledge to know how to do so (Ma, 1999). Such new educational standards are a direct result from the new Common Core State Standards that our government has put in place. Teachers teach what they know, and likewise do not teach what they do not know (Philipp).

During classroom instruction, in order for teachers to be able to guide students to conceptual discoveries, they themselves need a higher level of mathematical understanding. A problem arises when we ask the question, how can teachers follow students' suggestions and explorations if they themselves do not know enough higher level mathematics to perceive where the student may lead or what the student may ascertain (Davis,  Maher, & Noddings, 1992)? In order to improve the quality of a student's mathematical knowledge, we must recognize that there is a direct relationship to improving the quality of a teacher's mathematical knowledge (Ma, 1999). However there exists a difference between what mathematics teachers need to study in order to be an effective conceptual teacher at on-level curriculum in their classroom and the advanced topics that mathematicians study (Ma, 1999).

## IMPLICATIONS FOR TEACHER EDUCATION

Having established how little conceptual fractional understanding most pre-service mathematics teachers and mathematics students have, the researcher identifies strategies to help fix the awful cycle of procedural teaching, which leads to procedural learning. Converting mathematics teachers to teach and reason conceptually is the first and biggest step to getting students to learn and reason conceptually. Teacher preparation may serve as the force to break the cycle. Teacher preparation is the teacher's continued education and is a period in which change from procedural to conceptual teaching can be made (Ma, 1999). The shift to conceptual teaching would leave the mathematics teachers asking new questions before and after lessons, including but not limited to "Does the student use and understand why each step in an algorithm work?", "Does the student have a notion of units and portions of units, fractions?", and "Can the student use reasoning in making comparisons between fractions?" (Nelso & Reys, 1976). When mathematic teachers speak of units, they are discussing another name for one of something (Eather, 2012). Overall teacher preparation is extremely important to teachers changing from procedural to conceptual teaching of fractions. We can begin to get mathematics teachers who use procedural teaching methods to become conceptually orientated by having them look at mathematics differently: by getting students to reason in particular ways instead of getting students to do in a particular way. Mathematics teachers need to develop more of a conceptual orientation where doing takes a back seat to understanding and where calculating falls behind sense making (Philipp).

There are different strategies a teacher can utilize in teaching conceptually. One such specific strategy would be for mathematics teachers to have students work in groups on activities using manipulatives. It has been highly recommended that students gradually internalize discussion held within their peer groups. From group work, students tend to challenge their own original ideas, examine and analyze their own mental work, and question their peers reasoning.

Thus the students conceptually teach themselves through exposure to a variety of strategies, which leads to a more conceptual understanding (Davis, Maher & Noddings, 1992).

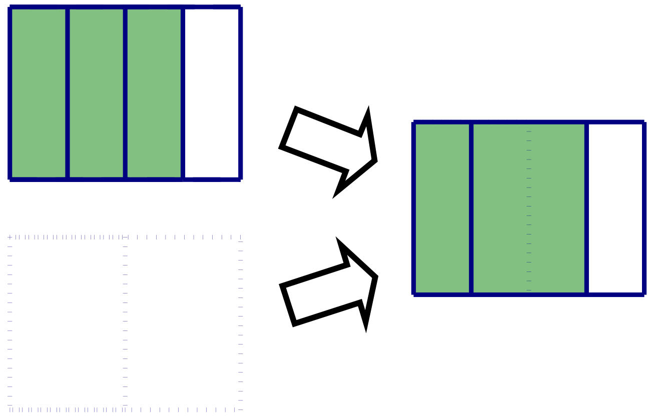
Another strategy teachers may make use of is to occasionally have students lead class. Textbooks are great guidelines for class, but mathematics teachers must be able to go with the students' conjectures and thoughts and not be rigid in their procedural teaching methods. When teachers allow the textbook to determine the next lesson, they assume that children learned everything from each page as it was intended. When basing lesson plans off the textbook, mathematics teachers should be able to look at a unit and decipher two to four big ideas that should be covered in order to avoid being too dependent on the book for lessons (Van De Walle, 2004). Teachers should be able guide the class, without hindering the student-generated ideas, while still allowing for a student-led classroom (Ma, 1999). In order to teach mathematics well, teachers must understand their students' thought processes. In order to provide a high quality mathematics education, teachers must be able to understand how students learn mathematics. A consequence of this includes a keen awareness of the individual mathematical development of each student (Van De Walle, 2004). This means understanding the little shortcut marks on students' papers in order to be able to further guide their mathematical thinking or correct a mistake. Teachers also need to set realistic expectations to know what they can or cannot accomplish with the material they present the students. Class time should be time for students to do most of the explaining, and the teacher does most of the listening. When students talk about their reasoning from a problem, they can usually work it out together as to the correct answer (Reinhart, 2000). Mathematics teachers should be able to correct students' errors, reassure them, and create a math environment using manipulatives and sound work habits that establishes relationships between mathematic objects and topics. A mathematics teacher's job entails steering student learning through a mathematical world where students are comfortable and at ease (Davis, Maher & Noddings, 1992).

There are three basic principles that teachers using conceptual teaching methods need to understand in regards to mathematics. The first principle is the importance of identifying and attending to the main mathematical concepts in a topic of study. Building on both students' and teachers' existing knowledge is the second principle that comes into play. Lastly, introducing symbols and procedures to students only after introducing the concepts they represent to the students first. In combination of all three of these principles of conceptual teaching, mathematics teachers will be able to teach their students so that they can see the subject as an interrelated connected web of ideas on which they can continue to build.

Mathematics teachers must begin by looking at the first principle where the importance lies in not sweating the details and smaller consequences of the main topics. Conceptual teachers need to focus on the big picture and connecting those big pictures to form mathematical relationships between concepts. When looking specifically at the division of fractions, conceptual teachers must be able to direct students' attention towards two main concepts. The first is being able to look at fractions through a measurement approach. We can discuss this main approach of the division of fractions using an example; let us look at the following.

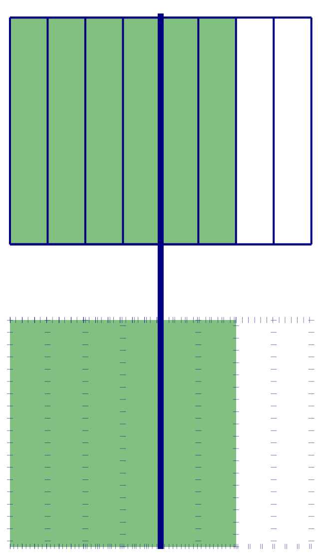
$$\frac{\frac{3}{4}}{\frac{1}{2}}$$

By using the measurement approach, we can look at this expression as how many groups of $\frac{1}{2}$ are in $\frac{3}{4}$. The following picture, where we overlap $\frac{3}{4}$ with a picture of a whole divided into



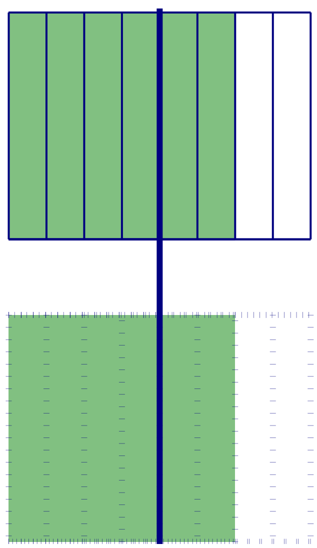
halves, suggests that there is one $\frac{1}{2}$ in $\frac{3}{4}$, with an amount leftover.

In determining how much is left over, we it is best to find a common partitioning of eighths. This way we will get a better idea of how much is leftover.



The heavy line shows how the halves match up with the smaller partitions in the $\frac{3}{4}$. Note that there are two small pieces leftover. In looking at the leftover part, we must interpret it in terms of $\frac{1}{2}$. This leftover part is also $\frac{1}{2}$ of a $\frac{1}{2}$, because it takes four pieces of this size to make a $\frac{1}{2}$.

Therefore since we are asking how many $\frac{1}{2}$'s are in $\frac{3}{4}$, we write our answer as $1\frac{1}{2}$. This means that there is one $\frac{1}{2}$'s in $\frac{3}{4}$ with a remainder of $\frac{1}{2}$. The second main concept is a partitive interpretation in which we look at unit changes during the division of fractions. Using the partitive interpretation, we look at our answer, we want to know how many objects or how much of an object are in the whole group. In our previous example, we ask how many objects are in the whole group. Note that we are still using the same results as before, just in a different interpretation.



Since we are looking at a partitive interpretation, we see that there is $\frac{6}{8}$ because we have six overlapping pieces out of our whole of eight pieces. If these two concepts or approaches are not explicitly paid attention to by differentiating between them, then students will more than likely overlook and never consider these concepts (Philipp). By teachers making note of these two commonly overlooked concepts, students are extremely more aware and have a deeper understanding of why division between fractions works and is not just a quick rule they can remember during tests.

## IMPLICATIONS

Awareness of pre-existing knowledge from both students and teachers is a tremendously important principle that conceptual teachers have to take into consideration when teaching. Teachers who deeply understand mathematics are put into a position to be able to guesstimate their students' mathematical understanding. These conceptual teachers need to be able to consider what conceptual knowledge the students bring to the table, recall real life situations and examples to bring the mathematics to their students' world, and recognize different contexts of problems in order to encourage mathematical discussion between students and between students and teachers. Real life situations and applicable context in problems helps teachers support students' mathematical learning, which is already fragile to begin with (Philipp).

Unlike the unending debate over whether the chicken or the egg came first, we know that conceptual teaching comes before procedural teaching. In order to produce a full mathematical understanding, particularly with fractions, students need to be able to grasp basic knowledge and concepts before procedures are taught and taken as rule. If procedures were taught before concepts, students would unlikely be able to make the connection to concepts (Philipp). In one specific researcher's interview of a 1$^{st}$ and 5$^{th}$ grader, when both students were asked three comparative questions of a fraction being more than a whole, less than a whole, or equal to a whole, both students were able to give correct answers. The difference between the students was that the 1$^{st}$ grade student was able to explain why such her answer was correct and how she could make it equal to a whole, where as the 5$^{th}$ grade student was not able to give a single explanation or reasoning behind his answers. The 1$^{st}$ grade student had just begun working with fractions and was being introduced to fractions using conceptual teaching methods with manipulatives. However, the 5$^{th}$ grader was taught procedurally, and therefore lacked any sort of concept of fraction operations beyond rules and procedures (Philipp). Students with solid conceptual understandings of fractions can make sense of fraction procedures without having to be explicitly taught.

Interactive teaching methods that use any concrete objects that allow students to explore an idea in an active, hands-on approach are referred to as teaching with manipulatives. The ideal conceptual classroom is one in which the teacher creates situations where the students are able to think using manipulatives (Davis, Maher & Noddings, 1992). In discovering how fractions make up a whole, teachers should use blocks for a manipulative. Realizing that different fractions can make up the same whole is a natural consequence of being able to use the blocks (Philipp). The direction of students learning with manipulatives largely depends on how the teachers steers the students. However, we must distinguish that the use of manipulatives does not ensure good learning or teaching. One reason we as teachers may be overstating the power of manipulatives is because we are "seeing" concepts that we already understand (Ball, 1992). We need teachers to be conceptually focused when teaching mathematics to be able to explore the "why" and "how" things happen in core of mathematics (Ma, 1999). While even the most hardcore conceptual method teachers can admit that every mathematical classroom need a little procedural teaching methods, it is extremely dangerous for students to rush straight to rules and algorithms (Ma, 1999; Van De Walle, 2004). Using manipulatives to illustrate examples in class can lay the foundation for fractions and enable students to make the transition into procedural knowledge. The usage of block manipulatives in showing students how fractional portions make up a unit and how varieties of fractional portions can make up one unit is for students to be able to conceptually connect a physical model to its respective symbolic representation. The goal of manipulatives is for students to be able to move between physical, verbal, and symbolic representations of fractions; thus avoiding the fear of fractions (Philipp).

The fear of fractions is a legitimate fear when students are taught procedurally. Without a full conceptual understanding of fractions, students have little foundation to build on in order to continue connecting fractional operations and relationships. In particular, when the pre-service teachers were asked on their test what their general disposition is towards fractions and why, all but four pre-service teachers had a negative disposition towards them and/or found them frightening. All of the negative comments the pre-service teachers articulated dealt with the fact that they were taught procedurally. Not liking fractions came from being confusing because the pre-service teachers do not feel like they ever truly understood them and the pre-service teachers had difficulty remembering all the different rules for all the different operations. Fears also included the pre-service teacher only being confident when they know the rule for the operation at hand and are able to make sure their answer makes sense. Other pre-service teachers had problems with the fact that there were too many rules to keep straight and that they only learned one method for solving each kind of fraction problem (MAED, 2012). All of these issues and negative emotions that the pre-service teachers have, and the researcher would feel confident in saying most students feel the same, stem from a lack of conceptual understanding, which was produced by their procedural teachers. Out of the 25 total pre-service teachers, only 4 either liked fractions because they remembered the rules and know how to solve the problems given or were neutral towards fractions, feeling neither positive nor negative emotion. The amount of pre-service teachers alone that are afraid of dealing with fractions should be enough for them to want to break the cycle of procedural teaching methods with their students. It should also be cause for some mathematics teachers to wake up and realize that their fractional teaching methods are not working.

## CONCLUSIONS

"But here comes the effort of thought. It is easier to see the conditions in their separateness, to insist upon one at the expense of the other, to make antagonists of them, then to discover a reality to which each belongs" (Ma, 1999). Conceptual teaching of fractions may seem like more work, but it is the kind of work that pays off in the end. A student's ability to be able to delve deeper into a mathematical topic that connects to multiple other concepts is integral in the learning process. Teachers are able to accomplish this through using conceptual teaching methods first and then by being able to segway into procedural instruction. Yes, using manipulatives to teach fraction relationships and make connections when performing fraction operations will take more time in the classroom. However there are no shortcuts to good teaching (Davis, Maher & Noddings, 1992). However, in order to fully benefit students, teachers do not only need to be able to teach conceptually, they themselves need to understand the topics conceptually. In order to connect mathematical topics together and see the relationships they form, it is necessary for a foundation of principle concepts to be understood (Nelso & Reys, 1976). Conceptual teaching methods of mathematics and especially fractions, is where this foundational understanding comes from that students are able to then develop different consequences. It is with teachers being guided by conceptual teaching methods in teaching mathematics that our students benefit the most. When it comes to fractions, everyone understands something and no one understands everything (Philipp). It is with this attitude that our teachers should look to gain conceptual understanding and strive for higher levels of mathematical knowledge. Only then will they be able to teach such conceptual methods. As Shulman (1986) said, “Those who can, do. Those who understand, teach.”

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