

Application of linear Algebra to image processing

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Abstract

In this work, we explore the history of image processing. We examine the connection between digital image processing and linear algebra. We show that digital image manipulations are applications of matrix operations. We investigate examples of applications of digital image processing in our daily lives.

Keywords: image, matrix, pixels

1 Introduction

A digital image is as a discrete representation of data possessing both special (layout) and intensity (color). Image processing is a method to perform some operations on an image to get an enhanced image or to extract some useful information from it. It is a signal processing in which input is an image and output is another image or any features associated with that image. Image processing is a rapidly growing technology. Image processing includes the following steps:

- Importing the image via.

- Analyzing and manipulating the image.
- Output in which result can be altered image or report that is based on image analysis.

There are two types of methods used for image processing:

- Analogue
- Digital image processing.

Analogue image processing can be used for hard copies like printouts and photographs. Digital image processing uses computers to manipulate images. There are three general phases of digital processing: pre-processing, enhancement and display, information extraction.

Digital image processing has two main tasks:

- Improvement of pictorial information.
- Processing of the image data for storage, transmission and representation for autonomous machine perception.

The smallest component of an image that could be manipulated is called a pixel. In this paper, we explore the history of image processing. We examine the connection between digital image processing and matrix theory. We show that digital image manipulations are applications of matrix operations. We investigate examples of applications of digital image processing in our daily lives.

2 Digital image processing

2.1 The history of digital image processing

Early signs of image processing can be dated back to the 1920s. A news company (The Bartlane Cable Picture Transmission Service) transferred images by submarine cable between London and New York. Pictures were sent via codes and reconstructed on a telegraph printer. Later during the space race era, image processing was used to understand outer space and the satellite operations.

Today, Nasa leads the field of image processing. Image processing made one of its earliest contributions to space by capturing the Ranger 7 probe, as well as the Apollo landing mission. NASA became one of the biggest developers of image processing.

Image processing is also useful for medical imaging, videophones, character recognition, photographic enhancements, law enforcement, and artistic effects. Medical doctors can use image processing to identify possible operations and procedures that could be done to patients. MRI and CAT scan are other applications of image processing. Sir Godfrey Hounsfield and Professor Cormack share a Nobel Prize for their invention in tomography, and the technology behind the computerized Axial Tomography (CAT) scan. The CAT scan focuses on a digital image through x-rays of the body to identify unusual things. Video chat has recently emerged on the scene as a popular topic for families, friends, and companies. Now, with our cellphones, we can create and edit photos. This is called mobile image processing. Mobile processing is growing rapidly.

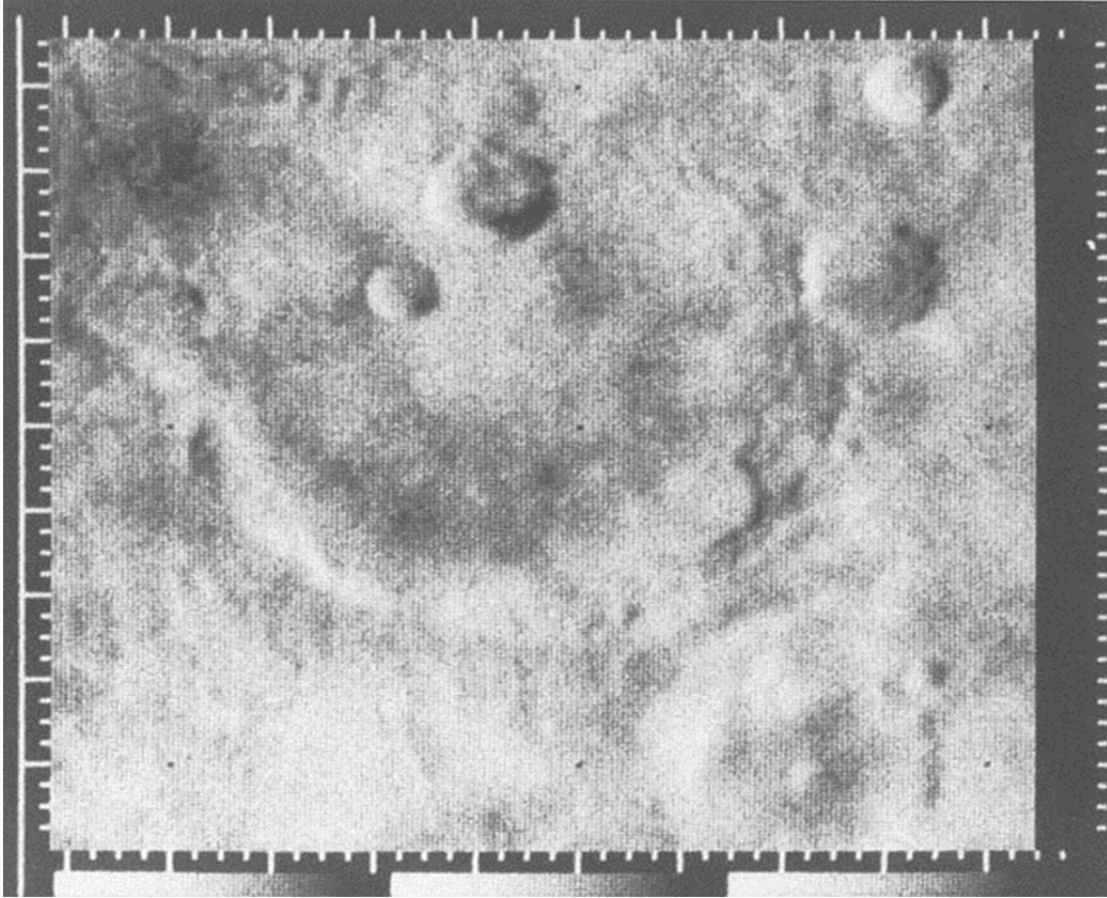
Other applications of image processing include crimes fighting by law enforcement through camera surveillance and facial recognition. The creation of image processing has allowed the law to protect and serve the country in a major way through stopping crime.

Tools and equipment for image processing were very expensive. But with the advancement of technology, many programs have been developed that allows manipulation of digital image at a lower cost

Over decades of research have made the image processing more accurate. Early picture obtained through submarine cables were not reconstructed properly. They were some alteration due to many errors that occurred during transportation of the image.



7.



A photo of the first closeups of Mars (164)

Today, digital image processing is a tool used in many other disciplines. One of the largest businesses who benefit from this tool is the entertainment industry. This industry focuses on the different ways you could alter photos and videos the display images to an audience. Image processing has also helped fight crime with the development of video surveillance.

2.2 The notion of pixel elements

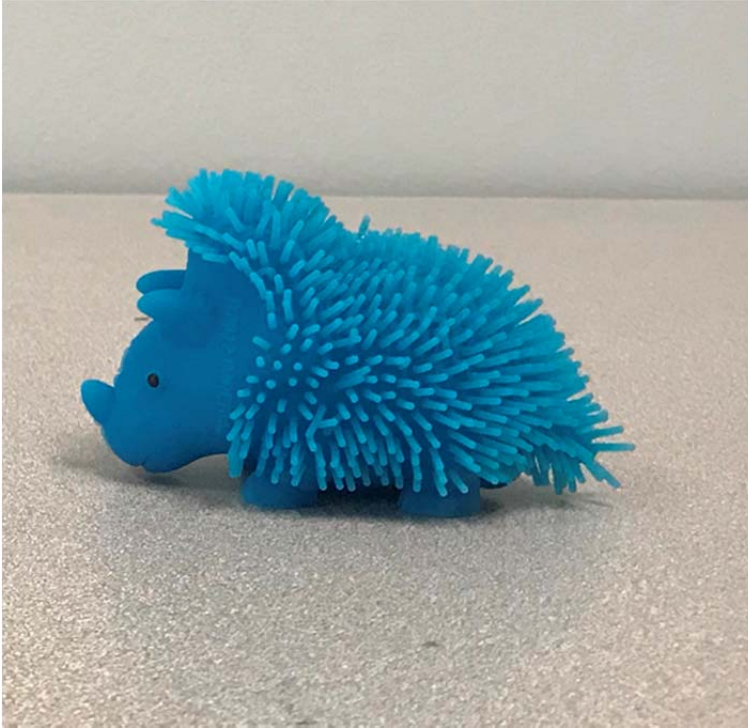
In a digital image, the smallest controllable element is a pixel. A pixel is the atom of the picture. The number of pixels in a picture depends on the quality of camera used to take the picture. Many computer systems contain between a 24-bit system to a 32-bit system. The greater the number of pixels, the better is the quality of the image.

Many images that cameras create are expressed in megapixels, which are the number of million pixels that are displayed. The higher the number of megapixels in an image allows for much better quality in a photo.

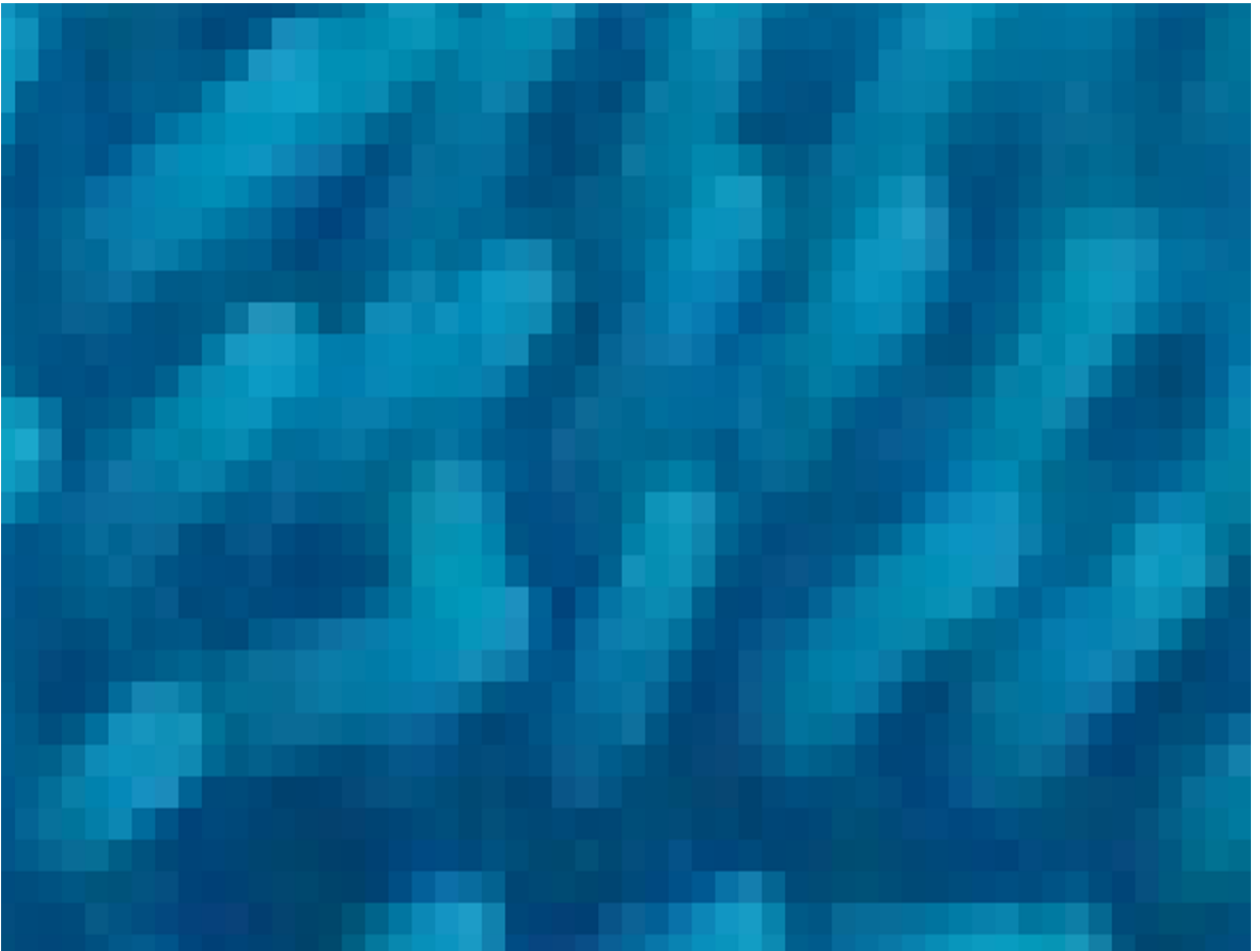
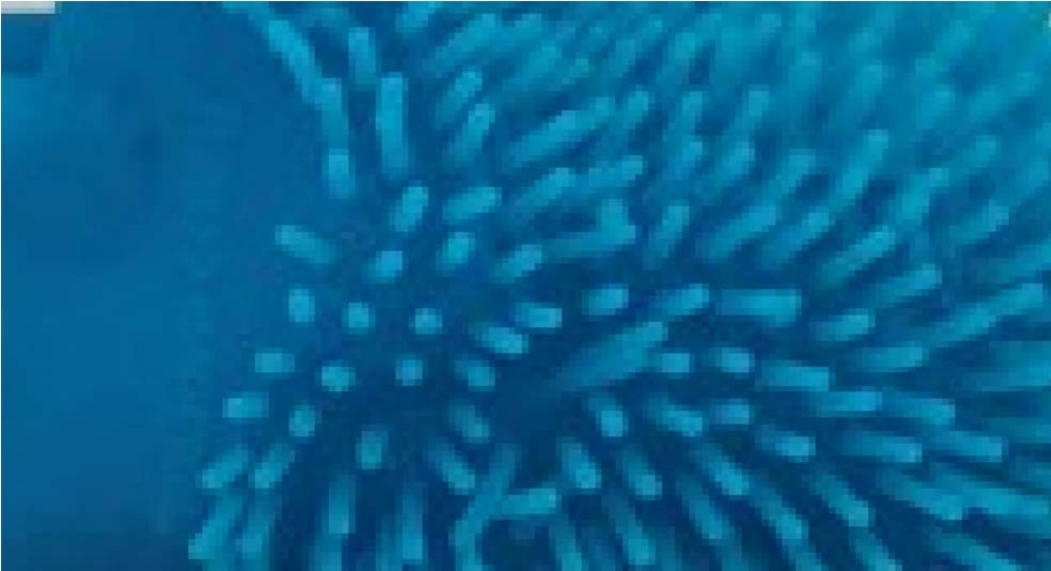
Pixel coloration is a product of color blending. Colors are generated with three base colors. They three base colors are a red, blue, and green. Each of the three base colors has 256 gradations. The color of a pixel is determined by the value of the red, green, blue component. This means that we can generated $256 \times 256 \times 256 = 16777216$ different types of pixel in RGB system

There are other color systems beside the RGB color system. For example, the CMYK (cyan-magenta-yellow-key) which is used mainly for printed color illustrations (hard copy). The RGB system is used mainly for computer displays.

Consider the following image



When we zoom in continuously, we observe little “squares” Those squares are pixels and their number depend on the type of device use to realize the picture.



2.3 An image is a matrix

In the RGB system, the image is automatically written in the form of a matrix whose entries are the pixels values. So any photo is associated to a matrix and vice-versa. MATLAB can convert image to a matrix and any matrix to an image. The table below gives the Red-Green-Blue values of some familiar colors.

| Colors | | Red | Blue | Green |
|-----------|--|-----|------|-------|
| Red | | 255 | 0 | 0 |
| Orange | | 255 | 0 | 128 |
| Yellow | | 255 | 0 | 255 |
| Green | | 0 | 0 | 255 |
| Turquoise | | 0 | 255 | 255 |
| Blue | | 0 | 255 | 0 |
| Purple | | 127 | 255 | 0 |
| Pink | | 255 | 153 | 51 |
| Black | | 0 | 0 | 0 |
| Grey | | 128 | 128 | 128 |
| White | | 255 | 255 | 255 |

The following website has a tool that helps generate pixel values for other colors.
http://www.rapidtables.com/web/color/RGB_Color.htm

We used MATLAB to generate a random matrix with 20x20x3. Here are the first entries of the matrix.

| | | | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0226 | 0.0579 | 0.1089 | 0.8229 | 0.1608 | 0.2470 | 0.4090 | 0.1109 | 0.3486 | 0.7350 |
| 0.9235 | 0.7603 | 0.1709 | 0.3578 | 0.9698 | 0.4888 | 0.9847 | 0.0251 | 0.1389 | 0.9008 |
| 0.2202 | 0.6035 | 0.4224 | 0.1886 | 0.7053 | 0.6909 | 0.4901 | 0.5955 | 0.3060 | 0.4615 |
| 0.6964 | 0.1513 | 0.9241 | 0.8661 | 0.5526 | 0.8640 | 0.3462 | 0.4890 | 0.5916 | 0.2983 |
| 0.8357 | 0.2821 | 0.3217 | 0.9236 | 0.6074 | 0.8935 | 0.2761 | 0.0310 | 0.4766 | 0.4152 |
| 0.4094 | 0.7284 | 0.1633 | 0.8831 | 0.0888 | 0.1146 | 0.1484 | 0.7418 | 0.5155 | 0.7230 |
| 0.4582 | 0.7833 | 0.4433 | 0.4519 | 0.8610 | 0.3010 | 0.6223 | 0.2642 | 0.5386 | 0.9980 |
| 0.2679 | 0.3711 | 0.7500 | 0.4683 | 0.8674 | 0.1592 | 0.5970 | 0.5258 | 0.1933 | 0.3535 |
| 0.4445 | 0.9890 | 0.4634 | 0.1272 | 0.3389 | 0.2071 | 0.3781 | 0.4750 | 0.6546 | 0.8157 |
| 0.7007 | 0.5412 | 0.6024 | 0.1471 | 0.8901 | 0.5296 | 0.6002 | 0.6407 | 0.7992 | 0.9526 |
| 0.7394 | 0.5295 | 0.4248 | 0.0632 | 0.0623 | 0.3633 | 0.8432 | 0.9846 | 0.6648 | 0.3636 |
| 0.4721 | 0.9319 | 0.7344 | 0.5392 | 0.5587 | 0.8204 | 0.4384 | 0.7511 | 0.0012 | 0.3528 |
| 0.6296 | 0.3325 | 0.1687 | 0.9405 | 0.0527 | 0.8143 | 0.5749 | 0.6143 | 0.7231 | 0.1058 |
| 0.8140 | 0.2608 | 0.4332 | 0.7249 | 0.1205 | 0.2273 | 0.4338 | 0.6689 | 0.2440 | 0.2235 |
| 0.5476 | 0.4537 | 0.7506 | 0.2187 | 0.1813 | 0.3258 | 0.9257 | 0.7906 | 0.2558 | 0.4723 |
| 0.4180 | 0.0013 | 0.9242 | 0.7244 | 0.1053 | 0.7043 | 0.0949 | 0.8745 | 0.5645 | 0.2952 |
| 0.8083 | 0.9989 | 0.0695 | 0.1173 | 0.3166 | 0.3886 | 0.0749 | 0.6815 | 0.0874 | 0.8635 |
| 0.9904 | 0.5022 | 0.5728 | 0.0450 | 0.2586 | 0.3605 | 0.6822 | 0.0265 | 0.8867 | 0.5484 |
| 0.7195 | 0.0808 | 0.1584 | 0.3373 | 0.7864 | 0.0744 | 0.4967 | 0.1792 | 0.0437 | 0.1009 |
| 0.1732 | 0.3343 | 0.8448 | 0.1394 | 0.9283 | 0.5438 | 0.4384 | 0.0037 | 0.4305 | 0.5413 |
| 0.3265 | 0.9144 | 0.4153 | 0.6656 | 0.1295 | 0.4223 | 0.5918 | 0.9406 | 0.8413 | 0.6277 |
| 0.3580 | 0.5904 | 0.0662 | 0.3064 | 0.3679 | 0.8637 | 0.9890 | 0.4483 | 0.6006 | 0.2661 |
| 0.0477 | 0.0424 | 0.0121 | 0.9008 | 0.3131 | 0.4537 | 0.4071 | 0.8346 | 0.5386 | 0.8727 |
| 0.2088 | 0.4000 | 0.6062 | 0.3852 | 0.0959 | 0.0950 | 0.4509 | 0.1040 | 0.3094 | 0.6763 |
| 0.6114 | 0.8727 | 0.7242 | 0.9393 | 0.1485 | 0.9097 | 0.7284 | 0.7514 | 0.2770 | 0.1529 |
| 0.8659 | 0.2414 | 0.6732 | 0.0076 | 0.4980 | 0.6920 | 0.7104 | 0.1759 | 0.1204 | 0.5539 |
| 0.2557 | 0.2966 | 0.6672 | 0.9578 | 0.5661 | 0.1898 | 0.9168 | 0.7824 | 0.0752 | 0.9830 |
| 0.1351 | 0.4706 | 0.2068 | 0.3551 | 0.0177 | 0.1665 | 0.9394 | 0.9833 | 0.9763 | 0.2230 |
| 0.4829 | 0.3744 | 0.9982 | 0.9092 | 0.4893 | 0.1168 | 0.1106 | 0.6189 | 0.8997 | 0.3566 |
| 0.4666 | 0.2443 | 0.7653 | 0.3920 | 0.1479 | 0.5335 | 0.0413 | 0.2694 | 0.5688 | 0.2472 |
| 0.6089 | 0.5417 | 0.0182 | 0.1340 | 0.7577 | 0.6142 | 0.2366 | 0.3663 | 0.8622 | 0.3888 |
| 0.8072 | 0.2605 | 0.0931 | 0.7384 | 0.7293 | 0.9062 | 0.4892 | 0.0134 | 0.3671 | 0.7171 |
| 0.4286 | 0.7725 | 0.8138 | 0.9625 | 0.5383 | 0.6982 | 0.0806 | 0.1427 | 0.4653 | 0.8145 |
| 0.8931 | 0.0497 | 0.8884 | 0.6304 | 0.0420 | 0.7396 | 0.4611 | 0.9928 | 0.1239 | 0.8995 |
| 0.4249 | 0.7105 | 0.0019 | 0.0589 | 0.0130 | 0.7696 | 0.6162 | 0.7180 | 0.3285 | 0.4314 |
| 0.4237 | 0.0904 | 0.5313 | 0.4089 | 0.2362 | 0.2033 | 0.5506 | 0.8666 | 0.5759 | 0.0755 |
| 0.6304 | 0.8015 | 0.5671 | 0.1016 | 0.5740 | 0.0811 | 0.0781 | 0.8797 | 0.5055 | 0.3053 |
| 0.1888 | 0.5530 | 0.5764 | 0.2124 | 0.8642 | 0.2030 | 0.6761 | 0.1406 | 0.8049 | 0.0994 |
| 0.4634 | 0.0383 | 0.1422 | 0.4206 | 0.4381 | 0.2532 | 0.8253 | 0.8806 | 0.1215 | 0.1047 |
| 0.9932 | 0.5278 | 0.3975 | 0.3847 | 0.4966 | 0.6372 | 0.6746 | 0.2192 | 0.8252 | 0.9736 |
| 0.6628 | 0.9092 | 0.0157 | 0.8756 | 0.9383 | 0.7457 | 0.3813 | 0.1228 | 0.2471 | 0.8927 |
| 0.1102 | 0.8255 | 0.9510 | 0.2977 | 0.6457 | 0.7658 | 0.3076 | 0.1514 | 0.3237 | 0.6352 |
| 0.0053 | 0.7077 | 0.2089 | 0.6073 | 0.0728 | 0.1625 | 0.7315 | 0.7957 | 0.2888 | 0.6465 |
| 0.5547 | 0.5832 | 0.0291 | 0.4069 | 0.1196 | 0.0970 | 0.0215 | 0.4281 | 0.7713 | 0.8773 |
| 0.4820 | 0.1428 | 0.8989 | 0.1661 | 0.2447 | 0.9913 | 0.9619 | 0.5810 | 0.9692 | 0.5138 |
| 0.5210 | 0.4811 | 0.3056 | 0.5377 | 0.2567 | 0.3330 | 0.4519 | 0.9396 | 0.9714 | 0.5696 |
| 0.7861 | 0.2507 | 0.0814 | 0.5828 | 0.6159 | 0.5456 | 0.3376 | 0.5338 | 0.7920 | 0.7096 |
| 0.5343 | 0.7442 | 0.6044 | 0.6605 | 0.5385 | 0.0553 | 0.2288 | 0.7996 | 0.3503 | 0.6006 |
| 0.2665 | 0.2956 | 0.6960 | 0.7229 | 0.5461 | 0.8163 | 0.7714 | 0.7114 | 0.8513 | 0.9994 |
| 0.8051 | 0.7913 | 0.8389 | 0.9150 | 0.2888 | 0.9726 | 0.7342 | 0.6805 | 0.9997 | 0.7199 |
| 0.6273 | 0.2870 | 0.0862 | 0.5046 | 0.5136 | 0.6266 | 0.1765 | 0.2138 | 0.5481 | 0.5455 |

| | | | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.7951 | 0.1383 | 0.7070 | 0.5456 | 0.7010 | 0.7275 | 0.7975 | 0.7006 | 0.2247 | 0.6545 |
| 0.1478 | 0.8161 | 0.8935 | 0.1653 | 0.3526 | 0.2922 | 0.9008 | 0.0662 | 0.2379 | 0.9087 |
| 0.4698 | 0.9339 | 0.8887 | 0.5280 | 0.9974 | 0.6390 | 0.8540 | 0.4414 | 0.5393 | 0.2033 |
| 0.7398 | 0.6145 | 0.7993 | 0.6166 | 0.1189 | 0.2833 | 0.5110 | 0.7681 | 0.5442 | 0.7519 |
| 0.4920 | 0.6521 | 0.6786 | 0.4573 | 0.4576 | 0.6021 | 0.3811 | 0.3787 | 0.2622 | 0.3522 |
| 0.4073 | 0.3769 | 0.7682 | 0.9750 | 0.1613 | 0.4902 | 0.1233 | 0.8433 | 0.0032 | 0.8302 |
| 0.6858 | 0.9532 | 0.6469 | 0.0771 | 0.5368 | 0.3016 | 0.1422 | 0.6342 | 0.1235 | 0.2054 |
| 0.3898 | 0.0806 | 0.3403 | 0.1188 | 0.9493 | 0.3802 | 0.6173 | 0.0460 | 0.4044 | 0.7237 |
| 0.2869 | 0.0473 | 0.4908 | 0.6450 | 0.3290 | 0.0075 | 0.0798 | 0.0838 | 0.0295 | 0.5635 |
| 0.5710 | 0.8050 | 0.2142 | 0.3695 | 0.0317 | 0.7609 | 0.3345 | 0.2963 | 0.1166 | 0.1558 |
| 0.9020 | 0.8279 | 0.6240 | 0.2548 | 0.1617 | 0.7061 | 0.3520 | 0.8990 | 0.8402 | 0.9797 |
| 0.3798 | 0.5342 | 0.0572 | 0.6467 | 0.5098 | 0.9734 | 0.4092 | 0.8084 | 0.6232 | 0.2592 |
| 0.8064 | 0.0243 | 0.5641 | 0.2934 | 0.0078 | 0.5702 | 0.5198 | 0.3536 | 0.8343 | 0.7933 |

MATLAB reads this random matrix as the following image



2.4 Some operations on images

Since MATLAB reads each image as a matrix, any matrix operation has an analogue for images.

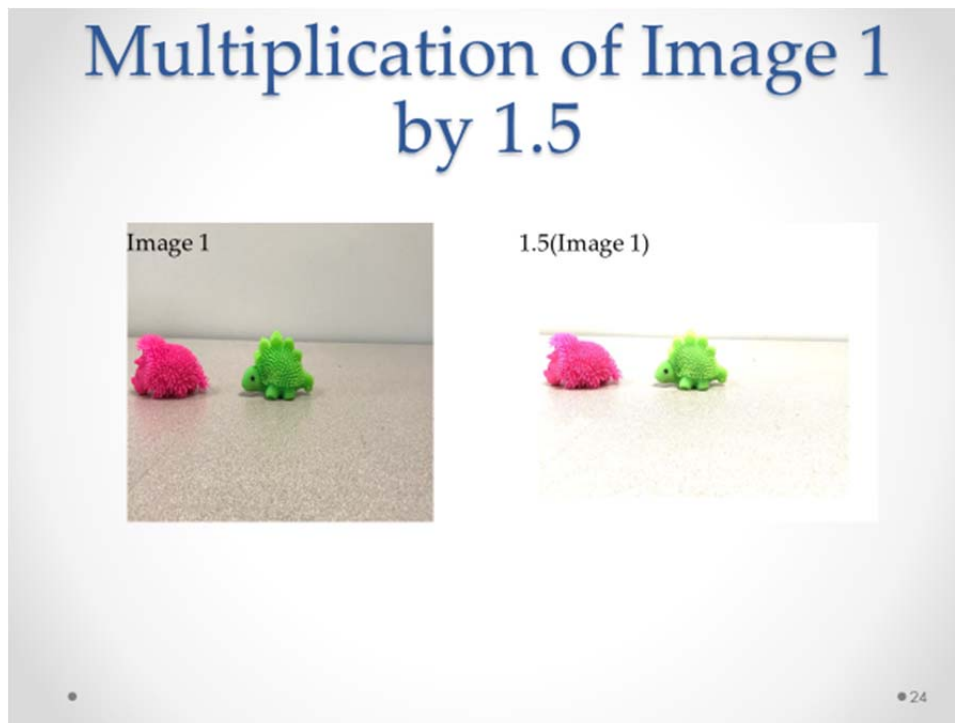
Consider the following two images that we label **Image 1** and **Image 2**.





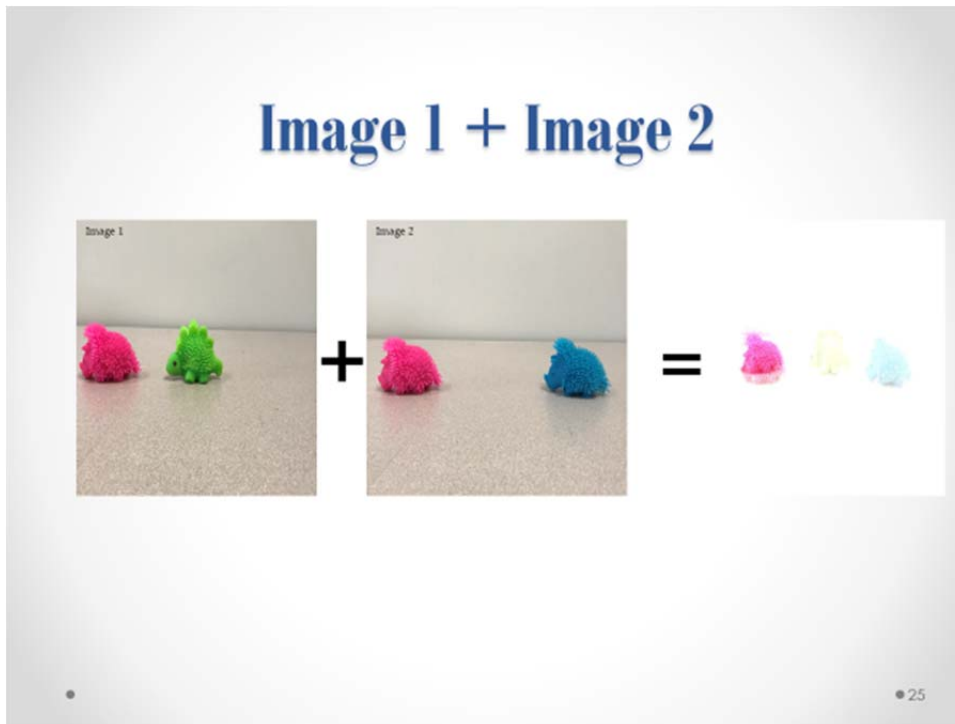
After extracting the matrices associated to **Image 1** and **Image 2** we can perform the following operations.

- Multiplication of an image by a constant
- Image addition
- Image subtraction
- Absolute value of an image

**MATLAB Code:**

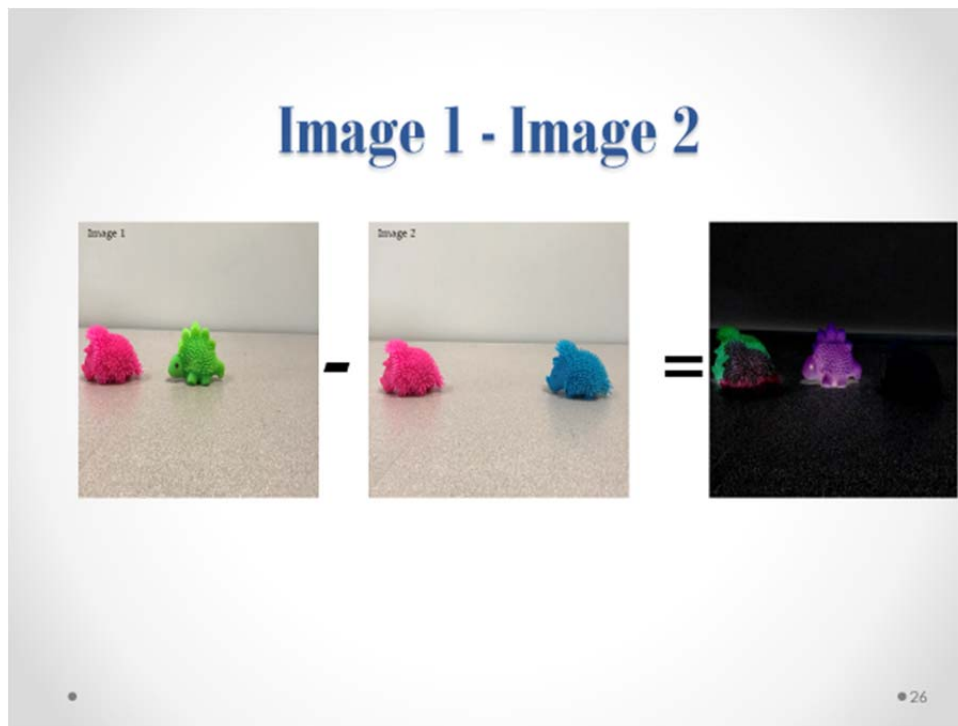
```
A= imread('image1.jpg');    %Read in 1st image  
Output = immultiply(A, 1.5) %Multiply image by 1.5
```

All the coefficients of the matrix associated to Image are multiplied by 1.5. The image obtained is lighter than the original image. This shows that multiplication by a constant is used to change the contrast of an image

**MATLAB Code:**

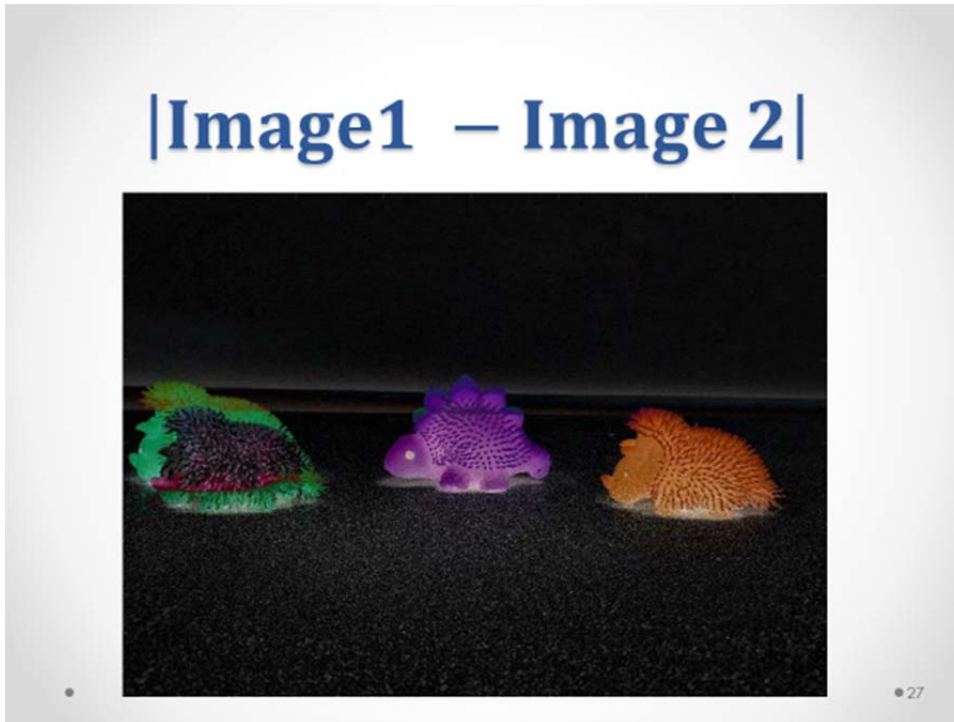
```
A= imread ('image1.jpg');    %Read in 1st image
B= imread ('image2.jpg');    %Read in 2nd image
imshow (A);                  %Display 1st image
imshow (B);                  %Display 2nd image
```

```
Output = imadd (A; B);      %Substract images
```

**MATLAB Code:**

```
A= imread ('image1.jpg');    %Read in 1st image
B= imread ('image2.jpg');    %Read in 2nd image
imshow (A);                  %Display 1st image
imshow (B);                  %Display 2nd image

Output = imsubtract (A; B);  %Subtract images
```


**MATLAB Code:**

```

A= imread ('image1.jpg');    %Read in 1st image
B= imread ('image2.jpg');    %Read in 2nd image
imshow (A);                  %Display 1st image
imshow (B);                  %Display 2nd image

Output = imabsdiff(A; B);    %Subtract images

```

3 Application of digital image processing

Over time, the ability for everyday people to use digital image processing has grown tremendously. For example, social media started as place where you could give public knowledge of your everyday situation. Today as a social media user, you have the freedom to crop, resize, and alter picture at your fingertips. One of the most common social media sites is Instagram. These media sites allow you to snap a photo and add or subtract anything in the photo that you like. More commonly, the most popular photos are filtered with some type of effects to the photo. These filters can vary in many ways, for example the fade effect, this filter allow your photo to

appear to be dimmer than the usual color of the image. There are also other features that allow you to add different image filters to your photo. The Instagram system operate similarly to our MATLAB code reading of jpeg images. The system reads in the image and map colors to their proper codes, then those codes will allow you to demonstrate special editing. Many filtering settings have their own filtering pattern.

Before programs such as Facebook, Instagram, snapchat, Photoshop was one of the image processing programs that would allow a person to create different edits to the images that would be working on. This process was created from computerized photography. This was one of the first popular site used to edit images and reconfigure photos of choice. Photoshop allowed the business of image filtering to grow as large as it is now. Instagram is an application of special effect filtering in its own way that benefited from photoshop. One of the more interesting features of different filters is the ability to include facial recognition. The programs that uses facial recognition works based on matrix observation. This is where a person image is read into the software, and the software generate a matrix for that face. When the matrix is created, the software is able to identify a consistency of a pattern in the face, which allows for the recognition of a person's face to be identified. The Eigenface technic is one of the many ways matrix theory is used to process image for face detection.

Amazon, one of the fastest growing companies uses facial recognition, selfie payment, which allows users to access their accounts instead of remembering a passcode. Facebook, another growing company uses facial recognition to grant access to accounts, making accessibility much easier. Video surveillance has become more beneficial for government purposes and private practice businesses. Real-time crime has become an issue the government has decided to stop. Surveillance system are creating program that can detect suspicious motion in real time to decrease crime activity. Image processing is used to monitor the surveillance over large areas, to maintain the safety of that space. Video Forensics considers the noise of a video surveillance.

In the early 1960s, NASA worked very close with image processing. The Mariner Mars orbit in 1971 was taken with a photo that contained a pixel count of 200 X 200. In order for those photos to be taken, the photos had to be converted by a team of scientists, who could make the image acceptable for the public to see and understand. The NASA team has created different programs that have improved over the years. Image processing allowed the space station to generate images that were monocolored and miles away to have a vivid description in photos that were taken.

Aerial photography had the ability to capture a large quantity of earth's surface and place into a detailed map description. Aerial photos are always very desirable, but are particularly useful for resource surveys. Many areas have no maps available to show the landscape of property or vegetation. Aerial photos show features such as field patterns and vegetation which are often omitted on maps. Overtime, Aerial photos can depict the changes to landscape in an area. Eventually, this would allow satellite based programs to improve their images and show image of better quality. The Hubble Telescope is another tool that was used to give space stations accurate depictions of things out in orbit. There are different ways that companies use image processing to benefit their companies. Some of the purposes are recreational, but others are to improve and generate new science. Image processing has changed over decades.

4 Conclusion

Image processing is still a growing field with endless applications. During the early stages of image processing, the software and the technology were not as accessible for common people to use (due to cost). But today, we see that almost all the aspects of our daily lives are being affected by the results of the many works done in the area of image processing. From monthly doctor visit, and weekly check-ups to the next post on Instagram or Snap Chat, image processing has to be used in different systems each day.

Image processing is also a tool for other research areas. From the exploration of the universe, the advancement in television, its increase in broadcast, video editing and storage efficiency has

caused a huge innovation in television and picture quality, to the human-made interfaces and cellular devices finger printing has shown some of the recent discoveries found.

This work has allowed us to understand one of the applications of mathematics, linear algebra in particular to engineering.

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